Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Anisotropic fractal model for the effective thermal conductivity of random metal fiber porous media with high porosity

Hangming Shen, Qian Ye*, Guoxiang Meng

School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China

A R T I C L E I N F O

Article history:
Received 21 June 2017
Received in revised form 17 July 2017
Accepted 1 August 2017
Available online xxxx
Communicated by M. Wu

Keywords:
Fractal
Random fiber porous media
Effective thermal conductivity
High porosity

A B S T R A C T

An anisotropic fractal model for the effective thermal conductivity (ETC) of random metal fiber porous media saturated in a low thermal conductivity fluid is proposed in this paper. The representative elementary volume (REV) was introduced. The cross section and the tortuous fiber in REV were considered as a fractal surface and a fractal curve respectively. The thermal tortuosity, maximum and minimum ETCs of REV were derived based on the fractal description under the assumption of one-dimensional heat conduction along the tortuous fractal fiber. The space angles between the REV and the heat flow direction were determined based on the experimental data to deduce the anisotropic ETC model of random metal fiber porous media with high porosity. The model prediction results have good agreement with the experimental data in literature.

1. Introduction

The random metal fiber porous media with high porosity have many advantages and have received attracted attention in recent years [1–3]. They have large specific surface area and can be regarded as one kind of heat transfer enhancement materials [4,5], such as the fine copper wire (diameter of 50 μm) which can be filled in the isothermal chamber to make the air temperature in the chamber almost constant during charging or discharging [6,7]. The ETC is considered as the most important thermophysical property in characterizing the heat transfer of porous media. Recently, many researchers used the formula $k_e = (1-\varphi)k_s + \varphi k_f$ to calculate the ETC of porous media in the heat transfer models [6,8,9], but it would affect the accuracy of the models. Therefore, the accurate evaluation of ETC is an important prerequisite to study the mechanism of heat transfer enhancement.

Due to the structural complexity of fiber porous media, the microstructure is difficult to describe so that it limited the study of ETC. Lee et al. [10] presented an improved theoretical model for the prediction of radiative properties in high porosity fiber thermal insulations. Qu et al. [2] proposed an ETC model of consolidated porous materials saturated with fluid under the assumption of octet-truss lattice unit cell, but this model was isotropic. Li et al. [11] studied the ETC of screen mesh layers and an analytical model depending on the contact conditions, mesh number and wire diameter was developed. Haruki et al. [12] studied on the ETC of various kinds of metal fiber materials experimentally. Several researchers [1,2,13,14] studied the ETC of sintered metal fibers in vacuum or saturated in low thermal conductivity fluid experimentally. However, the ETCs are in low porosity conditions. Zhao and Zhang et al. [15,16] studied on the ETC of high-alumina fibrous insulation under the condition of high temperature and low pressure numerically and experimentally, but the change of porosity was not considered. Wang et al. [17] used the lattice Boltzmann numerical method to study the ETC of general fibrous material.

In conclusion, although there are still some studies on the fiber porous media, the studies on the random metal fiber porous media with high porosity (>0.9) are few. Furthermore, there is little information on the characterization of microstructure for the fiber porous media. Numerical methods can realize microstructure reconstruction but they entail considerable computational burden with limited universal characteristics. In this paper, the REV was defined and the fractal theory was used to describe the microstructure of random fibers in the REV. The space angles between the REV and the heat flow direction were determined to deduce the anisotropic ETC of random metal fiber porous media. The fractal ETC model was validated by the experimental data from the present study and the open literature. It is concluded that the anisotropic fractal model proposed in this paper is accurate and universal.

* Corresponding author.
E-mail address: yeqian@sjtu.edu.cn (Q. Ye).

http://dx.doi.org/10.1016/j.physleta.2017.08.003
0375-9601/

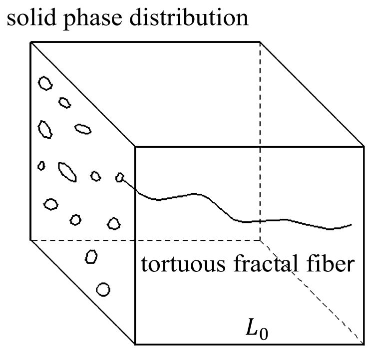

Fig. 1. The schematic diagram of REV.

2. Anisotropic fractal model

2.1. The fractal description of REV

A cubic REV (representative elementary volume) with side length of L_0 is defined. All the fibers in REV extend through the entire REV from the left surface to the right surface. The solid phase (fibers) distribution in the cross section and fiber tortuous path in the REV are illustrated in Fig. 1.

Yu et al. [18] used fractal theory to describe the relationship between the number and size of hole in the porous media, then established a fractal model of granular porous media. By analogy, the solid phase distribution in the cross section of REV also has the following fractal description.

$$N(>\lambda) = \left(\frac{\lambda_{\max}}{\lambda}\right)^{D_f} \tag{1}$$

where λ is the equivalent diameter of fiber in the cross section, $\lambda_{\max}$ is the maximum equivalent diameter, N is the number of fibers with equivalent diameter greater than λ, and D_f is the fractal dimension about cross section.

Differential for λ, the number of fibers with equivalent diameter between λ and $\lambda + d\lambda$ in the cross section were obtained.

$$dN(\lambda) = D_f \lambda_{\max}^{D_f} \lambda^{-D_f-1} d\lambda \tag{2}$$

The fractal path of the tortuous fiber can be described as follows [19,20]:

$$L(\lambda) = L_0{}^{D_T} \lambda^{1-D_T} \tag{3}$$

where $L(\lambda)$ is the fiber tortuous length along the heat flow direction, L_0 is the representative length which is also the side length of REV, and D_T is the fractal dimension that characterizes the fiber tortuous length.

The area of solid phase in the cross section is

$$A_p = \int_{\lambda_{\min}}^{\lambda_{\max}} \frac{\pi\lambda^2}{4} dN(\lambda) = \frac{\pi D_f \lambda_{\max}^{D_f}(\lambda_{\max}^{2-D_f} - \lambda_{\min}^{2-D_f})}{4(2-D_f)} \tag{4}$$

The area of cross section in the REV is

$$A = L_0^2 = \frac{A_p}{1-\varphi} \tag{5}$$

where φ is the porosity.

2.2. The ETC of REV

According to Fourier's law, the total heat applied to REV can be expressed as

$$Q_t = k_R A \frac{\Delta T}{L_0} \tag{6}$$

where k_R is the ETC of REV, and ΔT is the temperature difference the left surface and the right surface.

The one-dimensional heat conduction along one tortuous fractal fiber can be expressed as

$$Q_s = k_s \frac{\pi\lambda^2}{4} \frac{\Delta T}{L(\lambda)} \tag{7}$$

where k_s is the thermal conductivity of metal fiber.

The total heat transfer of all fibers in REV can be expressed as

$$Q_S = \int_{\lambda_{\min}}^{\lambda_{\max}} Q_s\left[dN(\lambda)\right] = \frac{k_s \pi \Delta T D_f \lambda_{\max}^{D_f}(\lambda_{\max}^{D_T-D_f+1} - \lambda_{\min}^{D_T-D_f+1})}{4L_0{}^{D_T}(D_T - D_f + 1)} \tag{8}$$

The total heat transfer of REV is conducted by solid-phase (fibers) heat and fluid-phase (air) heat.

$$Q_t = Q_S + Q_f \tag{9}$$

The fluid-phase heat can be expressed as

$$Q_f = k_f(\varphi A)\frac{\Delta T}{L_0} \tag{10}$$

where k_f is the thermal conductivity of air.

Put Eqs. (6), (8), (10) into (9).

$$k_R = \frac{k_s \pi D_f \lambda_{\max}^{D_f}(\lambda_{\max}^{D_T-D_f+1} - \lambda_{\min}^{D_T-D_f+1})}{4AL_0{}^{D_T-1}(D_T - D_f + 1)} + \varphi k_f \tag{11}$$

Then put Eq. (5) into (11). The maximum ETC of REV can be calculated by Eq. (12).

$$k_{R(\max)} = X(1-\varphi)k_s + \varphi k_f \tag{12}$$

where $X = \frac{(2-D_f)(\lambda_{\max}^{D_T-D_f+1} - \lambda_{\min}^{D_T-D_f+1})}{L_0{}^{D_T-1}(D_T-D_f+1)(\lambda_{\max}^{2-D_f} - \lambda_{\min}^{2-D_f})}$.

Also, the minimum ETC of REV can be calculated by Eq. (13) [21].

$$k_{R(\min)} = \frac{k_s k_f}{(1-\varphi)k_f + \varphi k_s} \tag{13}$$

Because the thermal conductivity of metal fiber is much larger than that of air, the thermal conductivity of air can be negligible. The maximum ETC of REV can be expressed by Eq. (14).

$$\frac{k_{R(\max)}}{k_s} = X(1-\varphi) \tag{14}$$

The coefficient X in Eq. (14) is the reciprocal of thermal tortuosity, stating the ratio of the distance traveled actually to the straight-line distance [22–24]. For example, the thermal tortuosity in the ETC models of metal foams is 0.33–0.35 [22,25,26].

2.3. Parameter determination

In order to determine the parameter D_f, some certain diameter circles are generated randomly in a plane by using the Matlab. The planes of different porosity can be obtained by controlling the number of circles and these planes can be regarded as the cross sections of REV under different porosity. The cross sections of REV have the property of self-similarity [27] and the fractal dimension D_f under different porosity can be calculated by the box-counting

Please cite this article in press as: H. Shen et al., Anisotropic fractal model for the effective thermal conductivity of random metal fiber porous media with high porosity, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2017.08.003