Physics Letters A 381 (2017) 3785–3797

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Multiple tolerances dilute the second order cooperative dilemma

Xiukai Sui [a], Bin Wu [b], Long Wang [a,*]

[a] Center for Systems and Control, College of Engineering, Peking University, Beijing 100871, China
[b] School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China

ARTICLE INFO

Article history:
Received 3 July 2017
Received in revised form 13 September 2017
Accepted 26 September 2017
Available online 9 October 2017
Communicated by C.R. Doering

Keywords:
Tolerant punishment
Second-order cooperative dilemma
Stochastic evolutionary dynamics

ABSTRACT

A peer punisher directly imposes fines upon defectors at a cost to himself. It is one of the mechanisms promoting cooperation, which is ubiquitous in nature. Typically, it is assumed that a peer punisher punishes provided that there is one defector in the group. The threshold that triggers punishment, however, is not necessarily one. The larger the threshold is, the more tolerant the peer punisher is. We study the evolutionary dynamics of those diverse tolerant peer punishment strategies in public goods game. We find that, i) less tolerant punishers prevail over tolerant ones; ii) large group size could enhance punishment, in contrast with the case in the first-order cooperative dilemma. Our analytical results are based on weak selection limit and large population size, which are verified by simulations. Our work sheds light on how punishment of diverse tolerance evolves.

1. Introduction

Public goods game (i.e. PGG) is a collective dilemma present in the social activities [1–4]. Although mutual cooperation achieves the maximal income for the whole community, the egoism drives individuals to free ride. Cooperation is exploited by defection resulting in "The Tragedy of the Commons" [5]. This problem can be solved by many mechanisms including direct and indirect reciprocity [6–13]. The last decade has seen an intensive study on how punishment, implying that one pays a cost to itself to incur a cost for the opponent, promotes cooperation [14–19]. In addition, human behavior experiments have shown that punishment is likely if it is optional [20,21].

Punishers bear extra costs, although punishment suppresses defection. In this case, punishers are worse off than those who contribute but not punish. Therefore costly punishment is an altruistic act. Based on evolutionary theory, the second-order free riders who contribute but not punish are beneficial over punishers. It yields the so-called "the second-order cooperative dilemma" [14,22]. In this case, we are tackling the dilemma between punishers and non-punishers. The evolution of peer punishment has been intensively addressed [14–17,19]. Here peer punishment means that individuals privately sanction a defector in the group [14–16]. Many mechanisms are found to sustain peer punishment, such as voluntary participation [4,14,15,23], spatial interactions [1,2,17,24], probabilistic sharing [25] and random explorations [15]. However, it has been typically neglected that people are not always sensitive to defection and sometimes show tolerance towards it. In fact, this idea of tolerance has been present in direct reciprocity. "tit for tat" (i.e. TFT) can be outperformed by "generous tit for tat" (i.e. GTFT) [26]. Here GTFT cooperates after a co-player cooperates but also cooperates with a certain probability after the opponent defects [26,27]. Thus a GTFT can be seen as a tolerant TFT. In the same way, peer punishers can also be insensitive to defection in the group. It may be attributed to two reasons: one is that a punisher does not recognize free riders. The other is that a punisher tolerates free riders when they are few even if the punisher has recognized them.

Inspired by these, we try to investigate how the tolerance alters the fate of punishment. For the sake of simplicity, we assume that individuals are with perfect recognition of all the others' strategies, and do not take into account the cost of this recognition [4,28]. In particular, we propose that each punisher's tolerance is captured by the number of defectors that triggers punishment. Thus a tolerant punisher cooperates if the number of defectors is below its tolerance value and punishes otherwise. In particular, our peer punishers degenerate to the classic cooperator, who cooperates without punishing, provided the tolerance is the group size; they degenerate to the classic peer punishers provided the tolerance is one. Thus the introduction of the tolerance unifies and generalizes previous strategies.

The more tolerant peer punishers pay less cost than the less tolerant ones in a group with no less than one defector. Thus, it seems that the more tolerant peer punishment strategy prevails, but is it true? In general, we tackle the condition under which peer

* Corresponding author.
E-mail address: longwang@pku.edu.cn (L. Wang).

https://doi.org/10.1016/j.physleta.2017.09.041
0375-9601/

Table 1
Definitions of symbols.

Symbol	Definition
N	Population size
M	Group size
r	Enhancement factor for the public goods game
a	Threshold number of defectors that triggers punishment in group interactions
υ	Cost of punishment
ω	Fine imposed on defectors
P_K	Average payoff of Player K
P_L	Average payoff of Player L
β	Intensity of selection, $\beta = 0$ means neutral evolution
μ	Mutation rate
n	Number of strategies
$\pi(\beta)$	Distribution of the strategies in the stationary state as a function of intensity of selection β
π_i	Average abundance of strategy i in the stationary state
P_a	Peer punishers with tolerance value a
C	Cooperation strategy
D	Defection strategy
$\omega_{cri}\|3$	Minimum fine for defectors such that punishment prevails for group size 3
$\omega_{cri}\|4$	Minimum fine for defectors such that punishment prevails for group size 4

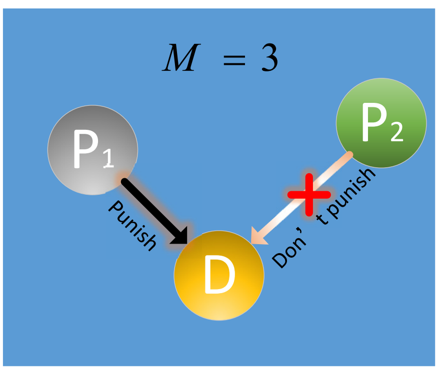

Fig. 1. Diversified tolerant peer punishers. P_a contributes to the common pool as a cooperator in the first-order social dilemma, punishes when there are at least a defector opponents, and does not punish otherwise in the second-order social dilemma. In the illustration, P_1, P_2 and D coexist in the group of size 3. In this case, P_1 contributes to the common pool and punishes the defector, since there is 1 defector in the group. P_2, however, does not punish the defector, since there is only one defector in the group, which is not sufficient to trigger its action of punishment. In fact, P_2 only punishes defectors if all the other group members are defectors.

punishment prevails over defection when the tolerance is taken into account.

The article is organized as follows. The model of tolerant punishment in finite populations is proposed in Section 2. In Section 3, we analyze the evolutionary results of competing strategies including tolerant punishment, cooperation and defection. Moreover, individual-based simulations are performed to verify the analytical results. In Section 4, we summarize and discuss the results.

2. Evolutionary process

We focus on M-player compulsory PGG in which all players enjoy the benefit in finite well-mixed populations [29,30] (see Table 1). The M individuals are randomly chosen from a population of N. Cooperators and all peer punishers contribute a fixed amount to the public good. We assume it equals to 1 without loss of generality. Defectors, however, contribute nothing. All contributions are summed up, multiplied by the enhancement factor r, and equally distributed by all the participants irrespective of their contributions. $1 < r < M$ is typically assumed to model the conflicts between group benefits and individual benefits. In this case, a group of cooperators are better off than a group of defectors, yet defectors outperform cooperators in any mixed groups. After the PGGs, peer punishers P_a with threshold value a would not punish the defectors in the group if the number of defectors is less than the threshold a, and punish otherwise. Then the payoff for each exploiter is reduced by ω, and the payoff for each punisher by υ. Suppose that υ is strictly smaller than ω ($\upsilon < \omega$). As one extreme case of $a = 1$, the punishers are nothing but unconditional peer punishers which have been intensively studied before [4,14,15,23]; As the other extreme case of $a = M$, the tolerant peer punishers degenerate to unconditional cooperators. Thus, the introduction of tolerance bridges unconditional cooperation and unconditional peer punishment, and puts these two strategies into one framework. Meanwhile, the diversity of tolerant punishment strategies is also induced by different thresholds of tolerance values. Thus the number of all possible tolerant punishment strategies represents the diversity of punishment, which equals to $M - 1$ (Fig. 1). We adopt the pairwise comparison rule with explorations to update the strategy of players [31]: One individual is chosen randomly, namely L. It explores or mutates to any other strategy with a given exploration rate μ. Otherwise, a player K is chosen from the rest of the population. And player L adopts the strategy of player K with probability $(1 + \exp(-\beta(P_K - P_L)))^{-1}$, where P_K and P_L denote the average payoffs of K and L [16]. Note that the average payoffs P_L and P_K depend on players' strategies and the frequencies of the strategies. The imitation strength β measures how important individuals deem the impact of the game in decision making. When the imitation strength β is large, a more successful player is almost always imitated, and the less successful one never is; When β vanishes, the updating of strategy is nearly random.

This evolutionary process is an ergodic stochastic system due to the presence of the exploration rate, i.e., non-vanishing μ [32–35]. This leads to the existence of average abundance in the long run [32,33]. The average abundance of the stochastic system for n strategies $\pi(\beta)$ is given by $\pi(\beta) = (\pi_1, \pi_2, \pi_3, \cdots, \pi_n)$, where π_i denotes the average abundance of strategy i. In the case of neutral selection, the average abundance of all strategies is the uniform distribution $1/n$. Here we define that strategy i is favored by selection if $\pi_i > 1/n$, and is disfavored if $\pi_i < 1/n$ [32,33]. In the following, we theoretically address which tolerant punishment strategy is favored.

3. Results and analysis

Firstly, we investigate how the evolutionary fate of the classic peer punishment strategy is altered by the presence of multiple