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On certain properties of nonlinear oscillator with coordinate-dependent mass

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ABSTRACT

A nonlinear model of the scalar field with a *coupling* between the field and its gradient is developed. It is shown, that such model is suitable for the description of phase transitions accompanied by formation of spatially inhomogeneous distributions of the order parameter. The proposed model is analogous to the mechanical nonlinear oscillator with the coordinate-dependent mass or velocity-dependent elastic module. Besides, for some values of energy the model under consideration has exact analytical solution. This model may be related to the spinodal decomposition, quark confinement, or cosmological scenario. All predictions can be verified experimentally.

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1. Introduction

Most physical processes are essentially nonlinear: thermal expansion, heat conductivity, lattice dynamics, nonlinear quantum optics phenomena, etc. In some cases it may lead to phase transitions or behavior that can be interpreted as a phase transition. Numerous works in this area indicate the importance of mentioned problems to physics. For example, phase transition for a simple class of models was studied by Langer [1]. Later, this method was applied in a field theory to describe the formation of new phase [2]. Other studies in this area include spinodal decomposition [3], phase transitions with formation of a new inhomogeneous state in condensed matter [1], alternative cosmological model [4], etc.

An appealing method for theoretical study of mentioned systems' class is the restatement of the problem in terms of nonlinear harmonic oscillators [5]. For example, stationary nuclear fission rate can be described by oscillator model with coordinate-dependent mass and specific potential [6]. It turns out that such analytical estimates agree with numerical simulations based on Langevine equation. Second example comes from nuclear physics as well: transition through the fission barrier potential in the WKB-approximation [7].

The examples above illustrate that nonlinear oscillator models can be used in various areas of theoretical physics. Naturally, this induces permanent interest in solving corresponding equations. Traditionally, behavior of nonlinear oscillators has been analyzed numerically or by perturbation methods [8]. However, the latter cannot be successfully applied to the number of models. For example, displacement phase transitions and dynamic soft-mode behavior require nonperturbative theories [1]. Thus, recent advances in this area include development of alternative analytical techniques without small parameter, e.g. variational approach [8]. Other outstanding results worth mentioning are the exact solution for the Hamiltonian with coordinate-dependent mass in semiclassical theory [9] and approximations of the effective action in case of such particle moving through one-dimensional scalar field [10].

In the present article we use the nonlinear oscillator model to describe phase transition accompanied by formation of spatially inhomogeneous distribution of the order parameter [5,11,12]. We modify the standard model of a scalar field by means of *coupling* between the field and its gradient. We expect this model to be related to spinodal decomposition [3] and cosmological scenario [4]. Moreover, quark confinement may be hypothetically a physical realization of this mechanistic model as well. We suppose, one can treat quarks as oscillators with increasing mass instead of specific interaction when bag's boundary is approached. Our goal is to answer two questions: are there any analytical solutions to this model (at least under some conditions) and whether this system exhibits chaotic behavior. First question can be answered positively if certain amount of energy is contained within the system. Cases without analytic solution can be easily treated numerically. Hereinafter we will use plots obtained by computer simulation to

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illustrate system's behavior at different parameters values (including the ones that lack analytic solution).

The question we state about possible chaotic behavior is not of pure curiosity. It has been recognized that chaotic behavior seems to be preferred by some natural systems and can be utilized for practical applications [13]. Besides, chaotic dynamics of isolated system is interesting on its own. For example, we know conventional techniques fail to detect chaos in such systems [14]. Situation may become significantly different if external perturbation of Hamiltonian is employed (e.g., small external force is applied). For instance, the nonlinear Schrodinger (NLS) equation, when perturbed, starts exhibiting different types of chaotic behavior and instabilities (homoclinic chaos, hyperbolic resonance, and parabolic resonance). Detailed analysis can be performed by constructing hierarchy of bifurcations [15]. Besides, numerical computation show that NLS driven by external force is subjected to similar chaotic phenomena [15]. This suggests that our model system can undergo bifurcation or acquire chaotic properties as well.

2. Model with order parameter and its gradient coupling

Let us consider a continuous system. In the context of phase transitions theory, we assume it to have the ground state described in terms of order parameter. The latter can be the subject to various geometrical representations. For instance, theories of condensed matter [1] or field theories [4] introduce order parameter in the form of scalar field.

Determining a stable state of a condensed matter, we can expect a non-uniform field distribution in some cases. Indeed, current experimental data suggests the existence of disordered configurations of the ground state. Their theoretical descriptions are mostly phenomenological, but fairly general. For example, [16] and [17] describe spatial distributions of the order parameter both before and after the phase transition.

The model can be improved by introducing term responsible for interaction between the order parameter and its gradient into the free-energy functional. In case of competing order parameters, the gradient terms may lead to inhomogeneous states [18]. Moreover, at instability threshold coefficients before the quadratic terms of the order parameter and its gradient can change their signs [3]. This means, some decomposition scenario is possible and system's state may undergo transition from disordered to modulated, patterned, or ordered-patterned. In other words, we can observe spinodal decomposition when scalar spatially-dependent order parameter rapidly changes. We expect this behavior to be related to cosmological model. For example, it can describe generation of spatially inhomogeneous states when temperature drops. Besides, temperature decrease can be the reason for a new bubble phase formation in the cosmological model.

The main idea of present contribution is to generalize the standard model of scalar field by introducing the interaction between the field and its gradient. We consider following form of the condensed matter free energy density [1]

$$f = a\left(\vec{\nabla}\varphi(\vec{r})\right)^2 + b\left(1-(\varphi(\vec{r}))^2\right)^2 + \\ + c\varphi^2(\vec{r})\left(\vec{\nabla}\varphi(\vec{r})\right)^2, \tag{1}$$

where a, b, and c are real constants ($a, b > 0$). The same expression is valid for three-dimensional action in field theory [4]. One can see, setting $c = 0$ transforms (1) to well-known relation of φ^4 *model* for single real scalar field.

There are two reasons why form (1) is favored over other possible expressions. First, we can prove this model is equivalent to the mechanical one — nonlinear oscillator with coordinate-dependent mass. It seems, renormalization problem can be solved in this new representation with QFT methods. Namely, one can employ covariant background field method [10] to calculate the one-loop quantum effective action for the particle with the coordinate-dependent mass moving slowly in one-dimensional configuration space. It makes (1) a promising model, because thus far we have been working with free energy expressed in terms of non-renormalizable fields, masses, and coupling constants.

The second reason we propose model (1) is the variety of its solutions. We will describe these solutions and show they can give rise to spatially inhomogeneous scalar field distributions and topological structures of the new phase.

3. Order parameter-space — space–time duality

Let us define free energy from its density (1) as follows $F[\varphi(\vec{r})] = \iiint f\left(\varphi(\vec{r}), \vec{\nabla}\varphi(\vec{r})\right) d\vec{r}$, where integration is performed over the entire system. We write $F[\varphi(\vec{r})]$ here to emphasize F is treated as functional of φ (and a subject to variational calculus we will apply).

Now we can use variational calculus to minimize $F[\varphi(\vec{r})]$ by calculating functional derivative $\delta F/\delta\varphi(\vec{r})$ and setting it to zero. This is a multidimensional problem, so we obtain Euler–Lagrange–Ostrogradsky equation

$$\frac{\partial f}{\partial \varphi} - \sum_i \frac{\partial}{\partial r_i}\frac{\partial f}{\partial\left(\partial\varphi/\partial r_i\right)} = 0,$$

where r_i are components of $\vec{r}$. Before we proceed, let us rewrite the last equation in a different form

$$\frac{\partial f}{\partial \varphi} - \vec{\nabla}\cdot\frac{\partial f}{\partial\vec{\nabla}\varphi} = 0, \tag{2}$$

where $\partial f/\partial\vec{\nabla}\varphi$ is a derivative of a scalar with respect to a vector and $\cdot$ means scalar product.

If $\vec{\nabla}$ in equation (2) was one-dimensional derivative, we could call that dimension "time" and designate it t, while F could be renamed to L and called Lagrangian. If so, we could obtain a regular equation we are used to in classical mechanics. Actually, this is exactly what will happen, after we make use of symmetry and reduce the number of spatial variables to one.

Now we use equations (1) and (2) to obtain the following expression

$$\left(a + c\left(\varphi(\vec{r})\right)^2\right)\Delta\varphi(\vec{r}) + c\varphi(\vec{r})\left(\vec{\nabla}\varphi(\vec{r})\right)^2 + \\ + 2b\varphi(\vec{r})\left(1-(\varphi(\vec{r}))^2\right) = 0. \tag{3}$$

We are interested in bubble formation, thus spherical symmetry can be applied. For one- and two-dimensional cases this means we can write $d\varphi/dr$ instead of $\vec{\nabla}\varphi$ and $d^2\varphi/dr^2$ in place of $\Delta\varphi$. Three-dimensional case is a little bit more complicated, since term $(2/r)d\varphi/dr$ is introduced by Laplace operator. But in this case one can imply a thin-wall approximation [4,2], which essentially means neglecting $(2/r)d\varphi/dr$ term. Thus, we write for all one to three dimensions

$$\left(a + c\left(\varphi(r)\right)^2\right)\frac{d^2\varphi}{dr^2} + c\varphi(r)\left(\frac{d\varphi}{dr}\right)^2 + \\ + 2b\varphi(r)\left(1-(\varphi(r))^2\right) = 0, \tag{4}$$

where r is the only coordinate left.

Now we leave cosmological model for a moment and switch gears to the mechanical one — oscillator with coordinate-dependent mass.

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