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[Physics Letters A](http://dx.doi.org/10.1016/j.physleta.2017.08.019) ••• (••••) •••-•••

Contents lists available at [ScienceDirect](http://www.ScienceDirect.com/) 1 67

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20 and the contract of the con 21 ANIILLE INFO ADSINALI A R T I C L E I N F O A B S T R A C T

22 88 *Article history:* Received 1 May 2017 Received in revised form 5 August 2017 Accepted 7 August 2017 Available online xxxx Communicated by P.R. Holland *Keywords:* Nonclassicality

Wigner function

- Higher-order squeezing
- Higher-order antibunching Non-Gaussian states

33 Nonclassical volume 99 Nonclassical volume

23 Article history:
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sub-Poissonian photon statistics, higher-order antibunching. It is observed that both photon added and 25 Accepted 7 August 2017
subtracted squeezed coherent states are highly nonclassical as they satisfy criteria for all of the above 26 Available online xxx
Communicated by B.P. Holland and a set of other criteria including negativity of Wigner function, Klyshko's ⁹² 27 Criterion and Agarwal's (A₃) parameter. Further, the amount of nonclassicality present in these two types 93 28 94 of states has been compared quantitatively using a measure of nonclassicality known as nonclassical 29 95 volume. Variation in the amount of nonclassicality with the number of photon(s) added/subtracted is 30 96 also investigated, and it is found that the addition of photons makes the squeezed coherent state more 31 Higher-order squeezing **than 200 starts of the subtraction** of photons. with specific focus on the higher-order nonclassicalities, such as higher-order squeezing, higher-order

32 by the order antibunching
32 by September 2017 Published by Elsevier B.V. 98

1. Introduction

40 A state that cannot be expressed as a mixture of coherent states to be essential for the realization of protocols for discrete [\[13\]](#page--1-0) and 106 ⁴¹ is referred to as a nonclassical state, and usually negativity of continuous variable quantum cryptography [9], quantum telepor- 107 42 Glauber–Sudarshan *P*-function defines a nonclassical state [\[1,2\].](#page--1-0) tation [14], dense-coding [15]. As a consequence of these useful the 43 However, P-function cannot be measured directly in any experi- developments, in the recent years, nonclassical states of light have 105 ⁴⁴ ment. This fact led to the construction of several other operational been recognized as a valuable resource in quantum information 110 45 111 criteria for the detection of nonclassicality, such as negativity of 46 Wigner function [3], zeros of Q-function [4], and a set of moment been proposed to generate nonclassical states. Two basic opera- 112 47 based criteria [5]. These operational criteria are only sufficient for tions that can lead to nonclassical states are adding photons to 113 ⁴⁸ the detection of nonclassicality. Thus, any of these criteria would or subtracting photons from a quantum state [\[16\].](#page--1-0) In what fol-
114 ⁴⁹ fail to detect the presence of nonclassicality in some cases. Still lows, we aim to investigate the effect of these two processes on 115 ⁵⁰ these criteria are used for long to study the possibilities of gen-
⁵⁰ the squeezed coherent states. ⁵¹ eration of nonclassical states and to propose their applications ^{Today}, there exists a rich variety of nonclassical states of light, 117 52 (see [\[6\]](#page--1-0) and references therein). However, the interest on nonclas-charge there in reality or (at least) in principle [6]. Our objective is to the 53 sical states has been intensified in the recent past with the intro- search for nonclassicality in these states or superpositions of these 119 54 duction of several new applications of nonclassical states $[7-15]$. States so that the states can be used for the applications men- 120 ⁵⁵ For example, applications of squeezed state are reported in the de-**the fill and the State and State and State** Menonclassical 121 56 tection of gravitational waves in the famous LIGO experiment $[7,8]$ hature of light present in photon added/subtracted squeezed co- 122 57 and in the realization of schemes for continuous variable quan- herent states through some particular manifestations of the non- 123 $\,$ 58 tum key distribution [\[9\]](#page--1-0) in particular, and schemes for continuous classical nature of light, such as lower- and higher-order pho- 124 59 variable quantum communication in general [10], teleportation of ton antibunching, sub-Poissonian distribution of photon numbers, 125 A state that cannot be expressed as a mixture of coherent states However, *P* -function cannot be measured directly in any experiment. This fact led to the construction of several other operational Wigner function [\[3\],](#page--1-0) zeros of *Q* -function [\[4\],](#page--1-0) and a set of moment based criteria [\[5\].](#page--1-0) These operational criteria are only sufficient for the detection of nonclassicality. Thus, any of these criteria would fail to detect the presence of nonclassicality in some cases. Still variable quantum communication in general [\[10\],](#page--1-0) teleportation of

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64 130 <http://dx.doi.org/10.1016/j.physleta.2017.08.019>

 65 $0375-9601$ / \odot 2017 Published by Elsevier B.V.

38 **1. Introduction coherent states [\[11\];](#page--1-0) antibunching is reported to be useful in char-** 104 39 105 acterizing single photon sources [\[12\],](#page--1-0) entangled states are found continuous variable quantum cryptography [\[9\],](#page--1-0) quantum teleportation $[14]$, dense-coding $[15]$. As a consequence of these useful processing and quantum optics. Many experimental schemes have the squeezed coherent states.

60 126 squeezing and negativity of the Wigner function, Klyshko's criteria 61 127 [\[17\]](#page--1-0) and Agarwal's *A*³ parameter [\[18\].](#page--1-0) Today, there exists a rich variety of nonclassical states of light, either in reality or (at least) in principle $[6]$. Our objective is to search for nonclassicality in these states or superpositions of these states so that the states can be used for the applications mentioned above. Specifically, we would like to study the nonclassical nature of light present in photon added/subtracted squeezed coherent states through some particular manifestations of the nonclassical nature of light, such as lower- and higher-order pho-

62 62 62 E-mail address: anirban.pathak@jiit.ac.in (A. Pathak). The studies on nonclassical states of light, operator ordering 128 63 129 is considered to be one of the fundamental tasks as it plays a cru-

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E-mail address: anirban.pathak@jiit.ac.in (A. Pathak).

15 81 nonclassical phenomena. Earlier, some efforts had been made to ¹⁶ study the lower-order nonclassical properties of PSSCS [\[22\].](#page--1-0) How- **2. Quantum states of our interest 162 182** 17 83 ever, to the best of our knowledge, neither the existence of higher-¹⁸ order nonclassicality in PSSCS and PASCS, nor the variation of This paper is focused on two types of quantum states, namely 84 19 amount of nonclassicality present in them and reflected through a FASCS and PSSCS. Before, we investigate nonclassical properties of 85 20 86 quantitative measure of nonclassicality have yet been investigated. 21 A set of recent studies on higher-order nonclassicalities [\[23–29\]](#page--1-0) begin with let us define PASCS. 22 88 have strongly established their relevance, especially in detecting ²³ weak nonclassicalities. The effect of addition and/or subtraction μ . Definition of PASCS 24 of photon(s) on the higher-order nonclassicalities have been stud-
24 of photon(s) on the higher-order nonclassicalities have been stud-25 ied earlier in the context of photon added coherent state $[30-32]$ The PASCS is defined as ²⁶ and a non-Gaussian entangled state generated by adding photons $\frac{1}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ 27 to both modes of a two-mode squeezed coherent state [\[33\].](#page--1-0) Pos- $\langle \Psi \rangle = N + \langle \alpha, z, m \rangle u \quad |\alpha, z \rangle$, ²⁸ sibility of observing higher-order nonclassical effects through the where $|\alpha, z\rangle = D(\alpha)S(z)|0\rangle$ is the squeezed coherent state (SCS), ⁹⁴ ²⁹ addition of photons to coherent or thermal state was investigated m is the number of photons added, |0\ is the vacuum state, ⁹⁵ 30 in [\[34\].](#page--1-0) A similar study on the effect of addition/subtraction of $D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^* a)$ is the displacement operator, and $S(z) = 96$ 31 photon(s) on the lower-order nonclassicality has been reported for $\exp(\frac{1}{2}z a^{\dagger}^2 - \frac{1}{2}z^* a^2)$ is the squeeze operator. Here, $z = r \exp(i\phi)$, $\frac{97}{9}$ ³² two-mode photon-added displaced squeezed states [\[35\],](#page--1-0) Gaussian where r is the squeezing parameter and ϕ is the squeeze angle 33 entangled state [\[36\],](#page--1-0) squeezed coherent state [\[22\],](#page--1-0) and two-mode The normalization constant N (α z m) is determined from the ⁹⁹ a comparison of nonclassicality between photon-added squeezed state (PSSCS). In the similar line, in [\[20\],](#page--1-0) a comparison of nonclasaddition of photons to coherent or thermal state was investigated photon(s) on the lower-order nonclassicality has been reported for squeezed vacuum state [\[37\].](#page--1-0)

35 In $[36,37]$ and in most of the other studies mentioned here, α obtain ³⁶ photon addition, photon subtraction, and their coherent superpo-**102** photon and their coherent superpo-³⁷ sition are considered as primary non-Gaussian operations (at the $\frac{N}{\sqrt{N}}$ ($\frac{2}{N}$ and $\frac{2}{N}$ and $\frac{2}{N}$ and $\frac{2}{N}$ and $\frac{2}{N}$ and $\frac{2}{N}$ and $\frac{2}{N}$ ($\frac{m!}{N}$ and $\frac{2}{N}$ and $\frac{2}{N}$ and ³⁸ single photon level) that form one of the most powerful tools for $(1 + (\alpha, 2, m) - 1)$ (4) (4) (4) (6) (1) 39 controlling nonclassicality in general, and enhancement of entan-⁴⁰ glement in particular. Here, we report the role of these two types $\frac{1}{2}$ ⁴¹ of operations in controlling nonclassicality in PASCS and PSSCS $\times |H_{m-l}(A)|^2$, (2) ¹⁰⁷ 42 108 with special attention towards higher-order nonclassicality. Apart 43 from the recently reported applications of the nonclassical states, α are α a ⁴⁵ Iron the recently reported applications of the honclassical states,

⁴⁴ the task is interesting because of the possibility that the states where *A* = *i*α exp(−*iφ/2*), and *H_n*(*x*) is the Hermite polynomial. 45 studied here can be experimentally realized. This is so because, Now the validity of the analytic expression of $N_+(\alpha, z, m)$ can 111 46 in [\[38,39\]](#page--1-0) a photon added coherent state (PACS) proposed by Agar- be established by noting that in the limiting cases $z=0$ and $z=12$ 47 wal and Tara [\[40\]](#page--1-0) was experimentally generated, and thus the basic $\alpha=0$ it reduces to the normalization constant for photon added 113 ⁴⁸ non-Gaussian operation required for the generation of PASCS and coherent state [40] $N_{\perp}(\alpha,0,m) = [m!I_{\perp}(\alpha-|\alpha|^2)]^{-1/2}$ and that 114 ⁴⁹ PSSCS is already achieved. With the successful experimental real-
of the photon added squeezed vacuum state [44] N₁ (0, z, m) = 1¹⁵ 50 ization of a set of techniques and concepts that involve photon $[m(\cosh r)^m P - (\cosh r)^{-1/2}]$ respectively. Here $I(\alpha)$ and $P(\alpha) = 116$ ⁵¹ addition and other non-Gaussian operations (see [\[41,42\],](#page--1-0) and [\[43\]](#page--1-0) $\frac{1}{2}$ are the Laguerre and Legendre polynomials, respectively, $\frac{117}{2}$ ⁵² and references therein), recently interest on the properties and ap-
 $\frac{118}{2}$ and references therein), recently interest on the properties and ap-
 $\frac{118}{2}$ and similar fashion, we may define PSSCS as follows. 53 119 plications of non-Gaussian states, like PASCS and PSSCS have been 54 increased considerably. The construction of the construction of PSSCS and the construction of PSSCS the task is interesting because of the possibility that the states studied here can be experimentally realized. This is so because, non-Gaussian operation required for the generation of PASCS and PSSCS is already achieved. With the successful experimental realization of a set of techniques and concepts that involve photon

55 121 The remaining part of the paper is organized as follows. In ⁵⁶ Sec. 2, we introduce the quantum states of our interest (i.e., PASCS The PSSCS is defined as 57 123 and PSSCS) and obtain analytic expressions for the required nor-⁵⁸ malization constants. In Sec. [3,](#page--1-0) we report two closed form analytic $|\psi\rangle = N_{-}(\alpha, z, m) \ddot{a}''|\alpha, z\rangle$, (3) ¹²⁴ 59 expressions for $\langle a^{\dagger}p q^q \rangle$, where the expectation values are com-
⁵⁹ expressions for $\langle a^{\dagger}p q^q \rangle$, where the expectation values are com- 60 puted with respect to (a) PASCS and (b) PSSCS. In the subsequent 60 and 500 and 600 and 6 ⁶⁵ in Sec. [3](#page--1-0) to study the existence of higher-order antibunching and $\times |H_{\text{max}}(A)|^2$ (4) ¹³¹ 66 higher-order sub-Poissonian photon statistics in PASCS and PSSCS. The contract of the contr

¹ cial role in obtaining expressions for various states of light and also Subsequently, in Sec. 5, we report photon number distribution for ⁶⁷ ² in calculating the expectation values of the operators with respect PASCS and PSSCS and demonstrate that they satisfy Klyshko's crite- ⁶⁸ 3 to these states. A simple, unified approach for arranging quan- $\;$ rion for nonclassicality, which requires very limited knowledge of $\;$ 69 $\;$ ⁴ tum operators into ordered products (normal ordering, antinormal photon number distribution. Nonclassical properties of PASCS and ⁷⁰ 5 71 ordering, Weyl ordering) is the technique of *integration within an or-*⁶ dered product (IWOP) [\[19\].](#page--1-0) This technique is used here to perform and the existence of Hong–Mandel type higher-order squeezing in ⁷² ⁷ a comparison of nonclassicality between photon-added squeezed these states are reported in Sec. [7.](#page--1-0) In Sec. [8,](#page--1-0) we first report analytic 73 ⁸ coherent state (PASCS) and photon-subtracted squeezed coherent expressions for Wigner function for PASCS and PSSCS and investi- 74 ⁹ state (PSSCS). In the similar line, in [20], a comparison of nonclas- gate the existence of nonclassicality in these states through the ⁷⁵ ¹⁰ sicality between photon-added and photon-subtracted squeezed negative values of the Wigner function, then we provide a quan- ⁷⁶ ¹¹ vacuum states was performed. In Ref. [\[21\]](#page--1-0) nonclassical properties titative measure of nonclassicality present in these states using 77 ¹² of photon subtracted squeezed state was also studied. The present nonclassical volume and use the same for comparison of nonclassi- 78 ¹³ study aims to conduct a similar but much more rigorous compari- cality present in these states as the number of photons added/sub- ⁷⁹ ¹⁴ son between PASCS and PSSCS with primary focus on higher-order tracted is varied. Finally, the paper is concluded in Sec. 9. $\frac{80}{2}$ Subsequently, in Sec. [5,](#page--1-0) we report photon number distribution for PASCS and PSSCS and demonstrate that they satisfy Klyshko's criterion for nonclassicality, which requires very limited knowledge of photon number distribution. Nonclassical properties of PASCS and PSSCS reflected through Agarwal's criterion are reported in Sec. [6](#page--1-0) and the existence of Hong–Mandel type higher-order squeezing in expressions for Wigner function for PASCS and PSSCS and investinegative values of the Wigner function, then we provide a quantitative measure of nonclassicality present in these states using nonclassical volume and use the same for comparison of nonclassicality present in these states as the number of photons added/subtracted is varied. Finally, the paper is concluded in Sec. [9.](#page--1-0)

2. Quantum states of our interest

This paper is focused on two types of quantum states, namely PASCS and PSSCS. Before, we investigate nonclassical properties of these states, it would be apt to properly define these states. To begin with let us define PASCS.

2.1. Definition of PASCS

The PASCS is defined as

$$
|\psi\rangle = N_{+}(\alpha, z, m) \hat{a}^{\dagger m} |\alpha, z\rangle, \qquad (1)
$$

34 squeezed vacuum state [37]. μ **hormalization condition** $\langle \psi | \psi \rangle = 1$. Using the IWOP technique, we where $|\alpha, z\rangle = D(\alpha)S(z)|0\rangle$ is the squeezed coherent state (SCS), *D*(α) = exp($\alpha a^{\dagger} - \alpha^* a$) is the displacement operator, and *S*(*z*) = where *r* is the squeezing parameter and ϕ is the squeeze angle. The normalization constant $N_+(\alpha, z, m)$ is determined from the obtain

$$
N_{+} (\alpha, z, m) = \left[\left(\frac{\sinh 2r}{4} \right)^{m} \sum_{l=0}^{m} \frac{(m!)^{2} (2 \coth r)^{l}}{l! \{(m-l)!\}^{2}} \times |H_{m-l}(A)|^{2} \right]^{-1/2},
$$
\n(2)

be established by noting that in the limiting cases $z = 0$ and $\alpha = 0$ it reduces to the normalization constant for photon added coherent state $[40]$ $N_+(\alpha, 0, m) = [m!L_m(-|\alpha|^2)]^{-1/2}$ and that of the photon added squeezed vacuum state $[44]$ N_+ $(0, z, m)$ = $[m!(\cosh r)^m P_m(\cosh r)]^{-1/2}$, respectively. Here, $L_n(x)$ and $P_n(x)$ are the Laguerre and Legendre polynomials, respectively.

In a similar fashion, we may define PSSCS as follows.

2.2. Definition of PSSCS

The PSSCS is defined as

$$
|\psi\rangle = N_{-}(\alpha, z, m)\,\hat{a}^{m}|\alpha, z\rangle,\tag{3}
$$

where $N = (\alpha, z, m)$ is the normalization constant,

61 sections these expressions are used to investigate the existence of nonclassically using various criteria that are based on moments of the annihilation and creation operators. Specifically, in Sec. 4, we use a set of moment-based criteria and the expressions obtained is:\n
$$
N = \left[\left(\frac{\sinh 2r}{4} \right)^m \sum_{l=0}^m \frac{(m!)^2 (2 \tanh r)^l}{l! \left((m-l)! \right)^2} \right]
$$
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$$
N = \left[\left(\frac{\sinh 2r}{4} \right)^m \sum_{l=0}^m \frac{(m!)^2 (2 \tanh r)^l}{l! \left((m-l)! \right)^2} \right]
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N = \left[\left(\frac{\sinh 2r}{4} \right)^m \sum_{l=0}^m \frac{(m!)^2 (2 \tanh r)^l}{l! \left((m-l)! \right)^2} \right]
$$
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$$
N = \left(\frac{2 \tanh r}{4} \right)^{1/2} \left(\frac{2 \tanh r}{4} \right)
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$$
\times \left|H_{m-l}(A)\right|^2 \qquad (4) \qquad \frac{1}{1}
$$

Please cite this article in press as: K. Thapliyal et al., Comparison of lower- and higher-order nonclassicality in photon added and subtracted squeezed coherent states, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2017.08.019

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