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Unidirectional reflection and invisibility in nonlinear media with an incoherent nonlinearity

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ABSTRACT

We give explicit criteria for the reflectionlessness, transparency, and invisibility of a finite-range potential in the presence of an incoherent (intensity-dependent) nonlinearity that is confined to the range of the potential. This allows us to conduct a systematic study of the effects of such a nonlinearity on a locally periodic class of finite-range potentials that display perturbative unidirectional invisibility. We use our general results to examine the effects of a weak Kerr nonlinearity on the behavior of these potentials and show that the presence of nonlinearity destroys the unidirectional invisibility of these potentials. If the strength of the Kerr nonlinearity is so weak that the first-order perturbation theory is reliable, the presence of nonlinearity does not affect the unidirectional reflectionlessness and transmission reciprocity of the potential. We show that the expected violation of the latter is a second order perturbative effect.

An important difference between real and complex scattering potentials is that the reciprocity in reflection is generally broken for a complex potential. An extreme example is a potential $v(x)$ that is reflectionless only from the left or right. If, in addition, $v(x)$ has perfect transmission property, i.e., the transmitted (left- or right-going) waves are not affected by the presence of $v(x)$, it is said to be unidirectionally invisible [1–3]. The aim of the present letter is to explore the consequences of introducing a weak incoherent nonlinearity on the behavior of a unidirectionally invisible potential.[1]

Consider an isotropic nonmagnetic medium with translational symmetry along the y- and z-axes. The interaction of this medium with electromagnetic waves is described by its relative permittivity $\hat{\varepsilon}(x)$. A normally incident z-polarized TE wave that propagates in such a medium has an electric field of the form $\vec{E}(\vec{r},t)=E_0 e^{-ikct}\psi(x)\hat{e}_z$, where $\vec{r}$ is the position vector, E_0 is a constant amplitude, k and c are respectively the wavenumber and the speed of light in vacuum, $\hat{e}_j$ is the unit vector along the j-axis with $j\in\{x,y,z\}$, and $\psi(x)$ solves the Helmholtz equation, $\psi''(x)+k^2\hat{\varepsilon}(x)\psi(x)=0$. This is equivalent to the Schrödinger equation:

$$-\psi''(x)+v(x)\psi(x)=k^2\psi(x), \tag{1}$$

for the optical potential:

$$v(x):=k^2[1-\hat{\varepsilon}(x)]. \tag{2}$$

For a nonlinear medium with incoherent nonlinearity, where $\hat{\varepsilon}$ depends on $|\vec{E}|$, the role of (1) is played by its nonlinear generalization. For a medium forming a slab that is placed between the planes $x=0$ and $x=L$, this has the form

$$-\psi''(x)+v(x)\psi(x)+\gamma\,\chi(x)\mathcal{F}(|\psi(x)|)\psi(x)=k^2\psi(x), \tag{3}$$

where γ is a real coupling constant,

$$\chi(x):=\begin{cases}1 & x\in[0,L],\\ 0 & x\notin[0,L],\end{cases} \tag{4}$$

$$v(x)=\mathfrak{z}\,f(x)\chi(x)=\begin{cases}\mathfrak{z}\,f(x) & x\in[0,L],\\ 0 & x\notin[0,L],\end{cases} \tag{5}$$

$\mathfrak{z}$ is a real coupling constant that we have introduced for future use, $f(x):=\mathfrak{z}^{-1}k^2[1-\hat{\varepsilon}(x)]$, and $\mathcal{F}:[0,\infty)\to\mathbb{C}$ is a function representing the nonlinear behavior of the medium.[2] For a Kerr medium, $\mathcal{F}(|\psi|):=|\psi|^2$.

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[1] Here by "incoherent" we mean that the nonlinear term in the wave equation does not depend on the phase of its solutions ψ, i.e., it is a function of $|\psi|$.

[2] We also assume that $\mathfrak{z}$ and γ are so that $|f(x)|$ and $|\mathcal{F}(|\psi|)|$ are bounded by numbers of order 1.

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Equation (3) is a nonlinear Schrödinger equation with a confined nonlinearity. The scattering theory defined by this equation has been of interest in laser physics [4,5] where the behavior of the lasing modes are linked with the corresponding nonlinear spectral singularities (real poles of the transmission and reflection coefficients). The study of the scattering features of nonlinear media modeled by a nonlinear Schrödinger equation has a long history [6]. The research activity in this subject is however mostly focused on situations where the nonlinearity has an infinite range, see for example [7] where the authors consider the transmission resonances in an extended Kerr medium containing a finite number of delta-function barrier potentials. For a recent review of the physical aspects of nonlinear Schrödinger equations involving a complex potential, see [8].

The principal example of a unidirectionally invisible potential is

$$v(x) = \mathfrak{z}\,\chi(x)e^{iKx}, \tag{6}$$

where $K = \pi/L$, [1,9–14]. This potential turns out to be unidirectionally invisible from the left for $k = 2K = 2\pi/L$ provided that $|\mathfrak{z}| \ll K^2$. The latter condition is an indication that the unidirectional invisibility of this potential is a first-order pertubative effect. We therefore call it "perturbative unidirectional invisibility"; it persists provided that the first Born approximation is applicable [15]. For sufficiently large values of $|\mathfrak{z}|/k^2$, the potential (6) loses this property [12,15]. This turns out to be a common feature of an infinite class of locally periodic potentials of the form

$$v(x) = \mathfrak{z}\,\chi(x)\sum_{n=-\infty}^{\infty} c_n e^{inKx}. \tag{7}$$

Specifically, if $c_{-s} = c_0 = 0 \neq c_s$ for some positive (respectively negative) integer s, then the potential (7) displays perturbative unidirectional left (respectively right) invisibility for $k = 2sK = 2\pi s/L$, [15]. If $c_{\pm s} = 0$, the potential is bidirectionally reflectionless, and if in addition $c_0 = 0$ it is bidirectionally invisible for this wavenumber.

Remarkably the potential (6) displays exact (nonperturbative) unidirectional invisibility for certain values of $\mathfrak{z}/k^2$ that are not necessarily small [16]. Other examples of potentials possessing exact unidirectional invisibility are given in [3,17–21].

In this letter we explore the phenomenon of unidirectional invisibility for the scattering processes defined by the nonlinear Schrödinger equation (3) with $|\gamma| \ll k^2$. Our starting point is the approach to nonlinear scattering theory that is developed in Ref. [22]. This is based on the observation that the scattering solutions of (3) that correspond to right- and left-incident waves are respectively given by

$$\psi_{k-}(x) = \begin{cases} N_- e^{-ikx} & \text{for} \quad x < 0, \\ \xi_k(x) & \text{for} \quad 0 \le x \le L, \\ \dfrac{e^{ik(x-L)}F_+(k) - e^{-ik(x-L)}F_-(k)}{2ik} & \text{for} \quad x > L, \end{cases} \tag{8}$$

$$\psi_{k+}(x) = \begin{cases} \dfrac{e^{ikx}G_+(k) - e^{-ikx}G_-(k)}{2ik} & \text{for} \quad x < 0, \\ \zeta_k(x) & \text{for} \quad 0 \le x \le L, \\ N_+ e^{ik(x-L)} & \text{for} \quad x > L, \end{cases} \tag{9}$$

where ξ_k and ζ_k are the solutions of (3) in $[0, L]$ satisfying

$$\xi_k(0) = N_-, \qquad \xi_k'(0) = -ikN_-, \tag{10}$$

$$\zeta_k(L) = N_+, \qquad \zeta_k'(L) = ikN_+, \tag{11}$$

$F_\pm$ and $G_\pm$ are the Jost functions determined by

Table 1
Conditions for reflectionlessness, transparency, and invisibility for an incident plane wave with wavenumber k.

	From right	From left
Reflectionlessness	$\tilde{\mathcal{X}}(2k) = 0$	$\tilde{\mathcal{Y}}(-2k) = 0$
Transparency	$\tilde{\mathcal{X}}(0) = 0$	$\tilde{\mathcal{Y}}(0) = 0$
Invisibility	$\tilde{\mathcal{X}}(2k) = \tilde{\mathcal{X}}(0) = 0$	$\tilde{\mathcal{Y}}(-2k) = \tilde{\mathcal{Y}}(0) = 0$

$$F_\pm(k) := \xi_k'(L) \pm ik\xi_k(L), \qquad G_\pm(k) := \zeta_k'(0) \pm ik\zeta_k(0), \tag{12}$$

and $N_\pm$ are nonzero constants.

In view of (8) and (9), we can identify the right/left reflection and transmission coefficients, $R^{r/l}$ and $T^{r/l}$, as follows [22]:

$$\begin{aligned} R^r &= -\frac{e^{-2ikL}F_+(k)}{F_-(k)}, & T^r &= -\frac{2ike^{-ikL}N_-}{F_-(k)}, \\ R^l &= -\frac{G_-(k)}{G_+(k)}, & T^l &= \frac{2ike^{-ikL}N_+}{G_+(k)}. \end{aligned} \tag{13}$$

In order to simplify these relations, first we note that ξ_k and ζ_k fulfill

$$\xi_k(x) = N_- e^{-ikx} + \int_0^x \mathcal{G}(x,x')[\gamma\mathcal{F}(|\xi_k(x')|) + \mathfrak{z}f(x')]\xi_k(x')dx', \tag{14}$$

$$\zeta_k(x) = N_+ e^{ik(x-L)} + \int_L^x \mathcal{G}(x,x')[\gamma\mathcal{F}(|\zeta_k(x')|) + \mathfrak{z}f(x')]\zeta_k(x')dx', \tag{15}$$

where $\mathcal{G}(x,x') := \sin[k(x-x')]/k$ is the Green function for the equation $\psi'' + k^2\psi = 0$. Next, we introduce

$$\hat{\xi}_k(x) := N_-^{-1}e^{ikx}\xi_k(x), \qquad \hat{\zeta}_k(x) := N_+^{-1}e^{ik(L-x)}\zeta_k(x), \tag{16}$$

$$\mathcal{X}(x) := \chi(x)\left[\gamma\mathcal{F}(|N_-\hat{\xi}_k(x)|) + \mathfrak{z}f(x)\right]\hat{\xi}_k(x), \tag{17}$$

$$\mathcal{Y}(x) := \chi(x)\left[\gamma\mathcal{F}(|N_+\hat{\zeta}_k(x)|) + \mathfrak{z}f(x)\right]\hat{\zeta}_k(x), \tag{18}$$

and use (12) and (14)–(18) to express $F_\pm(k)$ and $G_\pm(k)$ in terms of $\mathcal{X}(x)$ and $\mathcal{Y}(x)$. Substituting the result in (13), we find

$$\begin{aligned} R^r &= \frac{\tilde{\mathcal{X}}(2k)}{2ik - \tilde{\mathcal{X}}(0)}, & T^r &= \frac{2ik}{2ik - \tilde{\mathcal{X}}(0)}, \\ R^l &= \frac{\tilde{\mathcal{Y}}(-2k)}{2ik - \tilde{\mathcal{Y}}(0)}, & T^l &= \frac{2ik}{2ik - \tilde{\mathcal{Y}}(0)}, \end{aligned} \tag{19}$$

where a tilde denotes the Fourier transform; e.g., $\tilde{\mathcal{X}}(k) := \int_{-\infty}^{\infty} e^{-ikx}\mathcal{X}(x)dx$.

The system is said to be left/right reflectionless (respectively transparent) if $R^{l/r} = 0$ (respectively $T^{l/r} = 1$). It is said to be left/right invisible if it is both left/right reflectionless and transparent, i.e., $R^{l/r} = 0$ and $T^{l/r} = 1$. The unidirectional invisibility from left/right refers to situations where the system is left/right invisible but fails to be right/left invisible. In light of (19), we can express the conditions for reflectionlessness, transparency, and invisibility in terms of $\tilde{\mathcal{X}}$ and $\tilde{\mathcal{Y}}$, as shown in Table 1.

A linear medium might violate reciprocity in reflection, but it respects reciprocity in transmission [23,24], i.e., $T^l = T^r$. An important feature of nonlinear media is that they can violate the reciprocity in transmission. This is of particular interest in attempts to devise an optical isolator (diode) [25].

According to Table 1, we can acquire a more explicit quantitative characterization of unidirectional invisibility for the media