Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Emergence of Fresnel diffraction zones in gravitational lensing by a cosmic string

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A R T I C L E I N F O

Article history:
Received 9 January 2017
Received in revised form 27 February 2017
Accepted 6 March 2017
Available online xxxx
Communicated by P.R. Holland

Keywords:
Cosmic strings
Topological defects
Gravitational lensing
Gravitational waves
Diffraction

A B S T R A C T

The possibility to detect cosmic strings – topological defects of early Universe, by means of wave effects in gravitational lensing is discussed. To find the optimal observation conditions, we define the hyperbolic-shaped Fresnel observation zones associated with the diffraction maxima and analyse the frequency patterns of wave amplification corresponding to different alignments. In particular, we show that diffraction of gravitational waves by the string may lead to significant amplification at cosmological distances. The wave properties we found are quite different from what one would expect, for instance, from light scattered off a thin wire or slit, since a cosmic string, as a topological defect, gives no shadow at all.

1. Introduction

The first direct detection of gravitational waves by the Laser Interferometer Gravitational-Wave Observatory (LIGO) [1] opened up a new way to observe the Universe. Along with gravitational wave detection, it was the first direct observation of binary black holes. With this success, there are many hopes that other previously invisible cosmological objects, which emit or scatter gravitational waves, will be observed in the near future.

In this paper we discuss the possibility to detect cosmic strings – topological defects that may have been formed in the early Universe [2,3] – by means of wave effects in the gravitational lensing taking into account the interference and diffraction. We emphasize the difference of wave diffraction on a topological defect from that on a compact object. For the wave effects to be detectable in a compact-mass gravitational lens, the wavelength λ should be comparable or larger than the Schwarzschild radius R_s of the lens [4]. In this case, the Fresnel number, which is the key parameter for the diffraction, is given by R_s/λ, and the diffraction scales like $O(\lambda/R_s)$. This scaling cannot be applied to a string, a non-compact object with conical topology. It has been shown recently for the plane-wave diffraction by string [5] that the Fresnel number is determined by the ratio $r\Delta^2/\lambda$, where r is the distance from the string to the observer and Δ is a constant related to the deficit angle of conical space, which is proportional to the linear mass of the string [6–8]. For the typical $\Delta \sim 10^{-7}$, low Fresnel numbers can be achieved at cosmological distances from the string, $r \sim 10^{14}\lambda$. As a result, the diffraction effects can be of the same order as the geometrical optics giving an additional amplification at the observation point [5]. This is a direct consequence of the conical topology, for which the metric is locally flat, but globally it forces the parallel geodesics to cross (when they pass on opposite sides of the string) at a large distance. On the other hand, the deflection angle, equal to Δ, is independent of the impact parameter [6,7]. Hence, the characteristic fringe width in the interference pattern $\sim \lambda/(2\Delta)$ does not vary with distance. This is another feature distinct from the compact-object lens, for which the interference fringe scales with distance as $\sim \lambda\sqrt{r/R_s}$ [9].

The objective of this paper is twofold. First, we study the question of how the Fresnel diffraction zones emerge under wave propagation in conical spacetime created by a straight cosmic string.[1] The diffraction pattern we have obtained is quite different from what one would expect from light scattered off a thin wire or slit [10,11], since the cosmic string, as a topological defect, gives no shadow. After an appropriately chosen coordinate transformation, we convert the problem of a single-source wave in conical space to a more tractable form with a locally Minkowskian line element

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[1] Actual strings are not straight and may contain loops, we refer to a straight-line segment of an infinitely long or closed string lying at the observer-source line of sight.

http://dx.doi.org/10.1016/j.physleta.2017.03.046
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and a limitation on the angular range. As a result, we obtain the interference and diffraction pattern analytically as a superposition of wave fields from two image sources illuminating two virtual half-plane screens.

Second, we take into account the curvature of the incident wavefront by considering the wave source at a finite distance from the string. This is a more general case with respect to our previous study [5]. By applying the uniform asymptotic theory of diffraction [12,13], we obtain analytical solutions for the wave field in the whole space including the lines of singularities at the boundaries of the double-imaging region. Away from the boundaries, the wave field is interpreted in the framework of Keller's geometrical theory of diffraction [14], which has demonstrated to be quite efficient in studying diffraction on a topological defect [5]. Our results allow to predict with high accuracy the location of the diffraction maxima both in coordinate space and in energy spectrum, along with the nodal and antinodal lines of geometrical-optics interference. We found it convenient to associate the diffraction maxima with what we call the "Fresnel observation zones", that help to localize the regions where the amplification due to the string is the highest and easier to observe. The boundaries between the zones are determined by hyperbolas in an equivalent Minkowskian space. In the limit of an infinitely distant source (incident plane wave), the hyperbolas convert to parabolas, all with a common focus at the string.

2. Wave equation in conical spacetime

We start with a spacetime metric for a static cylindrically symmetric cosmic string [6,8]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + (1-4G\mu)^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \tag{1}$$

where G is the gravitational constant, μ is the linear mass density of the string lying along the z-axis, (t, r, φ, z) are cylindrical coordinates, and the system of units in which the speed of light $c = 1$ is assumed. With a new angular coordinate $\theta = (1-4G\mu)\varphi$, the metric (1) takes a locally Minkowskian form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + \mathrm{d}z^2, \tag{2}$$

having, however, a limitation on the angular range. It is assumed here, that a wedge of angular size $8\pi G\mu$ is taken out and the two faces of the wedge are identified [3,6]. By introducing the deficit angle 2Δ with

$$\Delta = 4\pi G\mu, \tag{3}$$

the angular coordinate θ spans the range $2\pi - 2\Delta$.

We consider the question of finding a solution of the wave equation in background (1) corresponding to a time harmonic source, situated at a finite distance from the string. For the sake of simplicity, in order to keep the problem two-dimensional, we consider a line source parallel to the string. Our aim is to see how a wave emitted by a line source is diffracted in conical spacetime. The wave equation in background (1) for the scalar field $U(r, \varphi)$ is (see, e.g., [5,15,16])

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{\beta^2 r^2}\frac{\partial^2}{\partial \varphi^2} + \omega^2\right) U = 0, \tag{4}$$

where we denoted $\beta \equiv 1 - \Delta/\pi$. We assume that Eq. (4) is valid for electromagnetic waves, as well as for gravitational waves (in an appropriately chosen gauge) when the effect of gravitational lensing on polarization is negligible and both types of waves can be described by a scalar field [17]. Consider a line source E located at $\mathbf{r_0} = (r_0, \pi)$ and emitting a cylindrical wave described by

$$U = A\, H_0^{(1)}(k|\mathbf{r} - \mathbf{r_0}|), \tag{5}$$

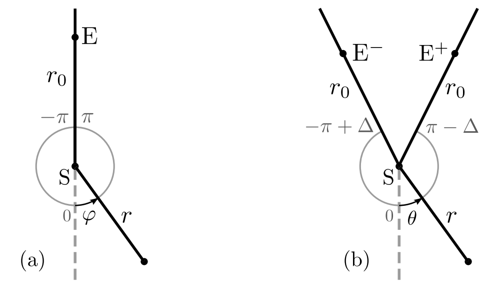

Fig. 1. Geometry of conical space at the $z = 0$ plane for two equivalent backgrounds with point S indicating the location of the string: (a) polar coordinates (r, φ) with a source E; (b) Minkowskian coordinates (r, θ) with deficit angle 2Δ and two image sources E^-, E^+.

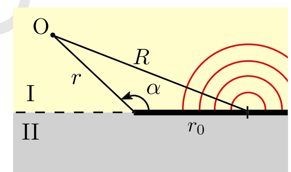

Fig. 2. Cylindrical wave emitted from a source on the upper surface of a half-plane screen (thick line). The space is split into two regions: illuminated (I), shadow (II).

where A is a normalization constant and $H_0^{(1)}$ is the Hankel function of the first kind which satisfies the Helmholtz equation (4) and corresponds to an outward-propagating solution [11]. It is advantageous to perform the angular transformation $\theta = \beta\varphi$ and to work in the Minkowskian geometry (2) with a wedge removed rather than in the metric (1), as done in Ref. [5] for an infinitely distant source. To conveniently perform the transformation, we put the origin at the string location S and join the point S with the emitting source E by a radial line [see Fig. 1(a)]. Then we assign the values $\varphi^- = -\pi$ to the left and $\varphi^+ = \pi$ to the right of the line SE that will be the cut line. Assuming that the emitting wave is symmetric (isotropic), we obtain a zero derivative $\partial_\varphi U = 0$ at the cut. After the angular transformation, the line SE converts to the wedge SE^-, SE^+, given by the angles $\pm(\pi - \Delta)$ [see Fig. 1(b)]. The two faces of the wedge should be identified since they represent the same plane in the spacetime (1). Thus, the propagation of a wave in conical spacetime can be represented as the propagation of two waves in flat geometry with a wedge removed. In our consideration, each emitting source lies on the corresponding face of the wedge. Our next step is to show that the problem posed in this section can be effectively treated in the framework of the canonical problem of diffraction on a perfectly conducting half-plane screen [5].

3. Uniform asymptotic theory of diffraction on a half plane

Let us consider a half-plane screen defined in polar coordinates (r, α) by: $\alpha = 0$ (upper surface) and $\alpha = 2\pi$ (lower surface). According to our geometry, the source is located on the upper surface of the screen at a distance r_0 from the edge, i.e., at $(r_0, 0)$ (see Fig. 2).

The emission field is a cylindrical wave that can be defined by [11]

$$U_i = \sqrt{\frac{\pi}{2}}\,\mathrm{e}^{\mathrm{i}\pi/4} H_0^{(1)}(kR) \approx \frac{\mathrm{e}^{\mathrm{i}kR}}{\sqrt{kR}}, \tag{6}$$

with $R = \sqrt{r^2 + r_0^2 - 2rr_0\cos\alpha}$ and the subscript "i" means "incident" field. The solution for the field in the whole space can be expressed as an integral [11,18–20]

Please cite this article in press as: I. Fernández-Núñez, O. Bulashenko, Emergence of Fresnel diffraction zones in gravitational lensing by a cosmic string, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2017.03.046