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Perspective Article

The feedback control research on straight and curved road with car-following model

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ABSTRACT

Taking account of the road consisting of curved part and straight part, an extended car-following model is proposed in this paper. A control signal including the velocity difference between the considered vehicle and the vehicle in front is taken into account. The control theory method is applied into analysis of the stability condition for the model. Numerical simulations are implemented to prove that the stability of the traffic flow strengthens effectively with an increase of the radius of curved road, and the control signal can suppress the traffic congestion. The results are in good agree with the theoretical analysis.

1. Introduction

Traffic jams play a great influence on traffic efficiency and energy economy. In order to alleviate increasingly serious traffic congestion, various traffic models have been built to investigate complex traffic phenomena and reveal the nature of traffic jams by many physicists and scholars. In general, traffic flow models [1–15] have been divided into three types: macroscopic models, microscopic models and mesoscopic models.

Microscopic traffic models [16–25] describe the moving behaviors of considered vehicles in detail, and car-following model is an representative microscopic model. The original car-following model was proposed by Reuschel and Pipes [5] in 1953. Later, Newell [13] put forward a car-following model described by a differential equation, which was called the optimal velocity function. But it had an obvious shortage that they cannot accurately describe the acceleration in the actual traffic. Bando [25] et al. proposed a car-following model named the optimal velocity model (for short, OVM) to overcome the shortcoming of infinite speed in 1995. The OVM is based on the idea that each vehicle has an optimal velocity according to the following distance of the preceding vehicle. Afterwards, many scholars and scientists extended the model with considering various traffic factors [26–31], such as the relative velocity difference and slope.

In 2006, Zhao and Gao [36] studied the coupled map car-following model by means of velocity feedback control. This method is simple and consistent with the reality. In 2007, Han [37] et al. presented modified coupled map car-following model for traffic flow and discovered that the model can effectively ease traffic jams to some degree. Subsequently, Ge [38] pointed out that control method can be applied to car-following model for congested traffic in 2012. Moreover, A.K. Gupta [39] analyzed the effect of feedback control in the lattice hydrodynamic model in 2015. Recently, although the control method [32–39] has been investigated in the coupled map car-following model, the investigations are few in car-following model.

As is known to all, not all roads are straight in the actual traffic situations. Although the highway system can be divided into a series of straight roads approximatively, the curved road is non-ignorable and can not be approximated as straight road. In real traffic flow, it is necessary to consider the effects of straight and curved road together. At the present time, there are few researches studying it by use of control method. So we put forward an extended traffic flow model based on the optimal velocity model considering the curved road. The effects of straight and curved road will be discussed from the viewpoint of control methods.

The rest of the paper is organized as follows. The extended optimal velocity model will be analyzed by control method in section 2. In section 3, a control signal will be added into the extended optimal velocity model and the control method is applied into analyzing the stability conditions. In section 4, the numerical simulations are made to support the theoretical results with and without control signal. The conclusions are given in section 5.

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2. The new extended model and its stability analysis

2.1. The new extended model

The car-following model can describe how vehicles follow one after another on a single lane. The typical OVM is presented as

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = a\left[V^{op}(\Delta x_n(t)) - v_n(t)\right] \tag{1}$$

where $a > 0$ is the sensitivity of a driver; $x_n(t)$ and $v_n(t)$ represent the position and velocity of the nth vehicle; $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$ means the headway difference between the preceding vehicle and the following vehicles;

The optimal velocity function $V^{op}(\Delta x_n(t))$ is presented as

$$V^{op}(\Delta x_n(t)) = \frac{v_{max}}{2}\left[\tanh(\Delta x_n(t) - h_c) + \tanh(h_c)\right] \tag{2}$$

where v_{max} means the maximum velocity, and h_c represents the safety distance.

Base on the OVM, a modified model is given to describe the movement of running vehicles on a curved road section

$$\frac{\mathrm{d}\dot{s}_n(t)}{\mathrm{d}t} = a\left[V^{op}(\Delta s_n(t)) - \dot{s}_n(t)\right] \tag{3}$$

As is well known, the relationship between radius and arc length is given as

$$s_n(t) = r \cdot \theta_n(t), \Delta s_n(t) = r \cdot \Delta\theta_n(t) \tag{4}$$

Equation (1) can be rewritten as follows

$$\frac{\mathrm{d}\dot{\theta}_n(t)}{\mathrm{d}t} = \frac{a}{r}\left[V^{op}(\Delta\theta_n(t) \cdot r) - v_n(t)\right] \tag{5}$$

where $s_n(t)$ and $\dot{s}_n(t)$ are the position and velocity of the nth vehicle on a curved road; $\Delta s_n(t) = s_{n+1}(t) - s_n(t)$ means the headway difference between the preceding vehicle and the following vehicles on the curved road; θ is the radian and r is the radius of the curved road section. Similarly, the optimal velocity function $V^{op}(\Delta s_n(t))$ is presented as

$$V^{op}(r \cdot \Delta\theta_n(t)) = \frac{r\omega_{max}}{2}\left[\tanh(r \cdot \Delta\theta_n(t) - s_c) + \tanh(s_c)\right] \tag{6}$$

where s_c represents the safety distance on a curved road section; ω_{max} means the maximum angular velocity.

Basing on the central force formula [2], ω_{max} is bound up with the friction coefficient

$$m\omega_{max}^2 r = \mu m g \tag{7}$$

where μ means friction coefficient; g is acceleration of gravity.

The ω_{max} is acquired as follow

$$\omega_{max} = \sqrt{\frac{\mu g}{r}} \tag{8}$$

In the real traffic situations the ω_{max} is less than the theoretical result, so a constant coefficient k $(0 < k \le 1)$ is introduced and the OVM is rewritten as

$$V^{op}(r \cdot \Delta\theta_n(t)) = k\frac{\sqrt{\mu g r}}{2}\left[\tanh(r \cdot \Delta\theta_n(t) - s_c) + \tanh(s_c)\right] \tag{9}$$

Considering the influence of complex road conditions composed of straight and curved roadway, the extended car-following model is proposed as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = a[pV^{op}(\Delta x_n(t)) + qV^{op}(r \cdot \Delta\theta_n(t)) - v_n(t)] \tag{10}$$

where p, q $(p + q = 1)$ are the reaction coefficients of straight road and curved road condition respectively. When $p = 1$ and $q = 0$, the proposed model is simplified as OVM.

2.2. Stability analysis

Control method will be applicable to the stability condition of the extended car-following model, the dynamic equation is rewritten as

$$\begin{cases} \frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = a[pV^{op}(\Delta x_n(t)) + qV^{op}(r \cdot \Delta\theta_n(t)) - v_n(t)] \\ \frac{\mathrm{d}\Delta x_n(t)}{\mathrm{d}t} = v_{n+1}(t) - v_n(t) \\ \frac{\mathrm{d}\Delta s_n(t)}{\mathrm{d}t} = (\omega_{n+1}(t) - \omega_n(t)) \times r = k(v_{n+1}(t) - v_n(t)) \end{cases} \tag{11}$$

where $\omega_n(t)$ and $\omega_{n+1}(t)$ mean angular velocity of the nth vehicle at time t and angular velocity of the $n+1th$ vehicle at time t;

We suppose that the leading vehicle is not effected by others and its velocity keeps steady at v_0, then the steady state is given by

$$\begin{bmatrix} v_n(t) & \Delta x_n(t) \end{bmatrix}^T = \begin{bmatrix} v_0 & V^{op^{-1}}(v_0) \end{bmatrix}^T \tag{12}$$

$$\begin{bmatrix} w_n(t) & \Delta s_n(t) \end{bmatrix}^T = \begin{bmatrix} w_0 & V^{op^{-1}}(w_0) \end{bmatrix}^T \tag{13}$$

The error system can be calculated around state (5) as

$$\begin{cases} \frac{\mathrm{d}\delta V_n(t)}{\mathrm{d}t} = a[p\Delta x_n(t)\Lambda_1 + qr\Delta\theta_n(t)\Lambda_2 - \delta v_n(t)] \\ \frac{\mathrm{d}\delta\Delta x_n(t)}{\mathrm{d}t} = \delta v_{n+1}(t) - \delta v_n(t) \\ \frac{\mathrm{d}\delta\Delta s_n(t)}{\mathrm{d}t} = k(\delta v_{n+1}(t) - \delta v_n(t)) \end{cases} \tag{14}$$

where $\delta v_n(t) = v_n(t) - v_0$, $\delta\Delta x_n(t) = \Delta x_n(t) - V^{op^{-1}}(v_0)$, $\delta\Delta s_n(t) = \Delta s_n(t) - \frac{\Delta V^{op^{-1}}(rw_0)}{r}$, partial derivatives $\Lambda_1 = \frac{\mathrm{d}V^{op}(\Delta x_n(t))}{\mathrm{d}\Delta x_n(t)}\big|_{\Delta x_n(t) = V^{op^{-1}}(v_0)}$, and $\Lambda_2 = \frac{\mathrm{d}V^{op}(\Delta s_n(t))}{\mathrm{d}\Delta s_n(t)}\big|_{\Delta s_n(t) = V^{op^{-1}}(w_0)}$.

Laplace transformation, we get

$$\begin{cases} SV_n(s) - V_n(0) = ap\Delta X_n(s)\Lambda_1 + aqr\Delta\theta_n(s)\Lambda_2 - aV_n(s) \\ S\Delta X_n(s) - \Delta X_n(0) = V_{n+1}(s) - V_n(s) \\ S\Delta S_n(s) - \Delta S_n(0) = k(V_{n+1}(s) - V_n(s)) \end{cases} \tag{15}$$

The matrix formulation of governing equations is given as

$$\begin{bmatrix} V_n(s) \\ \Delta X_n(s) \\ \Delta S_n(s) \end{bmatrix} = \begin{bmatrix} s^2 & ap\Lambda_1 s & aq\Lambda_2 s \\ -s & s^2 + as + kaq\Lambda_2 & -aq\Lambda_2 \\ -ks & -kap\Lambda_1 & s^2 + as + ap\Lambda_1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ k \end{bmatrix} \frac{V_{n+1}(s)}{p(s)} \tag{16}$$

where $V_n(s) = L(\delta v_n(t))$, $\Delta X_n(s) = L(\delta\Delta x_n(t))$, $\Delta S_n(s) = L(\delta\Delta s_n(t))$, $L(\cdot)$ is the Laplace transform, s is a complex variable and $p(s) = s^3 + as^2 + (ap\Lambda_1 + kaq\Lambda_2)s$. Then the transfer function can be acquired

$$G(s) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} s^2 & ap\Lambda_1 s & aq\Lambda_2 s \\ -s & s^2 + as + kaq\Lambda_2 & -aq\Lambda_2 \\ -ks & -kap\Lambda_1 & s^2 + as + ap\Lambda_1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ k \end{bmatrix} \frac{1}{p(s)} \tag{17}$$

Taylor's formula is applied into $G(s)$ and the result is accurately presented as follows:

$$G(s) = \frac{ap\Lambda_1 + kaq\Lambda_2}{s^2 + as + ap\Lambda_1 + kaq\Lambda_2} \tag{18}$$

Based on the stability theory, the traffic jam will never happen in the traffic flow system when $p(s)$ is stable and $\|G(s)\|_\infty \le 1$.