Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Second Law based definition of passivity/activity of devices

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ARTICLE INFO

Article history:
Received 25 May 2017
Received in revised form 16 August 2017
Accepted 17 August 2017
Available online xxxx
Communicated by C.R. Doering

Keywords:
Activity
Passivity
Second Law

ABSTRACT

Recently, our efforts to clarify the old question, if a memristor is a passive or active device [1], triggered debates between engineers, who have had advanced definitions of passivity/activity of devices, and physicists with significantly different views about this seemingly simple question. This debate triggered our efforts to test the well-known engineering concepts about passivity/activity in a deeper way, challenging them by statistical physics. It is shown that the advanced engineering definition of passivity/activity of devices is self-contradictory when a thermodynamical system executing Johnson–Nyquist noise is present. A new, statistical physical, self-consistent definition based on the Second Law of Thermodynamics is introduced. It is also shown that, in a system with uniform temperature distribution, any rectifier circuitry that can rectify thermal noise must contain an active circuit element, according to both the engineering and statistical physical definitions.

1. Introduction

A simplified engineering mathematical model about a complex physical system can provide mathematically exact statements. However there is a danger that the model is unphysical when it misses capturing an essential part of the physics of the problem. In such cases, the model may yield incorrect conclusions.

Recently, our efforts to clarify the old question: "Is a memristor a passive or active device [1]?" triggered debates between engineers, who have had advanced definitions of passivity/activity of devices, and physicists with significantly different views about this seemingly simple question. These debates motivated our efforts to test the well-known engineering concepts about passivity/activity in a deeper way, by challenging them with statistical physics.

In [2], the Authors present a survey of commonly used engineering classifications of passivity/activity of electronic devices. Then they introduce the most advanced, *so-called-thermodynamic* definition that does not have the weakness of the former (commonly used) ones. However, we have found that even this most advanced engineering definition fails in certain physical/practical cases. Under certain conditions, it mis-classifies well-accepted passive devices as active, especially when temperature differences or specific nonlinearities are present.

Because of the failures of the established engineering concepts, in this paper we introduce a deeper definition of passivity/activity that is based on statistical physics. The Second Law of thermodynamics makes these definitions possible without having particular knowledge about the structure, circuitry, or other fine details of the device. The new, statistical physical definitions are robust against those challenges where the commonly used engineering definitions fail according to the analysis in [2]. Moreover our statistical physical definitions always classify a resistor as a passive device even when the most advanced engineering definitions fail to do so.

Below, we first outline the most advanced engineering, *so-called-thermodynamic*, definition [2] of passivity/activity and, in Section 2, we show two thermodynamic situations where it fails. In Section 3, we introduce our new, *statistical-physical* definitions. Finally in Section 4, we show that any rectifier circuitry that can rectify thermal noise must contain at least one active circuit element.

For the sake of simplicity, we restrict the analysis to time-invariant systems and 1-*port*, that is, to 2-*contact* devices. The engineering approach [2] uses the "available energy" $E_\mathrm{A}(x)$ served by the device which is

$$E_\mathrm{A}(x) = \sup_{t\geq 0}^{(x)} \left[\int_0^t -u(\tau)i(\tau)\mathrm{d}\tau \right], \tag{1}$$

where u and i are voltage and current, respectively; the given x represents the initial conditions at time $t = 0$ (that is, all the

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http://dx.doi.org/10.1016/j.physleta.2017.08.039

initial voltages and currents in the system/circuitry between the two contacts); $\sup^{(x)}_{t\geq 0}$ indicates that the upper limit (supremum) is taken over all $t \geq 0$ long time intervals and all physically possible $\{u(t), i(t)\}$ trajectories. The earlier definition would say the device is claimed to be *strongly passive* from a "thermodynamic" point of view [2] if:

i) $E_\mathrm{A}(x)$ is finite for all physically allowed initial states x.

ii) There exists a specific initial state x_0 (so-called relaxed state), where $E_\mathrm{A}(x) = 0$.

We acknowledge the intent of the Authors of [2] to include thermodynamics in the definition of passivity/activity and the efforts to choose their wording very carefully. They talk about "physically possible" processes, and describe a quiescent "relaxed state" (probably an attempt to account for thermal equilibrium). However, from a fundamental physics point of view, the name, "*thermodynamic*", for the above engineering definition is obviously incorrect because *no thermodynamical measure*, such as *temperature*, *heat* or *entropy* appears in the above formula. In the next section, we point out two practical situations where even the above advanced engineering definition gives the wrong conclusion.

2. Examples where the engineering definition fails

The method we use to crack the engineering definition of activity and passivity is based on the Fluctuation–Dissipation Theorem (FDT) of statistical physics. In the classical physical limit, the FDT for a passive electrical impedance $Z(f)$ interrelates the power density spectrum $S_\mathrm{u}(f)$ of the thermal noise voltage generated by the impedance with the real part $\mathrm{Re}[Z(f)]$ of that impedance [3]:

$$S_\mathrm{u}(f) = 4kT\,\mathrm{Re}\big[Z(f)\big], \tag{2}$$

where Equation (2) is the Johnson–Nyquist formula; k is the Boltzmann constant; and T is the absolute temperature of the free-standing impedance in thermal equilibrium. Even though there are current discussions [3,4] about the FDT's prediction in the quantum region (at very low temperatures and/or very high frequencies), in the classical physical limit Equation (2) is verified by both careful experiments and everyday electrical engineering practice.

2.1. Resistors at different temperatures

A direct, well-known consequence [5,6] of Equation (1) is utilized in practice even in secure communications, that a circuit of two parallel-connected resistors with different temperatures will execute a non-zero mean power flow from the warmer resistor to the colder one. Let us suppose that, at our test of passivity/activity by means of the engineering definition above, the *device-in-question* is the R_1 resistor at T_1 temperature in Fig. 1 and the T_1 and T_2 temperatures are different and stabilized by an active temperature regulation. In the frequency band Δf, the mean power flow from resistor R_1 to resistor R_2 is given as [5,6]:

$$\langle P_{12}\rangle = 4k(T_1 - T_2)\Delta f \frac{R_1 R_2}{(R_1 + R_2)^2}. \tag{3}$$

If the load resistor R_2 is colder than R_1, that is $T_2 < T_1$, then according to Equation (3):

$$0 < \langle P_{12}\rangle = \lim_{\substack{t\to\infty \\ t>0}} \frac{E_{12}}{t}, \tag{4}$$

thus

$$\lim_{\substack{t\to\infty \\ t>0}} E_{12} = \infty, \tag{5}$$

which, according Equation (1) and the related engineering definitions mean that R_1 is an active device contrary of the common understanding that linear resistors are passive devices. On the other hand, when the temperatures are interrelated in the opposite way, $T_1 < T_2$, then the R_1 is classified as a passive device. It is clear that the engineering definition based on Equation (1) is self-contradictory for classifying the activity/passivity of resistors.

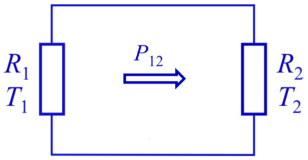

Fig. 1. Energy transfer between two resistors. The warmer resistor is heating the other one. According to Equation (1), the warmer resistor is an active element while the colder one is a passive one. In reality, both resistors are passive.

2.2. Devices with idealized rectifier characteristics

As an example for a nonlinear device, we consider a hypothetical "idealized rectifier" (e.g. an idealized diode that is noise-free, etc.) which is a device that rectifies all signals, including arbitrarily small signals, such as the thermal noise of a resistor. For our studies, any current–voltage characteristics that are continuous asymmetric functions are satisfactory provided they can hypothetically extract a non-zero DC voltage from thermal noise.

When the rectification of thermal noise is ignored, the engineering definition of passivity/activity (based on Equation (1)) yields the well-accepted conclusion that the idealized diode is a passive device. However, as it is shown in Section 4, in thermal equilibrium, feeding the hypothetical diode (and connected passive circuit elements) with thermal noise results in Equation (5), thus the diode circuitry is then an active device in this situation, according to the engineering definition. This paradox will be detailed and resolved in Section 4.

3. The new, statistical physical definition of passivity/activity

Here we introduce our definition of passivity/activity based on the Second Law of Thermodynamics, that is, the impossibility of Perpetual Motion Machines (PMM) of the second kind. Such machines would be able to extract energy from the heat in thermal equilibrium without using extra energy for the process, that is, they would be able to cause steady-state temperature differences and decrease the entropy in a closed system in thermal equilibrium. That extracted energy by the PMM would violate also the Energy Conservation Law. The new definition below is self-consistent and provides robust definition of passivity/activity even in situations when the advanced engineering definition given in Section 1 fails to do so.

Note: the *device-in-question* below may need external energy sources (bias) to provide its required signal response. Example: a transistor requires a power supply for its bias conditions to operate. We will address this problem below.

Our definition of activity/passivity is:

a) The *device-in-question* is *active* if the following condition holds. Suppose a *hypothetical-device*, which does not require an external energy source to function and it has the same signal-response characteristics as the device-in-question. In an isolated system originally in thermal equilibrium, such a hypothetical-device in a proper circuitry would be able to produce steady-state entropy reduction in the system where all the other elements are passive. In other words, such hypothetical device would violate the Second Law of Thermodynamics.

Note: for example, such an entropy reduction could be a steady-state, non-zero temperature gradient in the device's environment;

Please cite this article in press as: K.M. Sundqvist et al., Second Law based definition of passivity/activity of devices, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2017.08.039