Annals of Physics 375 (2016) 227–232

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Relativistic kinetic theory and non-gaussian statistical

Z.B.B. de Oliveira [a], R. Silva [a,b,*]

[a] Universidade Federal do Rio Grande do Norte, Departamento de Física, Natal-RN, 59072-970, Brazil

[b] Universidade do Estado do Rio Grande do Norte, Departamento de Física, Mossoró, 59610-210, Brazil

ARTICLE INFO

Article history:
Received 19 July 2016
Accepted 2 October 2016

Keywords:
Entropy
H-theorem
Non-gaussian statistics

ABSTRACT

The nonextensive statistical mechanics is extended in the special relativity context through a generalization of H-theorem. We show that the Tsallis framework is compatible with the second law of the thermodynamics when the nonadditive effects are consistently introduced on the collisional term of the Boltzmann equation. The proof of the H-theorem follows from using of q-algebra in the generalization of the molecular chaos hypothesis (Stosszahlansatz). A thermodynamic consistency is possible whether the entropic parameter belongs to interval $q \in [0, 2]$.

© 2016 Elsevier Inc. All rights reserved.

Boltzmann–Gibbs entropy has been extended in order to capture all properties of anomaly systems [1]. The Tsallis entropy has been widely investigated in both theoretical and applications context [2,3]. From theoretical standpoint, the Tsallis framework has shown consistency with some foundations, e.g. issues on the kinetic theory, ensemble theory and mathematics theorems were investigated.

An alternative calculation of the Maxwell distribution, which is based on the introduction of statistical correlations among velocity distributions, has been developed in the domain of classical kinetic theory [4] (see also [5]). The second law of thermodynamics in the context of Kinetic Theory

* Corresponding author at: Universidade Federal do Rio Grande do Norte, Departamento de Física, Natal-RN, 59072-970, Brazil.

E-mail addresses: zairabruna@gmail.com (Z.B.B. de Oliveira), raimundosilva@fisica.ufrn.br (R. Silva).

http://dx.doi.org/10.1016/j.aop.2016.10.009
0003-4916/© 2016 Elsevier Inc. All rights reserved.

has also been investigated in the classical [6], the relativistic [7], and also in the quantum-mechanical regimes [8]. All regimes have presented the following constraint on the nonextensive parameter $q \in [0, 2]$. The convexity property of the generalized relative entropy in the quantum regime [9], also provided same constraint on q. More recently, the third law of thermodynamics has shown viability with Tsallis entropy and a more restrictive bound upon the nonextensive parameter was calculated, $q \in [1, 2]$ [10]. In the ensemble theory, the problem of the optimization of the q-entropy with constraints was investigated in [2,11,12] (see also [13,14] that analysed other properties of Tsallis entropy). Some important limit theorems also were analysed through Tsallis framework, e.g. De Moivre (1733), Laplace (1744), Gauss (1809), Levy (1937) and Khinchin (1949) [15].

Some applications in high energy physics have been successfully considered using the distribution function which emerges of Tsallis statistics. For instance, it has been used in the description of transverse momentum spectra in high-energy pp and $p\bar{p}$ collisions at the LHC [16,17] and particle productions in pp and AA collisions at RHIC and LHC [18]. Indeed, the Tsallis statistics has been widely applied in the experimental measurements at RHIC [19] and LHC [20]. Issues related to cosmology and astrophysics have also been studied using these generalized approaches, e.g., entropic cosmology for a generalized black hole entropy and evolution of the universe [21,22], the entropy and the formation of the black hole [23–25], the direct dark matter detection rates [26], relativistic plasmas [27] and stellar rotational velocities [28].

Focusing on the framework of special relativity, we address a manifestly covariant approach for the Tsallis statistics which has as main goal a proof H-theorem. It has been already discussed in [7], but such proof followed a generalization of the molecular chaos hypothesis ("Stosszahlansatz") which is based on the combination of generalized functions (for nonrelativistic regime, see Ref. [6]). In this letter, by following a different analysis, we introduce statistical correlations on the molecular chaos hypothesis using the so-called extended algebra [29], and to complete the proof, we use a generalization of the 4-entropy flux which is consistent with the Tsallis statistics. As result, we show that the molecular collisions in the relativistic gas present a power law distribution for the states of local equilibrium. Furthermore, the kinetic version from the second law of thermodynamics limits the entropic parameter to interval $q \in [0, 2]$.

First of all, let us mention that in the proof of the standard relativistic H-theorem, the molecular chaos hypothesis ("Stosszahlansatz") and 4-entropy flux are the core of H-theorem. These are ingredients that pick up whole statistical properties from the proof of the H-theorem. Specifically, the "Stosszahlansatz" is associated with the assumption that any two colliding particles are uncorrelated, i.e. the two point correlation function of the colliding particles are factorized as $f(x, p, p_1) = f(x, p)f(x, p_1)$, where p and p_1, are the 4-momenta just before collision. The particles have 4-momentum $p \equiv p^\mu = (E/c, \mathbf{p})$ in each point $x \equiv x^\mu = (ct, \mathbf{r})$ of the space–time, with their energy satisfying $E/c = \sqrt{p^2 + m^2c^2}$. Forward, we shall show that the relativistic nonadditive entropy is naturally plausible with a generalization from molecular chaos when exact statistical correlations are introduced. Mathematically speaking, it means replace the usual product among the distribution functions in the chaos molecular hypothesis by the so-called q-product, i.e. the two point correlation function of the colliding particles should be given by $f(x, p, p_1) = f(x, p) \otimes_q f(x, p_1) \equiv [f^{1-q}(x, p) + f^{1-q}(x, p_1) - 1]^{1/1-q}$. Note that, the statistical correlations among the distributions functions are disregarded when the extensive limit is recovered, i.e. $q \to 1$. It should be emphasized that the validity of the chaos molecular hypothesis is yet a controversial issue [30].

In order to prove the H-theorem we assume a relativistic rarified gas containing N particles of mass m enclosed in a volume V. This gas is under the action of an external 4-force field F^μ, and its states must be characterized by a Lorentz invariant one-particle distribution function $f(x, p)$. As is widely known, $f(x, p)d^3xd^3p$ provides, at each time t, the number of particles in the volume element d^3xd^3p around the particles space–time position x and momentum p. As already discussed in Ref. [7], there exists one only way of introducing nonextensive effects on the temporal evolution of the relativistic distribution function $f(x, p)$, i.e.

$$p^\mu \partial_\mu f + mF^\mu \frac{\partial f}{\partial p^\mu} = C_{q^*}(f, f'). \tag{1}$$

Download English Version:

https://daneshyari.com/en/article/5496026

Download Persian Version:

https://daneshyari.com/article/5496026

Daneshyari.com