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# Microscopic model of quantum butterfly effect: Out-of-time-order correlators and traveling combustion waves

Igor L. Aleiner[a], Lara Faoro[b], Lev B. Ioffe[b,c,d,*]

[a] Columbia Physics Department, 530 W 120th Street, New York, NY, 10027, USA

[b] Laboratoire de Physique Theorique et Hautes Energies, CNRS UMR 7589, Universites Paris 6 et 7, place Jussieu 75252 Paris, Cedex 05, France

[c] National Research University Higher School of Economics, Russia

[d] Department of Physics and Astronomy, Rutgers University, 136 Frelinghuysen Rd, Piscataway 08854, NJ, USA

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ABSTRACT

We extend the Keldysh technique to enable the computation of out-of-time order correlators such as $\left\langle O(t)\tilde{O}(0)O(t)\tilde{O}(0)\right\rangle$. We show that the behavior of these correlators is described by equations that display initially an exponential instability which is followed by a linear propagation of the decoherence between two initially identically copies of the quantum many body systems with interactions. At large times the decoherence propagation (quantum butterfly effect) is described by a diffusion equation with non-linear dissipation known in the theory of combustion waves. The solution of this equation is a propagating non-linear wave moving with constant velocity despite the diffusive character of the underlying dynamics.

Our general conclusions are illustrated by the detailed computations for the specific models describing the electrons interacting with bosonic degrees of freedom (phonons, two-level-systems etc.) or with each other.



* Corresponding author at: Laboratoire de Physique Theorique et Hautes Energies, CNRS UMR 7589, Universites Paris 6 et 7, place Jussieu 75252 Paris, Cedex 05, France.

*E-mail address:* lev.b.ioffe.phys1@gmail.com (L.B. Ioffe).

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## 1. Motivation

In a chaotic classical system a small perturbation leads to the exponential divergence of trajectories characterized by Lyapunov time, $1/\Lambda$. As a result, the observables in two copies of the system experiencing different perturbations quickly become uncorrelated. In a many body system a *local* perturbation initially destroys the correlations locally, then the region where the correlations are destroyed quickly grows with time. Killing a butterfly in Ray Bradbury story [1] leads to the spreading perturbation until it reaches the size of the system (Earth in this story). This phenomena is known as butterfly effect.

The concept of butterfly effect can be generalized to a closed chaotic quantum system even though such generic system does not necessarily have a direct analogue of Lyapunov divergence of trajectories because quantum mechanics prohibits the infinitesimal shift of the trajectory. The convenient measure of the butterfly effect is provided by the out-of-time-order correlator (OTOC) that was first introduced by Larkin and Ovchinnikov [2], revived by Kitaev [3,4] and extensively discussed by a number of works recently [5–8]. OTOC is defined by

$$\mathcal{A}(t) = \left\langle O(t)\tilde{O}(0)O(t)\tilde{O}(0)\right\rangle, \tag{1}$$

where $O(t)$ and $\tilde{O}(t)$ are two local operators in Heisenberg picture. Physically, it describes how much the perturbation introduced by $\tilde{O}(0)$ changes the value of the $O(t)$. At large times $\mathcal{A}(t)$ goes to a zero, because the state created by the consecutive action of the operators $O(t)\tilde{O}(0)$ is incoherent with the state obtained when these operators act in a different order.[1] The anomalous time order in the correlator (1) implies the evolution backward in time, so it is not measurable by direct physical experiments on one copy of the system in the absence of a time machine such as implemented in NMR experiments [9]. One can view the decrease of the OTOC with time as the consequence of the dephasing between two initially almost identical Worlds evolving with the same Hamiltonian. In this respect it is different from the problems of fidelity [10] and Loschmidt echo ([9] and references therein) that study evolution forward and backwards with slightly different Hamiltonians. It is also different from a problem of the evolution of a particle along quasiclassically close trajectories appearing in studies of the proximity effects [2] or weak-localization [11] and quantum noise [12].

For physical systems the Hamiltonian is local, so that distant parts of a system are not interacting directly with each other. In this case, one may further distinguish the case when operators $O$ and $\tilde{O}$ act far from each other in real space. One expects that the correlator decreases after the significant delay needed for the perturbation to spread over the distance separating these operators. When correlators of this type decayed for *any* separation between the operators in the real space the coherence is completely lost. The decay of OTOC at long times for all subsystems (i.e. for all separations) for all operators $O$ and $\tilde{O}$ implies complete quantum information scrambling [13]. Note that the separation of the operators in space is equivalent to the separation into subsystems introduced in quantum information works. We are not going to discuss here quantum information implications of OTOC and the exact definition of quantum scrambling; we refer the reader to the literature that discussed its theory [14–18] and the possibility of its experimental measurement [19,20].

The goal of this work is to develop the analytic tools to study OTOC (1) for microscopic models that allow for the solutions for conventional correlators. The technique that we develop is essentially a straightforward extension of the Keldysh technique. We apply our technique to three models that are basic in condensed matter physics: (i) electrons interacting with localized bosonic degrees of freedom (Einstein phonons or simplified two level systems), (ii) electrons in the disorder potential and (iii) electrons weakly interacting with each other. We find that in models (i) and (iii) the mathematical description of the OTOC is similar to the description of the combustion waves. The small initial perturbation first grows exponentially remaining local and then starts to propagate with a constant

[1] Here we assume that operators $O$ have zero averages in all states. If not, the irreducible correlators have to be discussed. We also assume that operator $\left\langle O^2(t)\right\rangle \neq 0$.