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Deterministic linear optical quantum Toffoli gate

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ABSTRACT

Quantum Toffoli gate is a crucial part of many quantum information processing schemes. We design a deterministic linear optical quantum Toffoli gate using three degrees of freedom of a single photon. The proposed setup does not require any ancilla photons and is experimentally feasible with current technology. Moreover, we show that our setup can be directly used to demonstrate that hypergraph states violate local realism in an extreme manner.

Multiqubit gates play a critical role in many quantum information processing schemes [1]. As is well known, quantum Toffoli gate is a fundamental quantum gate for three-qubit systems, which enables universal quantum computation together with a one-qubit Hadamard gate [2,3]. Moreover, it is valuable in quantum algorithms like Shor's algorithm [4], fault tolerant quantum circuits [5,6], distributed quantum computation [7,8], and has critical applications as correcting operation in quantum error correction [9,10].

Quantum gates designed with minimized resources is crucial for quantum computation, which depends on the specific features of given physical systems. So far, several physical systems, including ion trap systems [11], superconducting circuits [12,13], nuclear magnetic resonance [9] and linear optics [14–16], have been proposed to successfully demonstrated the quantum Toffoli gate. It has been proved that it is sufficient for realizing universal quantum computing using linear optics [17,18]. However, up to now, all the theoretical and experimental schemes for Toffoli gate with linear optical system are probabilistic. In 2007, Fiurášek proposed a schemes for linear optical three-qubit Toffoli gate with an optimal success probability of 0.75% [19]. In 2013, Fiurášek et al. realized a linear optical Toffoli gate with a probability of 1/9, which is the maximum value achievable without the use of ancilla photons [15]. In 2013, Yu et al. proposed that the Toffoli gate can be decomposed into five two-qubit gates [20]. However, for linear optical system, the two-qubit gate is still probabilistic. Although there are much valuable works on the construction deterministic quantum Toffoli gate basing on the hybrid systems of photon and other quantum system [21–24]. However, the interaction between photons and other systems is still experimentally complex and challenging.

Here, we first proposed a deterministic quantum Toffoli gate for linear optical quantum system. Our strategy is to encode the information in multiple degrees of freedom (DOFs) of photon. The proposed setup does not require any ancilla photons and is experimentally feasible with current technology, leading to a simple and efficient way for implementing linear optical Toffoli gate. Furthermore, we extend our method to implement controlled–controlled-Z (CCZ) gate, which is equivalent to the Toffoli gate up to single-qubit Hadamard transform on the target qubit, and show that our setting can be directly used to demonstrate that hypergraph states violate local realism in an extreme manner [25].

In our scheme, three qubits are encoded into three DOFs of a single photon. Suppose the first and second qubits are encoded into the spatial and spin angular momentum (SAM) DOF of the photon, respectively, and the third qubit is represented by the orbital angular momentum (OAM) of the photon. Our Toffoli gate is designed for these three qubits (see Fig. 1). Before introducing the Toffoli gate, we first describe how to encode the quantum state using these three DOF of a single photon. Without losing the generality, we here take the separable state $|\varphi\rangle$ as an example:

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$$|\varphi\rangle = (\alpha|0\rangle^{s} + \beta|1\rangle^{s}) \otimes (\gamma|0\rangle^{p} + \delta|1\rangle^{p}) \otimes (\mu|0\rangle^{o} + \nu|1\rangle^{o}) \quad (1)$$

Here, $|0\rangle^s$ and $|1\rangle^s$ label two orthogonal spatial modes of the photon; $|0\rangle^p$ and $|1\rangle^p$ denote horizontal and vertical polarizations of the SAM; $|0\rangle^o$ and $|1\rangle^o$ refer to right-handed and left-handed OAMs of $+\hbar$ and $-\hbar$, respectively; and α, β, γ, δ, μ and ν are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = |\gamma|^2 + |\delta|^2 = |\mu|^2 + |\nu|^2 = 1$.

Fig. 2 shows the setup for preparing the state $|\varphi\rangle$. Assume the photon is emitted from the fiber coupler with the polarization initialized as $|0\rangle^p$, and then prepared to $|0\rangle^p|0\rangle^o$ after passing through a spiral phase plates (SPP), where the SPP is used to convert the zero-order OAM (the OAM of photon emitted from the fiber coupler is zero-order mode) into first-order mode $|0\rangle^o$. To prepare the OAM qubit $\mu|0\rangle^o + \nu|1\rangle^o$, we first prepare the SAM qubit $\mu|0\rangle^s + \nu|1\rangle^s$, and then transfer the polarization information to the OAM qubit using two Mach–Zehnder interferometers (MZI) with Dove prisms inside, where the Dove prism is used to operate the OAM with the following rule (assume that the Dove prism is rotated by an angle of $\alpha\pi$):

$$\begin{aligned} e^{i\theta}|0\rangle^{o} &\xrightarrow{\text{Dove}} e^{i(\theta+2\alpha\pi)}|1\rangle^{o} \\ e^{i\theta}|1\rangle^{o} &\xrightarrow{\text{Dove}} e^{i(\theta-2\alpha\pi)}|0\rangle^{o} \end{aligned} \quad (2)$$

Interested readers could find more detailed information about the operation of the OAM of photon in [26].

Specifically, after preparing the $(\mu|0\rangle^s + \nu|1\rangle^s) \otimes |0\rangle^o$ by a set of half-wave plates (HWP), quarter-wave plates (QWP), and SPP, the photon is sent into the first MZI with one arm of which supports a Dove prism placed at 0 angle. The incoming photon in the $(\mu|0\rangle^s + \nu|1\rangle^s) \otimes |0\rangle^o$ state is initially split, by PBS, into horizontal and vertical components, traveling along the two arms. The OAM is rotated to $|1\rangle^o$ for the $|1\rangle^s$ SAM component when passing through

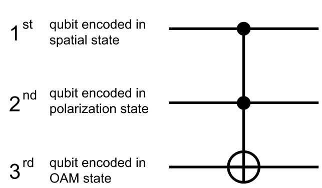

Fig. 1. Quantum Toffoli gate for three qubits encoded into three DOFs of a single photon.

the Dove prism in vertical polarization arm of the MZI. To make the optical path of both arms of MZI equal, a quartz crystal is placed in the horizontal polarization arm of MZI to compensate optical path of Dove prism, and three HWPs at $\pi/4$ (two in MZI and one at the output port of MZI) are placed to flip the polarization of photon to make the photon of the horizontal (vertical) polarization arm reflect (transmit) out of the MZI. After passing the first MZI, the output photon is transformed into $\mu|0\rangle^s|0\rangle^o + \nu|1\rangle^s|1\rangle^o$. In fact, the first MZI can be summarized as a CNOT gate between the SAM (control qubit) and OAM (target qubit), which is denoted as SAM-OAM-CNOT:

$$\begin{aligned} &(\mu|0\rangle^{s} + \nu|1\rangle^{s}) \otimes |0\rangle^{o} \\ &\xrightarrow{\text{MZI}} \frac{1}{\sqrt{2}}\mu|0\rangle^{s}|0\rangle^{o} + \nu|1\rangle^{s}|1\rangle^{o} \end{aligned} \quad (3)$$

Next, the photon passes through a HWP at $\pi/8$, and then enters into the second MZI with each arm of which supports a HWP at $\pi/4$ and a Dove prism, which are placed at $\pi/8$ and $-\pi/8$ angle in the horizontal polarization arm and vertical polarization arm, respectively. Finally, the output photon passes through a QWP at $\pi/8$, a Dove prism placed at $\pi/8$ angle, and a phase-shifters for creating phase shift of $-\pi/2$. The overall transformations can be summarized as a CNOT gate between the OAM (control qubit) and SAM (target qubit), which is denoted as OAM-SAM-CNOT:

$$\begin{aligned} &\mu|0\rangle^{s}|0\rangle^{o} + \nu|1\rangle^{s}|1\rangle^{o} \\ &\xrightarrow{\text{HWP}} \frac{1}{\sqrt{2}}\mu(|0\rangle^{s} + |1\rangle^{s})|0\rangle^{o} + \frac{1}{\sqrt{2}}\nu(|0\rangle^{s} - |1\rangle^{s})|1\rangle^{o} \\ &\xrightarrow{\text{MZI}} \frac{1}{\sqrt{2}}\mu e^{-i\frac{\pi}{4}}(|0\rangle^{s} + i|1\rangle^{s})|1\rangle^{o} - \frac{1}{\sqrt{2}}\nu e^{i\frac{\pi}{4}}(|0\rangle^{s} + i|1\rangle^{s})|0\rangle^{o} \\ &\xrightarrow{\text{QWP}} \mu|0\rangle^{s}|1\rangle^{o} - i\nu|0\rangle^{s}|0\rangle^{o} \\ &\xrightarrow{\text{Dove}} |0\rangle^{s} \otimes (\mu|0\rangle^{o} + \nu|1\rangle^{o}) \end{aligned} \quad (4)$$

After preparing the OAM qubit $\mu|0\rangle^o + \nu|1\rangle^o$, a QWP and a HWP is use to transform the state $|0\rangle^s \otimes (\mu|0\rangle^o + \nu|1\rangle^o)$ into $(\alpha|0\rangle^s + \beta|1\rangle^s) \otimes (\mu|0\rangle^o + \nu|1\rangle^o)$. Finally, the photon is split by BS with the transmission $|\gamma|^2$ into two orthogonal spatial modes. We define the reflected and transmitted paths as $|1\rangle^s$ and $|0\rangle^s$, respec-

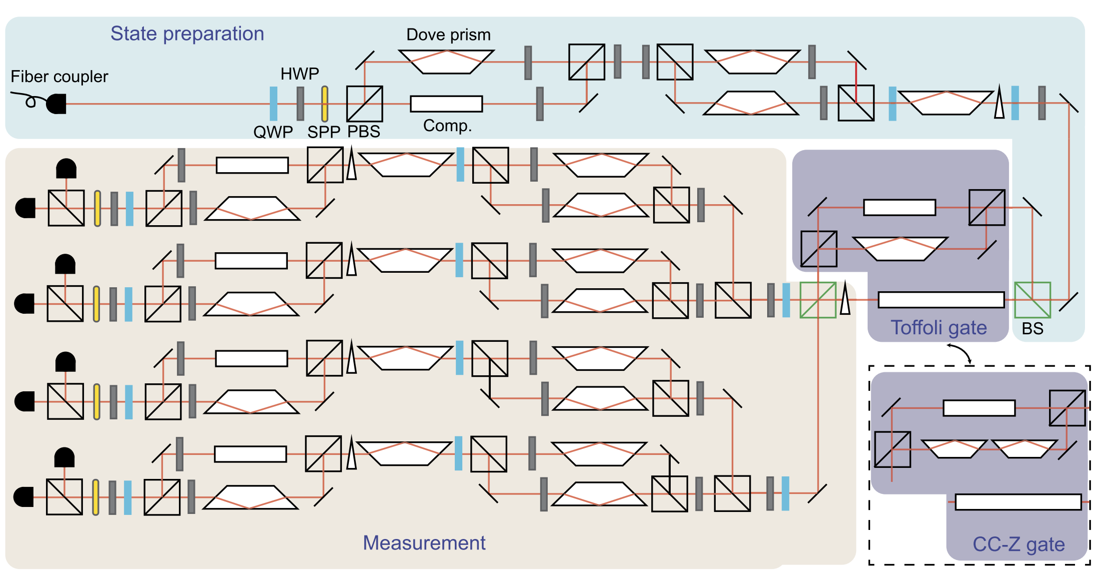

Fig. 2. (Color online). Proposal of experimental setup. The blocks with different colors represent the different basic stages. State preparation (blue block): three qubit encoding setup. Toffoli gate (purple block): Toffoli gate operating setup. CCZ gate (purple block): CCZ gate operating setup. Measurement (yellow block): qubit measuring setup.