Contents lists available at ScienceDirect
Physics Letters A
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Comparison of lower- and higher-order nonclassicality in photon added and subtracted squeezed coherent states

Kishore Thapliyal [a], Nigam Lahiri Samantray [b,1], J. Banerji [b], Anirban Pathak [a]

[a] Jaypee Institute of Information Technology, A-10, Sector 62, Noida, UP 201307, India
[b] Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

ARTICLE INFO

Article history:
Received 1 May 2017
Received in revised form 5 August 2017
Accepted 7 August 2017
Available online xxxx
Communicated by P.R. Holland

Keywords:
Nonclassicality
Wigner function
Higher-order squeezing
Higher-order antibunching
Non-Gaussian states
Nonclassical volume

ABSTRACT

Nonclassical properties of photon added and subtracted squeezed coherent states have been compared with specific focus on the higher-order nonclassicalities, such as higher-order squeezing, higher-order sub-Poissonian photon statistics, higher-order antibunching. It is observed that both photon added and subtracted squeezed coherent states are highly nonclassical as they satisfy criteria for all of the above mentioned nonclassicalities and a set of other criteria including negativity of Wigner function, Klyshko's criterion and Agarwal's (A_3) parameter. Further, the amount of nonclassicality present in these two types of states has been compared quantitatively using a measure of nonclassicality known as nonclassical volume. Variation in the amount of nonclassicality with the number of photon(s) added/subtracted is also investigated, and it is found that the addition of photons makes the squeezed coherent state more nonclassical than what is done by the subtraction of photons.

1. Introduction

A state that cannot be expressed as a mixture of coherent states is referred to as a nonclassical state, and usually negativity of Glauber–Sudarshan P-function defines a nonclassical state [1,2]. However, P-function cannot be measured directly in any experiment. This fact led to the construction of several other operational criteria for the detection of nonclassicality, such as negativity of Wigner function [3], zeros of Q-function [4], and a set of moment based criteria [5]. These operational criteria are only sufficient for the detection of nonclassicality. Thus, any of these criteria would fail to detect the presence of nonclassicality in some cases. Still these criteria are used for long to study the possibilities of generation of nonclassical states and to propose their applications (see [6] and references therein). However, the interest on nonclassical states has been intensified in the recent past with the introduction of several new applications of nonclassical states [7–15]. For example, applications of squeezed state are reported in the detection of gravitational waves in the famous LIGO experiment [7,8] and in the realization of schemes for continuous variable quantum key distribution [9] in particular, and schemes for continuous variable quantum communication in general [10], teleportation of coherent states [11]; antibunching is reported to be useful in characterizing single photon sources [12], entangled states are found to be essential for the realization of protocols for discrete [13] and continuous variable quantum cryptography [9], quantum teleportation [14], dense-coding [15]. As a consequence of these useful developments, in the recent years, nonclassical states of light have been recognized as a valuable resource in quantum information processing and quantum optics. Many experimental schemes have been proposed to generate nonclassical states. Two basic operations that can lead to nonclassical states are adding photons to or subtracting photons from a quantum state [16]. In what follows, we aim to investigate the effect of these two processes on the squeezed coherent states.

Today, there exists a rich variety of nonclassical states of light, either in reality or (at least) in principle [6]. Our objective is to search for nonclassicality in these states or superpositions of these states so that the states can be used for the applications mentioned above. Specifically, we would like to study the nonclassical nature of light present in photon added/subtracted squeezed coherent states through some particular manifestations of the nonclassical nature of light, such as lower- and higher-order photon antibunching, sub-Poissonian distribution of photon numbers, squeezing and negativity of the Wigner function, Klyshko's criteria [17] and Agarwal's A_3 parameter [18].

In the studies on nonclassical states of light, operator ordering is considered to be one of the fundamental tasks as it plays a cru-

E-mail address: anirban.pathak@jiit.ac.in (A. Pathak).
[1] Presently at: INRIM, Strada delle Cacce 91, I-10135 Torino, Italy.

http://dx.doi.org/10.1016/j.physleta.2017.08.019
0375-9601/© 2017 Published by Elsevier B.V.

cial role in obtaining expressions for various states of light and also in calculating the expectation values of the operators with respect to these states. A simple, unified approach for arranging quantum operators into ordered products (normal ordering, antinormal ordering, Weyl ordering) is the technique of *integration within an ordered product* (IWOP) [19]. This technique is used here to perform a comparison of nonclassicality between photon-added squeezed coherent state (PASCS) and photon-subtracted squeezed coherent state (PSSCS). In the similar line, in [20], a comparison of nonclassicality between photon-added and photon-subtracted squeezed vacuum states was performed. In Ref. [21] nonclassical properties of photon subtracted squeezed state was also studied. The present study aims to conduct a similar but much more rigorous comparison between PASCS and PSSCS with primary focus on higher-order nonclassical phenomena. Earlier, some efforts had been made to study the lower-order nonclassical properties of PSSCS [22]. However, to the best of our knowledge, neither the existence of higher-order nonclassicality in PSSCS and PASCS, nor the variation of amount of nonclassicality present in them and reflected through a quantitative measure of nonclassicality have yet been investigated. A set of recent studies on higher-order nonclassicalities [23–29] have strongly established their relevance, especially in detecting weak nonclassicalities. The effect of addition and/or subtraction of photon(s) on the higher-order nonclassicalities have been studied earlier in the context of photon added coherent state [30–32] and a non-Gaussian entangled state generated by adding photons to both modes of a two-mode squeezed coherent state [33]. Possibility of observing higher-order nonclassical effects through the addition of photons to coherent or thermal state was investigated in [34]. A similar study on the effect of addition/subtraction of photon(s) on the lower-order nonclassicality has been reported for two-mode photon-added displaced squeezed states [35], Gaussian entangled state [36], squeezed coherent state [22], and two-mode squeezed vacuum state [37].

In [36,37] and in most of the other studies mentioned here, photon addition, photon subtraction, and their coherent superposition are considered as primary non-Gaussian operations (at the single photon level) that form one of the most powerful tools for controlling nonclassicality in general, and enhancement of entanglement in particular. Here, we report the role of these two types of operations in controlling nonclassicality in PASCS and PSSCS with special attention towards higher-order nonclassicality. Apart from the recently reported applications of the nonclassical states, the task is interesting because of the possibility that the states studied here can be experimentally realized. This is so because, in [38,39] a photon added coherent state (PACS) proposed by Agarwal and Tara [40] was experimentally generated, and thus the basic non-Gaussian operation required for the generation of PASCS and PSSCS is already achieved. With the successful experimental realization of a set of techniques and concepts that involve photon addition and other non-Gaussian operations (see [41,42], and [43] and references therein), recently interest on the properties and applications of non-Gaussian states, like PASCS and PSSCS have been increased considerably.

The remaining part of the paper is organized as follows. In Sec. 2, we introduce the quantum states of our interest (i.e., PASCS and PSSCS) and obtain analytic expressions for the required normalization constants. In Sec. 3, we report two closed form analytic expressions for $\langle a^{\dagger p} a^q \rangle$, where the expectation values are computed with respect to (a) PASCS and (b) PSSCS. In the subsequent sections these expressions are used to investigate the existence of nonclassicality using various criteria that are based on moments of the annihilation and creation operators. Specifically, in Sec. 4, we use a set of moment-based criteria and the expressions obtained in Sec. 3 to study the existence of higher-order antibunching and higher-order sub-Poissonian photon statistics in PASCS and PSSCS. Subsequently, in Sec. 5, we report photon number distribution for PASCS and PSSCS and demonstrate that they satisfy Klyshko's criterion for nonclassicality, which requires very limited knowledge of photon number distribution. Nonclassical properties of PASCS and PSSCS reflected through Agarwal's criterion are reported in Sec. 6 and the existence of Hong–Mandel type higher-order squeezing in these states are reported in Sec. 7. In Sec. 8, we first report analytic expressions for Wigner function for PASCS and PSSCS and investigate the existence of nonclassicality in these states through the negative values of the Wigner function, then we provide a quantitative measure of nonclassicality present in these states using nonclassical volume and use the same for comparison of nonclassicality present in these states as the number of photons added/subtracted is varied. Finally, the paper is concluded in Sec. 9.

2. Quantum states of our interest

This paper is focused on two types of quantum states, namely PASCS and PSSCS. Before, we investigate nonclassical properties of these states, it would be apt to properly define these states. To begin with let us define PASCS.

2.1. Definition of PASCS

The PASCS is defined as

$$|\psi\rangle = N_+(\alpha, z, m)\,\hat{a}^{\dagger m}|\alpha, z\rangle, \tag{1}$$

where $|\alpha, z\rangle = D(\alpha)S(z)|0\rangle$ is the squeezed coherent state (SCS), m is the number of photons added, $|0\rangle$ is the vacuum state, $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ is the displacement operator, and $S(z) = \exp(\frac{1}{2}z a^{\dagger 2} - \frac{1}{2}z^* a^2)$ is the squeeze operator. Here, $z = r\exp(i\phi)$, where r is the squeezing parameter and ϕ is the squeeze angle. The normalization constant $N_+(\alpha, z, m)$ is determined from the normalization condition $\langle\psi|\psi\rangle = 1$. Using the IWOP technique, we obtain

$$N_+(\alpha, z, m) = \left[\left(\frac{\sinh 2r}{4}\right)^m \sum_{l=0}^{m} \frac{(m!)^2\,(2\coth r)^l}{l!\{(m-l)!\}^2} \times \left|H_{m-l}(A)\right|^2\right]^{-1/2}, \tag{2}$$

where $A = i\frac{\alpha\exp(-i\phi/2)}{\sqrt{\sinh 2r}}$, and $H_n(x)$ is the Hermite polynomial. Now the validity of the analytic expression of $N_+(\alpha, z, m)$ can be established by noting that in the limiting cases $z = 0$ and $\alpha = 0$ it reduces to the normalization constant for photon added coherent state [40] $N_+(\alpha, 0, m) = \left[m! L_m\left(-|\alpha|^2\right)\right]^{-1/2}$ and that of the photon added squeezed vacuum state [44] $N_+(0, z, m) = \left[m!\,(\cosh r)^m\,P_m(\cosh r)\right]^{-1/2}$, respectively. Here, $L_n(x)$ and $P_n(x)$ are the Laguerre and Legendre polynomials, respectively.

In a similar fashion, we may define PSSCS as follows.

2.2. Definition of PSSCS

The PSSCS is defined as

$$|\psi\rangle = N_-(\alpha, z, m)\,\hat{a}^{m}|\alpha, z\rangle, \tag{3}$$

where $N_-(\alpha, z, m)$ is the normalization constant,

$$N_-(\alpha, z, m) = \left[\left(\frac{\sinh 2r}{4}\right)^m \sum_{l=0}^{m} \frac{(m!)^2\,(2\tanh r)^l}{l!\{(m-l)!\}^2} \times \left|H_{m-l}(A)\right|^2\right]^{-1/2}. \tag{4}$$

Please cite this article in press as: K. Thapliyal et al., Comparison of lower- and higher-order nonclassicality in photon added and subtracted squeezed coherent states, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2017.08.019