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Finite temperature expansion dynamics of Bose–Einstein condensates in ring traps

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ABSTRACT

We explore the effects of finite temperature on the dynamics of Bose–Einstein condensates (BECs) after it is released from the confining potential. In addition, we examine the variation in the expansion dynamics of the BECs as the confining potential is transformed from a multiply to a simply connected geometry. To include the effects of finite temperatures we use the frozen thermal cloud approximation, and observe unique features of the condensate density distribution when released from the confining potential. We find that at $T \neq 0$, during the initial stages of expansion, the multiply connected condensate has more pronounced interference rings compared to the case of zero temperature. Such difference in the dynamical evolution is also evident for simply connected condensates.

1. Introduction

The time-of-flight measurement is an important experimental technique to detect Bose–Einstein condensation in dilute atomic gases, and probe their other properties as well. In this technique the atoms are left to expand by switching off the external confining potential, and the atoms are then imaged with optical methods. The technique has been used to observe a plethora of diverse phenomena in the Bose–Einstein condensates (BECs) of dilute atomic gases. Few examples are the experimental observation of the non-equilibrium many-body phenomenon based on matter wave interference patterns [1], dynamical quasicondensation of hard-core bosons [2], and the observation of thermally activated vortex pairs in a quasi-2D Bose gas leading to a crossover from a Berezinskii–Kosterlitz–Thouless phase to a vortex-free BEC [3]. In fermionic atomic species, the technique has been used to probe the superfluidity of strongly interacting Fermi mixtures [4] and, to measure the p-wave Feshbach resonances for fermionic atoms [5].

Toroidal condensates, which are multiply connected BECs, are near ideal systems to study phenomena related to persistent superflows. In these systems too, the time-of-flight measurements play an important role to detect and probe the phenomena of interest. In experiments, toroidal condensates have been obtained with the use of harmonic potential in combination with a Gaussian potential [6], Laguerre-Gaussian beams [7–10], combination of an RF-dressed magnetic trap with an optical potential [11,12], magnetic ring traps [13–16], time-averaged ring potentials [17,18], coincident red and blue detuned laser beams [19], and employing digital micromirror devices [20]. The toroidal or ring condensates also serve as model-systems in the field of atomtronics. These systems have been used to implement superconducting quantum interference devices (SQUIDs) [21,22], and phase-slips in rf SQUID have been modelled by employing blue-detuned laser beam along the axis of toroidal condensates [23]. In both of these experiments, the phase-difference along the condensate has been measured from the free expansion dynamics. It is found that for toroidal BECs with finite circulation the images after expansion have a central hole whose area is proportional to the winding number. On the other hand, there is finite density at the center when there is no circulation [23,24]. These are well understood and in good agreement with theoretical simulations using zero temperature time-dependent Gross–Pitaevskii (GP) equation [24].

For the present work, we consider a toroidal BEC obtained with a confining potential consisting of a harmonic and Gaussian potential [6]. This configuration offers the possibility to probe the effects associated with the transition from multiply to simply connected BEC due to relative shift in the component trapping potentials. In experiments, this is an important consideration since the trap centers, extremum of the harmonic and Gaussian trapping potentials, never coincide. This is due to gravitational sagging, and deviations of the optical elements and external fields from perfect alignment. These experimental realities establish the need to theoretically probe the effects of geometry on the expansion dy-

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namics of a quasi-2D BEC. In particular, we wish to investigate the modification of the expansion dynamics and the nature of superflow as the condensate changes from multiply to simply connected topology. It is of interest to find out what is the nature of the expansion when the BEC is simply connected. In which case there is no superflow present in the BEC. This forms the motivation of our present study. To validate the experimental results, and considering the deviations from an ideal case, it is also pertinent to examine the role of thermal fluctuations on the expansion dynamics. So that, it is possible to distinguish and identify the effects of relative shift in the trapping potential, and those emerging from the thermal fluctuations.

To examine the thermal fluctuations in the toroidal BEC as a function of the separation between the trap centers we use the Hartree–Fock–Bogoliubov theory with Popov (HFB–Popov) approximation. The increase in the separation of the trap centers induces a topological transformation in the condensate density profiles. The condensate density profile is modified from toroidal or multiply connected to a bow-shaped or simply connected geometry, the profile of the quantum fluctuations also exhibit a similar transformation. The thermal fluctuations, in contrast, remain multiply connected [25]. These differences in the structure of the condensate and thermal density profiles affect the dynamics of the condensate cloud during expansion at finite temperature. We examine this in detail using frozen thermal cloud approximation. It is to be mentioned here that, previous works using the classical field approximation have shown that thermal fluctuations affect the radial and axial condensate widths during expansion [26,27].

2. Theoretical methods

The grand-canonical Hamiltonian of an interacting quasi-2D BEC system is

$$\hat{H} = \iint dxdy\, \hat{\Psi}^\dagger(x,y,t) \times \left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) + V(x,y) - \mu + \frac{U}{2}\hat{\Psi}^\dagger(x,y,t)\hat{\Psi}(x,y,t)\right]\hat{\Psi}(x,y,t), \tag{1}$$

where $\hat{\Psi}$ and μ are the Bose field operator of a scalar BEC, and the chemical potential, respectively. We consider an external confining potential of the form $V(x,y) = (1/2)m\omega_x^2(x^2+y^2) + V_0 e^{-[(x-\Delta_x)^2+y^2]/2\sigma^2}$, which is a superposition of the harmonic oscillator and Gaussian potential with strength V_0. This choice of the potential parameters implies that the aspect ratio in the transverse direction $\alpha = \omega_y/\omega_x = 1$. The case of $V_0 = 0$, $\Delta_x = 0$ then produces a symmetric 2D harmonic trap, whereas $V_0 \gg 0$ makes it a toroid. Let Δ_x represent the separation between the centers of the harmonic and Gaussian potentials, we can consider the separation as along the x-axis through an appropriate rotation of the coordinates. This separation accounts for the non-coincidence of the trapping potential centers. This deviation is natural, since in experiments trap centers never coincide because of gravitational sagging, and optical axes are not perfectly aligned. In a quasi-2D system, as $\omega_z \gg \omega_x$ the condensate is considered to be in the ground state along z. The axial degrees of freedom can be integrated out and the excitations along the transverse direction only contribute to the dynamics. The atoms of the bosonic species with mass m and scattering length a interact repulsively through the s-wave binary collisions with strength $U = 2g\sqrt{2\pi\lambda}$, where $g = 4\pi\hbar^2 a/m$ and $\lambda = \omega_z/\omega_x$.

2.1. Gapless Hartree–Fock–Bogoliubov–Popov formalism

To compute the equilibrium density profiles of BEC at finite temperatures, we use the gapless HFB–Popov theory. In this theory the Bose field operator $\hat{\Psi}$ is decomposed into a condensate part represented by $\phi(x,y,t)$, and the fluctuation part denoted by $\tilde{\psi}(x,y,t)$. That is $\hat{\Psi} = \phi + \tilde{\psi}$, and the condensate part $\phi(x,y,t)$ solves the generalized GP equation

$$\hat{h}\phi + U\left[n_c + 2\tilde{n}\right]\phi = 0. \tag{2}$$

In the above equation the single-particle or the non-interacting part of the Hamiltonian is $\hat{h} = (-\hbar^2/2m)\left(\partial^2/\partial x^2 + \partial^2/\partial y^2\right) + V(x,y) - \mu$ with

$$n_c(x,y) \equiv |\phi(x,y)|^2,\ \tilde{n}(x,y) \equiv \langle \tilde{\psi}^\dagger(x,y,t)\tilde{\psi}(x,y,t)\rangle,$$

and $n(x,y) = n_c(x,y) + \tilde{n}(x,y)$ as the local condensate, non-condensate or thermal, and total density, respectively. To determine the non-condensate density, the fluctuation operator $\tilde{\psi}$ is represented through a superposition of Bogoliubov quasiparticle as

$$\tilde{\psi} = \sum_j \left[u_j(x,y)\hat{\alpha}_j e^{-iE_j t/\hbar} - v_j^*(x,y)\hat{\alpha}_j^\dagger e^{iE_j t/\hbar}\right], \tag{3}$$

with j denoting the energy eigenvalue index of a quasiparticle mode having energy E_j. The quasiparticle annihilation (creation) operators $\hat{\alpha}_j$ $(\hat{\alpha}_j^\dagger)$ satisfy the usual Bose commutation relations. The functions u_j and v_j are the Bogoliubov quasiparticle amplitudes corresponding to the jth energy eigenstate, and solves the following pair of coupled Bogoliubov–de Gennes (BdG) equations

$$\begin{aligned}(\hat{h} + 2Un)u_j - U\phi^2 v_j &= E_j u_j,\\ -(\hat{h} + 2Un)v_j + U\phi^{*2} u_j &= E_j v_j.\end{aligned} \tag{4}$$

Following these definitions, the thermal or the non-condensate density at temperature T is

$$\tilde{n} = \sum_j \{[|u_j|^2 + |v_j|^2]N_0(E_j) + |v_j|^2\}, \tag{5}$$

where $\langle\hat{\alpha}_j^\dagger\hat{\alpha}_j\rangle = (e^{\beta E_j} - 1)^{-1} \equiv N_0(E_j)$ with $\beta = 1/k_B T$, is the Bose factor of the jth quasiparticle state with energy E_j at temperature T. More details of the derivations, and numerical scheme to solve the generalized GP and coupled Bogoliubov-de Gennes (BdG) equations self-consistently are given in our previous works [25,28–31]. In the calculations, the spatial and temporal variables are scaled as x/a_{osc}, y/a_{osc} and $\omega_x t$ respectively, where $a_{osc} = \sqrt{\hbar/m\omega_x}$. It must be mentioned here that, in the HFB–Popov formalism, the properties that we obtain are equivalent to ensemble-average. This is, however, not the case in other methods like Stochastic Gross–Pitaevskii formalism or truncated Wigner approximation [32,33]. In these approaches each computation represents a member of an ensemble. So the ensemble average of properties are computed by averaging over several independent realizations. Such an averaging is not required in the HFB–Popov method, and the results we have given are equivalent to the ensemble average.

2.2. Frozen thermal cloud approximation

To study the dynamics of BEC at finite temperatures, we solve the time dependent GP equation with frozen thermal cloud approximation. In this approximation the dynamics of the thermal cloud is ignored, and the condensate atoms move in the presence of a static cloud of non-condensate atoms which are in a state of thermal equilibrium with the initial state, and obeys Bose–Einstein distribution function. This, in the expansion dynamics of the condensate after the removal of the trapping potential, is equivalent to