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Bandgap properties in simplified model of composite locally resonant phononic crystal plate

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ABSTRACT

This paper extends the traditional plane wave expansion (PWE) method to calculate the band structure of the proposed simplified model of composite locally resonant phononic crystal (LRPC) plate. Explicit matrix formulations are developed for the calculation of band structure. In order to illustrate the accuracy of the results, the band structure calculated by PWE method is compared to that calculated by finite element (FE) method. In addition, in order to reveal the bandgap properties, band structures of the "spring–mass" simplified model of stubbed-on LRPC plate, "spring–torsional spring–mass" simplified model of stubbed-on LRPC plate and "spring–torsional spring–mass" simplified model of composite LRPC plate with and without the viscidity considered are presented and investigated in detail.

1. Introduction

As is well-known, plates are extensively used as the containment structures in many areas such as marine, transport, aerospace engineering and civil construction projects [1,2]. Vibrations are mostly propagated along the containment structures from the vibration sources and structural noises are produced by the radiation of the vibrations. The proposition of PC concept provides a new idea for the investigation on the theory of vibration insulation and noise reduction. Over the past two decades, the propagation of elastic waves in PCs has attracted a lot of attentions which mainly focus on the calculation methods and bandgap properties, but the application researches particularly in the field of vibration insulation and noise reduction are still immature. Bragg scattering [3–6] and locally resonant (LR) [7–10] are considered as the two main mechanisms for the creation of acoustic band gaps, which the frequency range of band gaps based on the first mechanism is almost two orders of magnitude higher than that based on the second mechanism [7]. Hence, studies on plates with the design idea of LRPC introduced will provide a new idea for restraining the structure vibration and reducing the noise in the unmanageable low frequency range [11–14] of some industrial products.

In recent years, bandgap properties of plates with the design idea of LRPC introduced have been researched. By etching holes periodically in a solid matrix plate and then filling them with scatters, the so-called filled-in system is formed; by stubbing resonant units periodically onto the free surfaces of the plate, the stubbed-on system is formed [15]. Hsu et al. [16] and Xiao et al. [17] investigated the vibration band gaps of epoxy base plates with filled-in rubber resonant units and filled-in rubber-coated heavy mass resonant units by using PWE method, respectively. Similarly, the three-component and two-component stubbed-on systems constructed by periodically depositing rubber stubs with and without Pb capped on the surface of the base plate was studied by using FE method by Oudich et al. [18]. Besides, Xiao et al. [13] researched the flexural wave propagation and vibration transmission in an LR thin plate with a two-dimensional periodic array of attached spring–mass resonators. Zhao et al. [19] proposed a double-vibrator (rubber–steel–rubber–steel layers) three-component pillared PC plate on the basis of the traditional univibrator (rubber–steel layers) three-component pillared PC plate and studied the propagation characteristics of band gaps of flexural vibration and longitudinal vibration in the two-layer stubbed-on system. By revisiting the filled-in and stubbed-on structures, Ma et al. [15] proposed a new structure with the three-layered spherical resonant units, which opens a large sub-wavelength full acoustic band gap. By combining the filled-in and stubbed-on units, Li et al. [20] investigated the propagation characteristics of Lamb waves in an LRPC plate with the combined resonant unit. Based on this, Li et al. [21] further researched the expansion of LR complete acoustic band gaps in two-dimensional PCs using a double-sided stubbed composite PC plate with composite stubs. Recently, Qian et al. [22]

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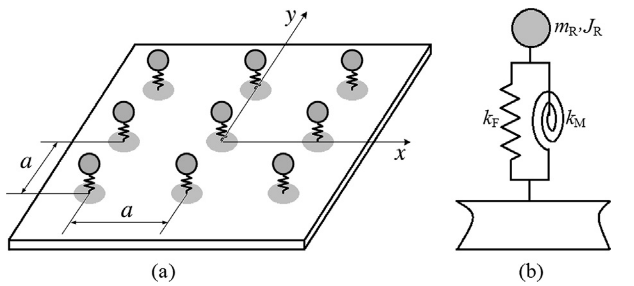

Fig. 1. (a) Simplified model of composite LRPC plate, and (b) simplified model of resonant unit attached on the plate in a unit cell.

investigated the propagation characteristics of flexural waves in the LRPC double panel structure made of a two-dimensional periodic array of a spring–mass resonator surrounded by n springs connected between the upper and lower plates.

In this paper, we investigate the propagation characteristics of flexural vibration in the proposed "spring–mass" simplified model of stubbed-on LRPC plate, "spring–torsional spring–mass" simplified model of stubbed-on LRPC plate and "spring–torsional spring–mass" simplified model of composite LRPC plate with and without the viscidity considered. The traditional PWE method is extended and formulized to treat such PC plates. All the results are expected to be of theoretic significances and engineering application prospects in the field of vibration and noise reduction.

2. Model and method

The simplified model of composite LRPC plate is composed by periodically attaching the simplified model of resonant unit onto the composite plate, as sketched in Fig. 1(a). The simplified model of resonant unit is composed of spring, torsional spring and mass. The composite plate is formed by etching holes periodically in the matrix plate and then filling them with another soft material. Here, the viscidity of the soft material in the composite plate is not considered at first, the plate and mass are connected by the spring and torsional spring in each unit cell, as shown in Fig. 1(b).

In this paper, the thickness of the plate is assumed to meet the requirement of thin plate. Besides, only the x-axis is considered as the direction that the mass twists around in order to simplify the derivation. According to the bending theory of thin plate, the governing equation of the model shown in Fig. 1(a) can be written as:

$$\begin{cases} \dfrac{\partial^2}{\partial x^2}\left[D(\mathbf{r})\left(\dfrac{\partial^2 W_1(\mathbf{r})}{\partial x^2}+\mu(\mathbf{r})\dfrac{\partial^2 W_1(\mathbf{r})}{\partial y^2}\right)\right] \\ \quad +2\dfrac{\partial^2}{\partial x\partial y}\left[D(\mathbf{r})\big(1-\mu(\mathbf{r})\big)\dfrac{\partial^2 W_1(\mathbf{r})}{\partial x\partial y}\right] \\ \quad +\dfrac{\partial^2}{\partial y^2}\left[D(\mathbf{r})\left(\dfrac{\partial^2 W_1(\mathbf{r})}{\partial y^2}+\mu(\mathbf{r})\dfrac{\partial^2 W_1(\mathbf{r})}{\partial x^2}\right)\right] \\ \quad -\omega^2\rho(\mathbf{r})h(\mathbf{r})W_1(\mathbf{r}) \\ \quad =\displaystyle\sum_{\mathbf{R}} Q_1(\mathbf{R})+\frac{\partial}{\partial x}\sum_{\mathbf{R}} M_1(\mathbf{R}) \\ -\omega^2 m_R W_R(\mathbf{R})=Q_R(\mathbf{R}) \\ -\omega^2 J_R\theta_R(\mathbf{R})=M_R(\mathbf{R}), \end{cases} \tag{1}$$

where,

$$\begin{cases} Q_1(\mathbf{R})=-k_F\big[W_1(\mathbf{R})-W_R(\mathbf{R})\big]\delta(\mathbf{r}-\mathbf{R}) \\ Q_R(\mathbf{R})=k_F\big[W_1(\mathbf{R})-W_R(\mathbf{R})\big], \end{cases} \tag{2}$$

Table 1
Parameters used in calculations.

E_1 (GPa)	μ_1	ρ_1 ($\mathrm{kg\,m^{-3}}$)	E_2 (GPa)	μ_2	ρ_1 ($\mathrm{kg\,m^{-3}}$)
77.6	0.35	2730	1.175×10^{-4}	0.469	1300

k_F ($\mathrm{N\,m^{-1}}$)	m_R (kg)	k_M ($\mathrm{N\,m^2}$)	J_R ($\mathrm{kg\,m^2}$)	a (m)	h (m)	R_s (m)
4×10^5	0.1	100	0.01	0.1	0.002	4×10^{-6}

$$\begin{cases} M_1(\mathbf{R})=k_M\left[\dfrac{\partial}{\partial x}W_1(\mathbf{R})\delta(\mathbf{r}-\mathbf{R})-\theta_R(\mathbf{R})\delta(\mathbf{r}-\mathbf{R})\right] \\ M_R(\mathbf{R})=-k_M\left[\dfrac{\partial}{\partial x}W_1(\mathbf{R})-\theta_R(\mathbf{R})\right]. \end{cases} \tag{3}$$

In the equations, $W_1(\mathbf{r})$ is the transverse displacement of the plate, $\rho(\mathbf{r})$ is density of the plate, $h(\mathbf{r})$ is the thickness of the plate, $D(\mathbf{r})=E(\mathbf{r})h(\mathbf{r})^3/12(1-\mu(\mathbf{r})^2)$ is the bending stiffness of the plate, $E(\mathbf{r})$ and $\mu(\mathbf{r})$ are the elasticity modulus and Poisson ratio of the plate. Because the composite plate is composed by two different materials, the density, bending stiffness, elasticity modulus and Poisson ratio of the hard and soft materials are represented by (ρ_1, D_1, E_1, μ_1) and (ρ_2, D_2, E_2, μ_2), respectively. For the simplified model of resonant unit, all parameters are denoted by spring stiffness k_F, mass m_R, torsional spring stiffness k_M and rotational inertia J_R. Moreover, the radius of the soft material is R_s, and the lattice constant is a.

By means of a series of spatial Fourier expansions, finally equation (1) can be expressed by a matrix formulation (see Appendix A for details) as

$$\left(\begin{bmatrix} [UG]+k_F[QG]-k_M[SG] & -k_F[PG] & k_M[RG] \\ -k_F[PG]^T & k_F & 0 \\ k_M[RG]^T & 0 & -k_M \end{bmatrix} -\omega^2\begin{bmatrix} S[FG] & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & m_R & 0 \\ \mathbf{0} & 0 & J_R \end{bmatrix}\right) \times\begin{bmatrix} [W_1(G)] \\ W_R(0) \\ \theta_R(0) \end{bmatrix}=0. \tag{4}$$

Equation (4) represents a generalized eigenvalue problem for ω^2. Finally, the band structure of the proposed simplified model of composite LRPC plate can be obtained by solving the equation for each Bloch wave vector limited in the irreducible first Brillouin zone (1BZ). Besides, all the parameters used in calculations are displayed in Table 1.

3. Numerical results and analyses

3.1. "Spring–mass" simplified model of stubbed-on LRPC plate

If the plate is composed of only one material and the torsional springs are ignored in the simplified model of composite LRPC plate shown in Fig. 1(a), the model can be further reduced to "spring–mass" simplified model of stubbed-on LRPC plate.

Because the plate is composed of one material, equation (A.3) can be simplified as

$$\xi(\mathbf{G})=\begin{cases} \xi_1, & \mathbf{G}=0 \\ 0, & \mathbf{G}\neq 0. \end{cases} \tag{5}$$

Substituting equation (5) into equations (A.10)–(A.15), we obtain that

$$[AG]_{ij}=\begin{cases} D_1(\mathbf{k}+\mathbf{G_i})_x^4, & i=j \\ 0, & i\neq j, \end{cases} \tag{6}$$

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