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A bilayer Double Semion model with symmetry-enriched topological order

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ABSTRACT

We construct a new model of two-dimensional quantum spin systems that combines intrinsic topological orders and a global symmetry called flavour symmetry. It is referred as the bilayer Doubled Semion model (bDS) and is an instance of symmetry-enriched topological order. A honeycomb bilayer lattice is introduced to combine a Double Semion Topological Order with a global spin–flavour symmetry to get the fractionalization of its quasiparticles. The bDS model exhibits non-trivial braiding self-statistics of excitations and its dual model constitutes a Symmetry-Protected Topological Order with novel edge states. This dual model gives rise to a bilayer Non-Trivial Paramagnet that is invariant under the flavour symmetry and the well-known spin flip symmetry.

1. Introduction

Topological effects have a long history in condensed matter physics and in strongly correlated systems in particular. The quantum Hall effects (integer and fractional) and the Haldane phase are two of the most outstanding examples [1–6]. Yet, topological effects have acquired a more prominent role in the last years when the notions of topological orders have emerged and pervaded many previous seemingly unrelated areas of physics.

Topological orders (TOs) are quantum phases of matter that exhibit ground state degeneracy when the system is on a lattice with non-trivial topology, like a torus. Their excitations are anyons [7–10] that may exist both in the bulk and at the boundaries of the system. These properties are robust against

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arbitrary local perturbations at zero temperature. Other features of TOs are the existence of a gap in the spectrum and long-range entanglement. A key ingredient in the construction of these TOs is a gauge group of symmetry $\mathcal{G}$. The elements of this group act locally on a set of k spins leaving the Hamiltonian invariant. Thus, in concrete models, the Hamiltonian is composed of k-local operators.

In the last decade, another class of topological orders has appeared with the discovery of topological insulators and superconductors [11–27]. These are topological phases where the fermion character of the degrees of freedom plays a fundamental role due to the conservation of fermion parity. More recently, bosonic versions of these phases have been discovered and they are now understood as symmetry-protected topological orders (SPTOs) [28–31]. These are quantum phases of matter with a gap to excitations in the bulk that have trivial anyonic statistics. Whereas at the boundary, they may exhibit edge/surface states with unusual properties like being gapless and may have anyonic statistics. These properties are robust under local perturbations that respect a global symmetry group $\mathbb{G}$, but not for arbitrary ones as in the TOs. The global group $\mathbb{G}$ may act as an internal group of symmetries on the spins of the system, i.e. like an on-site symmetry, or may act as an spatial symmetry group of the lattice [32]. SPTOs are short-range entanglement phases since they do not have TO in the bulk.

Given these two relevant classes of topological phases it is a natural question whether it is possible to combine them into a single quantum phase of matter. The idea of unifying two classes of symmetries is not purely ascetic. We know that when a unification happens to be possible, it comes with benefits in a better understanding of physical phenomena underneath and with rewards in the form of novel effects not present in the initial theories separately.

In the case of a topological order, the additional global symmetry produces the fractionalization of the topological charge. The paradigmatic example is the fractional quantum Hall effect (FQHE) [33–35]. Namely, the Laughlin state with filling factor $\nu = 1/m$ is an example of TO with global symmetry $\mathbb{G} = U(1)$ representing charge conservation. The quasiparticle excitations are Abelian anyons and moreover, they have fractional charge with respect to the electron charge: $q = e/m$, m being the filling factor. Another modern example of TO is the Doubled Semion (DS) model with spin degrees of freedom that has a gauge group $\mathcal{G} = \mathbb{Z}_2$. In general, the fractionalization class of an anyon describes its characteristic type of topological charge fractionalization.

Thus, the combination of intrinsic topological orders TOs with global symmetries produces new topological phases of matter that are now called symmetry-enriched topological orders (SETOs). They have been recently studied in several remarkable works. Their aim is to classify novel SET phases [30,36], or finding new mechanisms to produce them [29,37], or constructing explicit realizations of these phases [32,38] etc.

A recent fundamental discovery by Levin and Gu [29] introduces a new example of SPTO with global spin flip symmetry. The non-trivial braiding statistics of the excitations implies the existence of protected edge modes while bosonic or fermionic statistics yield no edge modes. There is a duality transformation between spin models and string models. Namely, the Kitaev and the DS models are referred to as string models since their anyon excitations are carried by string operators. The excitations are attached at the endpoints of these strings and consequently they appear in pairs.

The string-flux mechanism proposed by Hermele [37] is another explicit instance of how fractionalization of the topological charge occurs when a global symmetry is preserved: an anyon of a topological ordered phase with global symmetry may carry fractionalized quantum numbers with respect to the original TO without that global symmetry. This phenomenon occurs when the string attached to the anyon sweeps over a background pattern of fluxes in the ground state of the TO model.

Although these two different methods presented above act on different kind of systems, i.e. Levin and Gu introduced a new type of paramagnet that is a SPTO and Hermele worked out a general mechanism to obtain charge fractionalization, they bear some similarities. The Non-Trivial paramagnet derived in the Levin–Gu method has plaquette operators with opposite sign to the operators for the Trivial case. It can be interpreted as a pattern of fluxes as demanded by the string-flux mechanism of Hermele.

At this point, a fundamental question arises: how to make compatible the duality method of Levin–Gu and the string-flux mechanism of Hermele. Both of them provide us with very useful explicit techniques to fractionalize the topological charge in spin models with TO. However, to do so we face two main obstacles that look unsurmountable at first sight: