ARTICLE IN PRESS

COMREN:3352

C. R. Physique ●●● (●●●●) ●●●–●●●

Contents lists available at ScienceDirect

Comptes Rendus Physique

www.sciencedirect.com

Prix de la femme scientifique de l'année 2015 (ministère chargé de l'Enseignement supérieur et de la Recherche)

Geometric aspects of ordering phenomena

Aspects géométriques des phénomènes d'ordre

Leticia F. Cugliandolo

Sorbonne Universités, Université Pierre-et-Marie-Curie, Laboratoire de physique théorique et hautes énergies, tour 13, 5e étage, 4, place Jussieu, 75005 Paris, France

A R T I C L E   I N F O

Article history:
Available online xxxx

Keywords:
Coarsening
Percolation
Dynamic scaling

A B S T R A C T

A macroscopic system prepared in a disordered phase and quenched across a second-order phase transition into an ordered phase undergoes a coarsening process whereby it orders locally in one of the equilibrium states. The study of the evolution of the morphology of the ordered structures in two dimensions has recently unveiled two interesting and generic features. On the one hand, the dynamics first approach a critical percolating state via the growth of a new lengthscale and satisfying scaling properties with respect to it. The time needed to reach the critical percolating state diverges with the system size, though more weakly than the equilibration time. On the other hand, once the critical percolating structures established, the geometrical and statistical properties at larger scales than the one established by the usual dynamic growing length remain the ones of critical percolation. These observations are common to different microscopic dynamics (single spin flip, local and non-local spin exchange, voter) in pure or weakly disordered systems. We discuss these results and we refer to the relevant publications for details.

R É S U M É

Si un système macroscopique préparé dans une phase désordonnée est refroidi brusquement à une température inférieure à celle où, à l'équilibre, il y a une transition du second ordre, il subit alors un processus de coarsening au cours duquel il prend localement l'une des structures ordonnées stables à l'équilibre. L'étude de l'évolution de la morphologie des structures ordonnées en deux dimensions a récemment révélé deux propriétés génériques intéressantes. D'une part, la dynamique approche d'abord un état critique de percolation grâce à la croissance d'une nouvelle échelle de longueur, et vérifie des relations d'échelle vis-à-vis de celle-ci. Le temps nécessaire pour rejoindre l'état critique de percolation diverge avec la taille du système, moins faiblement que le temps nécessaire pour atteindre l'équilibre. D'autre part, après avoir atteint l'état critique de percolation, les propriétés géométriques et statistiques aux échelles plus longues que la longueur dynamique de croissance habituelle demeurent celles de la percolation critique. Ces observations sont communes aux différents types microscopiques de dynamique (retournement de spin simple, échange de spin local ou non, électeur) dans les systèmes purs ou faiblement

E-mail address: leticia@lpthe.jussieu.fr.

http://dx.doi.org/10.1016/j.crhy.2016.10.002

désordonnés. On discute ces résultats et on renvoie aux publications originales pour davantage de détails.

1. Introduction

Understanding the out-of-equilibrium evolution of collective phenomena in complex systems is a hard task. In many interesting cases of current interest, such as glassy materials, there is no clear comprehension of the mechanisms whereby these systems progress and, concomitantly, which are the microscopic rearrangements that lead to the slow but steady approach to equilibrium, when this asymptotic state is possible. There are, however, some lucky out of equilibrium relaxing systems in which the time-dependent microscopic configurations can be followed with numerical simulations or experimental techniques and simple mechanisms can thus be identified. Consequently, a better understanding of their dynamics can be reached. Among these are the systems that coarsen [1–6].

Coarsening or phase ordering kinetics is the process whereby an open system orders locally in its equilibrium states until the maximal order compatible with thermal fluctuations, conservation laws and boundary conditions is achieved. This phenomenon occurs when a macroscopic system is taken across a continuous phase transition by changing (abruptly, or slowly though not quasi-statically) one of its parameters or the environmental conditions. In this note, we focus on classical systems though, in principle, similar questions could be asked in quantum problems. Typical examples of coarsening systems are magnets taken from their paramagnetic to their ordered phase or, say, water and oil mixtures taken into demixing conditions.

The characterisation of coarsening has been largely circumscribed to the one of the space–time correlation [1–4,6] and linear response [4,7,8] functions, while the direct analysis of the morphology of the spatial structures has remained, in comparison, much less developed. However, this is a problem of fundamental but also practical interest since, from a materials science point of view, not only the average grain size but also its stochastic counterpart, the size distribution function, influences many material properties.

The morphology of critical *equilibrium* systems has been well characterised by a plethora of studies of the statistical and fractal properties of different kinds of geometrical structures [9–13]. In particular, the distributions of domain sizes, Fortuin–Kasteleyn cluster areas, interfaces lengths, winding angles, etc. are known. In contrast, a similar characterisation of systems evolving *out of equilibrium* after quenches across and to a critical point has not been performed yet, with the exception of two prominent cases. One is the distribution of droplets in Ostwald ripening, the process whereby the droplets of a minority phase diluted in a majority one organise, and has been the focus of much attention since the publication of the celebrated Lifshitz–Slyozov–Wagner theory [14,15]. The other one is the characterisation of small domains in polycrystalline formation [16], magnetic grains [17] soap froths [18,19] and biological tissues [20,21] usually done using kinetic Potts models.

In recent years, an impressive theoretical, numerical and experimental effort has been devoted to the measurement of the *density of defects* left in a system after its slow quench through a second-order phase transition [22] (the defects could be domain walls, vortices, or others depending on the system). However, little is known about the *size distribution, geometric properties and spatial organisation of the defects* inherited from such slow quenches.

A series of works aim at start filling this hole. In this article, I will shortly summarise recent advances in this line of research. I will focus on two-dimensional spin models, where most of the studies have been performed. I will describe results for the ferromagnetic 2d Ising and Potts spin models with microscopic dynamics satisfying detailed balance, and the planar voter model that goes beyond the physically constrained dynamic rules. The effects of quenched disorder will be shortly mentioned. A few words on 3d systems, including spin models and continuous field theories, will also be written. A final discussion section closes the paper with ideas for future research.

1.1. Models

The classical *Ising model* is defined by the Hamiltonian

$$\mathcal{H} = -J \sum_{\langle ij \rangle} s_i s_j \tag{1}$$

with $J > 0$. The bimodal spin variables take values $s_i = \pm 1$. The energy function of the classical *Potts model* is given by

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \delta_{s_i s_j} \tag{2}$$

with $s_i = 0, \ldots, q-1$. In both cases, the sum runs over nearest-neighbours on a finite-dimensional lattice. Both models undergo a second-order phase transition at a finite temperature as long as $2 \leq q \leq 4$ and $d \geq 2$. The Ising model is the