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Bubble merger and scaling law of the Rayleigh–Taylor instability with surface tension

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ABSTRACT

We present a theoretical model to study the effects of surface tension on the growth of single and multiple bubbles in the Rayleigh–Taylor instability. The asymptotic solution for a single bubble is obtained and is expressed in terms of the Eötvös number. The bubble merger process is also demonstrated from the model. We find the contrasting effects of surface tension: it reduces the growth of a single bubble, but enhances the mixing rate of multiple bubbles at a late time. The bubble merger of Rayleigh–Taylor instability follows the same scaling law of the growth of mixing zone even when surface tension exists, but the growth coefficient in the scaling law increases with surface tension. A comparison with an experimental result is in good agreement.

1. Introduction

Rayleigh–Taylor (RT) instability occurs when a heavy fluid is supported by a lighter fluid in a gravitational field [1,2]. A characteristic of RT instability is fingers, known as bubble and spike, of each phase extending into the region occupied by the opposite phase. The RT instability with initial random perturbations results in the bubble merger and turbulent mixing [3]. The RT instability plays an important role in many fields such as inertial confinement fusion, astrophysical supernova and supersonic combustion. To investigate the dynamics of this instability, extensive researches have been carried out in last decades [3,4].

The RT instability arises commonly with surface tension. The evolution of RT instability with surface tension exhibits a variety of interesting behaviors such as capillary waves, pinching and break-up [5,6]. At the linear stage of the instability, it is well known that surface tension produces a cut-off wave number and stabilizes high modes [7]. Sohn [8] studied effects of surface tension on the nonlinear evolution of RT instability and found that surface tension reduces the asymptotic velocity of a single-mode bubble. However, there have been only a few studies on the multi-mode RT instability when surface tension exists [9–12]. Moreover, some of these studies reported contrasting results on the growth rate of mixing zone. Cherfils and Mikaelian [9] first discussed the effect of surface tension (and also viscosity, etc.) on the RT mixing. Applying a simple diffusion model, they showed that surface tension affects the growth of RT mixing at early times, but has little effect at late times. From LEM (Linear Electric Motor) experiments, Dimonte and Schneider [10] commented that the mixing rate of RT instability is increased when surface tension is given, attributing its increase primarily to the enhanced meniscus at the wall. Young and Ham [11] conducted full numerical simulations for the RT mixing with surface tension and showed that surface tension reduces the effective mixing rate. These contrasting results of the previous studies are the motivation of our work. In this Letter, we investigate effects of surface tension on the growth of single and multiple bubbles of RT instability, from a theoretical model.

A central issue in the turbulent mixing by RT instability is a scaling law for the growth of the mixing zone. It has been found that the bubble front in the RT mixing grows self-similarly as

$$h_b = \alpha_b \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} g t^2, \tag{1}$$

and the coefficient α_b is generally insensitive to the density ratio [4,10,13,14], where ρ_1 and ρ_2 are the densities of heavy and light fluids, and g is the gravitation acceleration. The spike has a similar scaling law as the bubble, and the growth rate of spikes is similar to that of bubbles for small density ratios. However, an asymmetry has been observed between the growth rate of bubbles and the growth rate of spikes for moderate to large density ratios,

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i.e. $\alpha_s > \alpha_b$, and α_s increasing with the density ratio. When surface tension is present on the interface, one may question several issues on the RT mixing, which are not discovered yet: Does the bubble growth of the RT mixing with surface tension follow the scaling law (1)? If it does, how much and why does the growth coefficient α_b increase or decrease? Is it consistent with the motion of a single-mode bubble? We will address these questions in this Letter.

Theoretical models for comprehensive descriptions of the motion of bubbles at unstable interfaces are the potential flow models proposed by Layzer [15] and Zufiria [16]. Layzer [15] presented a model, based on the approximate description of the flow near the bubble tip and described the nonlinear evolution of the RT bubble of infinite density ratio. Since Layzer's work, it has been studied in various contexts; for example, finite density ratio [17,18], surface tension and viscosity [8], magnetic field [19], and viscoelasticity [20]. A limitation of the Layzer model is that it fails to give a quantitatively correct prediction for the bubble curvature [21]; the asymptotic bubble curvature from the model is always a constant, regardless of physical effects. In fact, the bubble curvature is an important parameter because it sets a length scale and plays a key role in the bubble merger process in the RT mixing. Note that other limitations of the Layzer model were reported by Mikaelian [22].

Another type of the potential-flow model is a source-flow model by Zufiria [16], which describes the bubble as the potential with a point source. This model provides more accurate prediction for the bubble motion [21]. The source-flow model also has been extended to various cases: the finite density contrast [23,24] and viscosity [25]. In this Letter, we apply the source-flow model to the unstable interface with surface tension and present the solution of a single bubble and the evolution of multiple bubbles.

2. Single bubble model

We consider an interface with surface tension, in a vertical channel, between two fluids with different densities in two dimensions. The fluids are assumed as incompressible, inviscid and irrotational. The upper fluid is heavier than the lower fluid, i.e. $\rho_1 > \rho_2$. From the assumption of potential flows, there exist complex potentials $W_i(z) = \phi_i + \mathrm{i}\psi_i$ for $i = 1, 2$, where ϕ and ψ are the velocity potential and the stream function. The location of the bubble tip is $Z(t) = X(t) + \mathrm{i}Y(t)$, in the laboratory frame of reference, with $Y(t) = D/2$, where D is the width of the channel. The bubble moves in the x direction with the velocity of the bubble tip U. (See Fig. 1 in Ref. [24].) We choose a frame of reference $(\hat{x}, \hat{y})$ comoving the bubble tip. In the moving frame, the interface near the bubble tip is approximated by

$$\eta(\hat{x}, \hat{y}, t) = \hat{y}^2 + 2R(t)\hat{x} = 0, \tag{2}$$

where $R(t)$ is the radius of curvature. Following the Zufiria–Sohn model [23], we take the complex potentials

$$W_1(\hat{z}) = Q_1 \ln(1 - e^{-k(\hat{z}+H)}) - U\hat{z}, \tag{3}$$

$$W_2(\hat{z}) = Q_2 \ln(1 - e^{-k(\hat{z}-H)}) + (K - U)\hat{z}, \tag{4}$$

where Q_i, $i = 1, 2$, represent the source strength, $H > 0$ represent the distance from the source, and $k = 2\pi/D$ is the wave number.

The evolution of the interface is determined by the kinematic condition and the Bernoulli equation,

$$\frac{D\eta}{Dt} = 2\frac{dR}{dt}\hat{x} + 2Ru_i + 2\hat{y}v_i = 0, \quad i = 1, 2, \tag{5}$$

$$\rho_1\left[\frac{\partial\phi_1}{\partial t} + \frac{1}{2}|\nabla\phi_1|^2 + \left(g + \frac{dU}{dt}\right)\hat{x}\right] - \rho_2\left[\frac{\partial\phi_2}{\partial t} + \frac{1}{2}|\nabla\phi_2|^2 + \left(g + \frac{dU}{dt}\right)\hat{x}\right] = p_2 - p_1, \tag{6}$$

where u_i and v_i, $i = 1, 2$, are $\hat{x}$ and $\hat{y}$ components of the interface velocity taken from the upper and lower fluids. The kinematic condition implies the continuity of the normal component of fluid velocity across the interface. By the Laplace–Young boundary condition, the pressure jump across the curve is balanced by the interfacial force due to surface tension

$$p_2 - p_1 = \sigma\kappa, \tag{7}$$

where σ and κ represent the surface tension and curvature of the interface, respectively. The interface curvature is expressed as

$$\kappa = -\frac{\hat{x}_{yy}}{(1+\hat{x}_y^2)^{3/2}}. \tag{8}$$

Therefore, the Bernoulli equation becomes

$$\rho_1\left[\frac{\partial\phi_1}{\partial t} + \frac{1}{2}|\nabla\phi_1|^2 + \left(g + \frac{dU}{dt}\right)\hat{x}\right] - \rho_2\left[\frac{\partial\phi_2}{\partial t} + \frac{1}{2}|\nabla\phi_2|^2 + \left(g + \frac{dU}{dt}\right)\hat{x}\right] = -\sigma\frac{\hat{x}_{yy}}{(1+\hat{x}_y^2)^{3/2}}. \tag{9}$$

The derivation of equations for the source-flow model with surface tension is basically similar to that without surface tension [24], and thus only the resulting equations are given below.

Using Eqs. (2)–(4) and satisfying the kinematic condition (5) up to the first order in $\hat{x}$, we have

$$\frac{dX}{dt} = U = c_1 Q_1 = \tilde{c}_1 Q_2 + K, \tag{10}$$

$$\frac{dR}{dt} = -Q_1(3c_2 + c_3 R)R = -Q_2(3\tilde{c}_2 + \tilde{c}_3 R)R. \tag{11}$$

The first and second order equations in $\hat{x}$ of the Bernoulli equation (9) are given by

$$(c_1 + c_2 R)\frac{dQ_1}{dt} + Q_1(c_2 + c_3 R)\frac{dH}{dt} - Q_1^2 c_2^2 R + g = \mu\left[(\tilde{c}_1 + \tilde{c}_2 R)\frac{dQ_2}{dt} + \frac{dK}{dt} - Q_2(\tilde{c}_2 + \tilde{c}_3 R)\frac{dH}{dt} - Q_2^2\tilde{c}_2^2 R + g\right] + \frac{3\sigma}{\rho_1 R^2}, \tag{12}$$

$$\left(\frac{c_2}{2} + c_3 R + \frac{c_4}{6}R^2\right)\frac{dQ_1}{dt} + Q_1\left(\frac{c_3}{2} + c_4 R + \frac{c_5}{6}R^2\right)\frac{dH}{dt} + \frac{1}{2}F_1 = \mu\left[\left(\frac{\tilde{c}_2}{2} + \tilde{c}_3 R + \frac{\tilde{c}_4}{6}R^2\right)\frac{dQ_2}{dt} - Q_2\left(\frac{\tilde{c}_3}{2} + \tilde{c}_4 R + \frac{\tilde{c}_5}{6}R^2\right)\frac{dH}{dt} + \frac{1}{2}F_2\right] + \frac{15\sigma}{2R^3\rho_1}, \tag{13}$$

where

$$F_1 = Q_1^2\left[c_2^2 - 2c_2c_3 R + \left(c_3^2 - \frac{4}{3}c_2c_4\right)R^2\right],$$

$$F_2 = Q_2^2\left[\tilde{c}_2^2 - 2\tilde{c}_2\tilde{c}_3 R + \left(\tilde{c}_3^2 - \frac{4}{3}\tilde{c}_2\tilde{c}_4\right)R^2\right],$$

and $\mu = \rho_2/\rho_1$ denotes the density ratio. The expressions for c_i are given in Ref. [24] and $\tilde{c}_n(H) = c_n(-H)$. Equations (10)–(13) determine the evolution of a single bubble of arbitrary density ratio with surface tension.