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Testing quantum gravity with cosmology / Tester les théories de la gravitation quantique à l'aide de la cosmologie

Asymptotically safe cosmology – A status report ☆

La cosmologie asymptotiquement sûre : un rapport d'étape

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ABSTRACT

Asymptotic safety, based on a non-Gaussian fixed point of the gravitational renormalization group flow, provides an elegant mechanism for completing the gravitational force at sub-Planckian scales. At high energies, the fixed point controls the scaling of couplings such that unphysical divergences are absent, while the emergence of classical low-energy physics is linked to a crossover between two renormalization group fixed points. These features make asymptotic safety an attractive framework for the building of a cosmological model. The resulting scenarios may naturally give rise to a quantum-gravity-driven inflationary phase in the very early universe and an almost scale-free fluctuation spectrum. Moreover, effective descriptions arising from an improvement of the renormalization group permit a direct comparison to cosmological observations as, e.g., *Planck* data.

RÉSUMÉ

La sécurité asymptotique, basée sur un point fixe non gaussien du flux du groupe de renormalisation gravitationnel, fournit un mécanisme élégant pour décrire la force gravitationnelle aux échelles sub-planckiennes. Aux hautes énergies, le point fixe contrôle le dimensionnement des couplages de manière à ce que les divergences non physiques soient absentes, tandis que l'émergence d'une physique classique des basses énergies est liée au croisement de deux points fixes de groupes de renormalisation. Ces éléments font de la sécurité asymptotique un cadre attractif pour la construction d'un modèle cosmologique. Les scenarios résultants peuvent naturellement donner lieu à une phase inflationnaire contrôlée par la gravité quantique dans l'univers primordial et à un spectre de fluctuations presque sans échelle. De plus, les descriptions effectives issues d'une amélioration du groupe de renormalisation permettent une comparaison directe avec des observations cosmologiques telles que, par exemple, les données de *Planck*.

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1. Asymptotic safety: a brief introduction

It is well known that the quantization of general relativity based on the Einstein–Hilbert action results in a quantum field theory that is perturbatively non-renormalizable. This conclusion also holds if (non-supersymmetric) matter fields are added. The phenomenological success of general relativity then motivates to treat gravity as an effective field theory. This approach leads to a renormalizable theory of gravity in the sense that any quantum field theory becomes renormalizable if all possible counterterms compatible with its symmetries are included in the action [1]. While providing a consistent quantum theory for gravity, this construction falls short in terms of predictive power: while the effective field theory formulation works well at energy scales below the Planck scale where higher-derivative terms are suppressed by powers of the Planck mass, describing gravity at trans-Planckian scales requires fixing an infinite number of free coupling constants from experimental input.

In principle, asymptotic safety lives in the same space of theories as the corresponding effective field theory. It resolves the problem of “predictivity” encountered in the effective field theory framework by imposing the extra condition that the quantum theory describing our world is located within the UV critical hypersurface of a suitable renormalization group (RG) fixed point. This condition implies that the high-energy behavior of the theory is controlled by the fixed point that renders all dimensionless coupling constants finite at high energy. Fixing the trajectory uniquely then requires a number of experimental input parameters equal to the dimensionality of the hypersurface.

On this basis, the crucial elements for asymptotic safety providing a valid theory for quantum gravity can be summarized as follows. Firstly, the existence of a suitable RG fixed point has to be shown. Secondly, the predictive power of the construction must be determined. Finally, it has to be shown that the UV critical hypersurface develops a regime where classical gravity constitutes a good approximation. Starting from the pioneering work [2], these points have been investigated in a vast variety of highly sophisticated computations, putting the scenario on firm grounds [3–8]. In particular, the dimension of the UV critical hypersurface could be as low as three.

The prospect that asymptotic safety could be capable of describing gravitational force at all length scales makes the theory quite attractive for cosmological model building [9–35,35–38]. On the one hand, some or all of the free parameters appearing in the construction of asymptotic safety (including the value of the cosmological constant and Newton’s constant complemented by a low number of higher-derivative couplings) may be determined from cosmological data. On the other hand, asymptotic safety provides a framework for developing effective cosmological models and addressing questions related to a possible resolution of cosmological singularities. Typically, such investigations incorporate the effect of scale-dependent couplings through RG improvement techniques implemented either at the level of the equations of motion or the effective (average) action. While the resulting models are not based on the same level of rigor as the RG computations forming the core of the asymptotic safety program, they allow for the construction of interesting cosmological scenarios, e.g., in the framework of $f(R)$-type gravitational actions or dilaton-gravity theories.

The rest of the work is then organized as follows. We briefly review the computation of gravitational RG flows and the central results in Sect. 2, emphasizing the occurrence of a classical phase where general relativity is a good approximation. Cosmological models arising from RG improved equations of motion are discussed in Sect. 3, while Sect. 4 summarizes results obtained from (improved) effective actions. We close with a brief summary and outlook in Sect. 5.

2. Asymptotic safety: fixed points and classical regime

Testing asymptotic safety at the conceptual level requires the ability to construct approximations of the gravitational RG flow beyond the realm of perturbation theory. A very powerful framework for carrying out such computations is the functional renormalization group equation (FRGE) for the gravitational effective average action Γ_k [2]

$$\partial_k \Gamma_k[g, \bar{g}] = \frac{1}{2} \operatorname{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} \partial_k \mathcal{R}_k\right] \tag{1}$$

The construction of the FRGE uses the background field formalism, splitting the metric $g_{\mu\nu}$ into a fixed background $\bar{g}_{\mu\nu}$ and fluctuations $h_{\mu\nu}$. The Hessian $\Gamma_k^{(2)}$ is the second functional derivative of Γ_k with respect to the fluctuation field at a fixed background and $\mathcal{R}_k$ provides a scale-dependent mass term for fluctuations with momenta $p^2 \ll k^2$ with the RG scale k constructed from the background metric. The interplay of $\mathcal{R}_k$ in the numerator and denominator renders the trace both infrared and ultraviolet finite and ensures that the flow of Γ_k is actually governed by fluctuations with momentum $p^2 \approx k^2$. In this sense, the flow equation realizes Wilson’s idea of renormalization by integrating “short-scale fluctuations” with momenta $p^2 \ll k^2$ such that Γ_k provides an effective description of physics for typical scales k^2. A priori, one may then expect that the resulting RG flow may actually depend strongly on the choice of the background. As it was explicitly demonstrated in [39], this is not the case, however: if the flow is computed via early-time heat-kernel methods, the background merely serves as a book-keeping device for disentangling the flow of different coupling constants.

The arguably simplest approximation of the gravitational RG flow is obtained from projecting the FRGE onto the Einstein–Hilbert action approximating Γ_k by

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