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An improved lattice hydrodynamic model considering the influence of optimal flux for forward looking sites

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ABSTRACT

In this paper, a lattice hydrodynamic model is derived considering the delayed-feedback control influence of optimal flux for forward looking sites on a single-lane road which includes more comprehensive information. The control method is used to analyze the stability of the model. The critical condition for the linear steady traffic flow is deduced and the numerical simulation is carried out to investigate the advantage of the proposed model with and without the effect of optimal flux for forward looking sites. Moreover it indicates that the characteristic of the model can lead to a lower energy consumption in traffic system. The results are consistent with the theoretical analysis correspondingly.

1. Introduction

In recent decades, the urban traffic problems have attracted more and more attention of engineers and scholars due to the increasing automobiles and the modernization construction. And the traffic congestion becomes the main concern which leads to the economic and social problems such as traffic accident, air pollution and global warming. So it has been highly recommended to attach importance to the traffic congestion problem with mathematical physics and control theories. Up to present, a large number of traffic flow models are proposed including the car-following models [1–24], the cellular automaton models [25–27], the hydrodynamic models [21,28–54], and the gas kinetic models [55,56] to figure out the complicated constitution behind the traffic congestion phenomenon.

In 1995, a famous car-following model called optimal velocity model (OVM, for short) was put forward by Bando et al. [5]. Based on OVM, a multitude of car-following models have been extended with delayed feedback control from the view of control theory [6–11]. In 2006, Zhao and Gao [12] proposed a simple coupled-map(CM) car-following model considering the velocity feedback control to suppress the traffic jam. Subsequently, Han [13] and Ge [14] took into account the ITS with CM car-following model in 2007 and 2011 respectively. Then in 2014, Li [15] presented a dynamic collaboration model with feedback signals to suppress the traffic congestion. Recently, because of the economic and social problems caused by traffic congestion, the energy consumption problem is also carried out to build a more effective and accurate traffic model [16–20], which has found that the vehicle's energy consumption is related to its speed and acceleration. Nevertheless, there are seldom studies hammer at traffic jam problem for macroscopic models with control theory.

The lattice hydrodynamic model was firstly introduced by Nagatani [31,32] to summarize the variational relationships between collective variables. After that, the lattice hydrodynamic model was universally studied by considering different factors from the macroscopic viewpoint [33–46], but the investigation of control signal in lattice hydrodynamic models is rare. In 2015, Redhu [47] carried out the DFC method for lattice hydrodynamic model considering the flux change in adjacent time and Ge [48] presented a simple control method with the lattice hydrodynamic model by applying a decentralized delayed-feedback control. Recently, Li [49] studied the lattice hydrodynamic model performance based on delayed feedback control with a view of density change rate difference.

As we all know, in the actual traffic, the most important factors that affect the behavior of the current vehicle are the in-

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fluences of the leading vehicle. But up to now, no studies have ever tried to analyze the traffic jam for macroscopic model considering the delayed-feedback control influence of optimal flux for forward looking sites which has a significant impact on traffic movement, not to mention considering the energy consumption problem. Based on this, a new lattice hydrodynamic model with the delayed feedback control will be presented to investigate its influence on traffic flow. And particularly the energy consumption problem is taken into account to make the traffic system more stable.

The outlines of this paper are as follows: in Sec. 2, we proposed a new lattice hydrodynamic model by taking into account the delayed-feedback control on traffic flow dynamic. The linear stability analysis by using control theory is performed in Sec. 3. In Sec. 4, numerical simulation is carried out for lattice hydrodynamic model with and without using delayed-feedback control signal and the energy consumption contrast figure is also presented. Finally, conclusion is given in Sec. 5.

2. Lattice hydrodynamic model considering optimal flux for forward looking sites

To study the complex mechanism behind the traffic flow, Nagatani [31] put forward the lattice hydrodynamic model which analyzed the density wave of traffic flow on a unidirectional road as follows:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 \\ \partial_t(\rho v) = a\rho_0 V(\rho(x+\delta)) - a\rho v \end{cases} \tag{1}$$

where ρ and $\rho(x+\delta)$ are the local density at the position of x and $x+\delta$ at time t, respectively. ρ_0 means the local average density and δ is the average space headway ($\rho_0 = \frac{1}{\delta}$). a is the sensitivity of the driver. v is the local average speed and $V(\rho)$ represents the optimal speed of traffic flow at density of ρ.

In order to make it easier to proceed the further study, Nagatani modified the equation in a discretization way as follows:

$$\begin{cases} \partial_t \rho_j + \rho_0\left(\rho_j v_j - \rho_{j-1} v_{j-1}\right) = 0 \\ \partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j \end{cases} \tag{2}$$

where the road is divided into N lattice sites and j indicates the site of the road on the one-dimensional lattice, ρ_j and v_j severally indicates the local density and the local average speed on site j at time t.

Recently, Redhu [47] proposed a delayed-feedback control for Nagatani's model in which flow evolution equation was modified taking the difference between current state t and delayed one τ of site $j+1$. The feedback gain was designed to suppress the traffic jam in lattice hydrodynamic model which finally verified the feasibility of feedback signal in suppressing the traffic congestion. Here, we propose a delayed-feedback control for Nagatani's model in which flow evolution equation is modified as

$$\begin{cases} \partial_t \rho_{j+1} + \rho_0\left(q_{j+1} - q_j\right) = 0 \\ \partial_t(q_j) = a\rho_0 V(\rho_{j+1}) - aq_j + u_j \end{cases} \tag{3}$$

where q represents the product of ρ and v. The flow rate difference u_j is described as:

$$k[\rho_0 V^{op}(\rho_{j+2}) - q_{j+1}] \tag{4}$$

with k being the delayed-feedback gain, the control signal describes the influence of optimal flux for forward looking sites $j+1$ at time t. And because Eq. (2) shows the similarity with Bando's car-following model, we adopt the analogous optimal speed equation:

$$V^{op}(\rho) = (V_{max}/2)\left[\tanh(1/\rho - 1/\rho_c) + \tanh(1/\rho_c)\right] \tag{5}$$

where V_{max} and ρ_c denote the maximal speed and the critical safety density.

3. Linear stability analysis with delayed-feedback control

In this section, we investigated the effect of delayed-feedback control through linear stability analysis to suppress the traffic jam. We suppose that the desired density of vehicles and comprehensive flow are ρ^* and q^* respectively, so the steady state of the following vehicles is:

$$[\rho_n(t), q_n(t)]^T = [\rho_n^*, q_n^*]^T \tag{6}$$

Then, we take into account an error system around steady state (6), that is:

$$\begin{cases} \partial_t \delta\rho_{j+1} + \rho_0\left(\delta q_{j+1} - \delta q_j\right) = 0 \\ \partial_t(\delta q_j) = a\rho_0\Lambda_1\delta\rho_{j+1} - a\delta q_j + k[\rho_0\Lambda_2\delta\rho_{j+2} - \delta q_{j+1}] \end{cases} \tag{7}$$

where $\Lambda_1 = \frac{\partial V(\rho_{n+1})}{\rho_{n+1}}|_{\rho_{n+1}=\rho^*}$, $\Lambda_2 = \frac{\partial V^{op}(\rho_{n+2})}{\rho_{n+2}}|_{\rho_{n+2}=\rho^*}$, $\delta\rho_j = \rho_j - \rho^*$, $\delta q_j = q_j - q^*$.

After Laplace transformation for the traffic system (7), the model can be described as:

$$\begin{cases} sP_{j+1}(s) - \rho_{j+1}(0) + \rho_0[Q_{j+1}(s) - Q_j(s)] = 0 \\ sQ_j(s) - q_j(0) = a\rho_0\Lambda_1 P_{j+1}(s) - aQ_j(s) \\ +k[\rho_0\Lambda_2 P_{j+2}(s) - Q_{j+1}(s)] \end{cases} \tag{8}$$

The matrix formulation of governing equations is given as

$$\begin{bmatrix} P_{j+1}(s) \\ Q_j(s) \end{bmatrix} = \begin{bmatrix} -(s+a) & \rho_0 \\ -a\rho_0\Lambda_1 & s \end{bmatrix} \begin{bmatrix} 0 & -\rho_0 \\ -k\rho_0\Lambda_2 & k \end{bmatrix} \begin{bmatrix} P_{j+2}(s) \\ Q_{j+1}(s) \end{bmatrix} \times \frac{1}{p(s)} \tag{9}$$

where $L(\rho_{j+1}) = P_{j+1}(s)$, $L(\rho_{j+2}) = P_{j+2}(s)$, $L(q_{j+1}) = Q_{j+1}(s)$, $L(q_j) = Q_j(s)$, $L(\cdot)$ is the Laplace transform, s is a complex variable and $p(s) = -s^2 - as^2 + a\rho_0^2\Lambda_1$. Then the transfer function can be acquired

$$G(s) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} -(s+a) & \rho_0 \\ -a\rho_0\Lambda_1 & s \end{bmatrix} \begin{bmatrix} 0 & -\rho_0 \\ -k\rho_0\Lambda_2 & k \end{bmatrix} \frac{1}{p(s)} \tag{10}$$

Taylor's formula is applied into $G(s)$ and the result is accurately presented as follows:

$$G(s) = \frac{[k\rho_0\Lambda_2 s \ - ks - a\rho_0^2\Lambda_1]}{s^2 + as - a\rho_0^2\Lambda_1} \tag{11}$$

According to the control theory, it's known that traffic jam will never happen in the traffic flow system when $p(s)$ is stable and $\|G(s)\|_\infty \le 1$. The flux of traffic flow can be steady through the fine tuning of the parameter λ. The stability state can be confirmed by Hurwitz stability criterion if $a > 0$ and $\Lambda_1 < 0$.

Subsequently, $G(s)$ must be smaller than 1 for all positive ω^2 to ensure stability. The stability criterion of the lattice model is given by:

$$\begin{cases} \|G(s)\|_\infty = \sup_{\omega\in[0,\infty)} |G(j\omega)| \le 1 \\ |G(j\omega)| = \sqrt{G(jw)G(-jw)} = \sqrt{\frac{a^2\rho_0^4\Lambda^2}{\left(a\rho_0^2\Lambda + \omega^2\right)^2 + \omega^2(a-\lambda\omega^2)^2}} \le 1 \end{cases} \tag{12}$$

The sufficient condition is given as

$$w^2 + 2a\rho_0^2\Lambda_1 - k^2\rho_0^2\Lambda_2^2 \ge 0, \omega \in [0, +\infty) \tag{13}$$