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Maxwell meets Reeh–Schlieder: The quantum mechanics of neutral bosons

Margaret Hawton [a,*], Vincent Debierre [b,*]

[a] Department of Physics, Lakehead University, Thunder Bay, ON, P7B 5E1, Canada
[b] Max Planck Institute for Nuclear Physics, Saupfercheckweg 1, 69117, Heidelberg, Germany

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A B S T R A C T

We find that biorthogonal quantum mechanics with a scalar product that counts both absorbed and emitted particles leads to covariant position operators with localized eigenvectors. In this manifestly covariant formulation the probability for a transition from a one-photon state to a position eigenvector is the first order Glauber correlation function, bridging the gap between photon counting and the sensitivity of light detectors to electromagnetic energy density. The position eigenvalues are identified as the spatial parameters in the canonical quantum field operators and the position basis describes an array of localized devices that instantaneously absorb and re-emit bosons.

1. Introduction

In nonrelativistic quantum mechanics (QM) the wave function is the projection of a particle's state vector onto a basis of position eigenvectors and its absolute square is the positive definite probability density. However, many experimental tests of QM are performed on photons and there is currently no well-defined relativistic QM [1–5] of photons or the neutral Klein–Gordon (KG) bosons often considered in their place for simplicity [6,7]. Recently it was claimed that the photon wave function [8] and Bohmian photon trajectories [9] were observed using weak measurements. This interpretation, justified by the analogy between the paraxial and Schrödinger equations, is disputed and an alternative interpretation based on the electromagnetic field has been presented [6,10,11]. In quantum optics most theorists deny the existence of number density and instead base calculations on energy density [12–15], although QM based on number density has been proposed [16]. The photon wave function and its application to emission by an atom and Bohmian trajectories is reviewed in [17,18]. Two sources of nonlocality have contributed to the perception that there is no relativistic QM or number density: Wave functions are assumed to be of positive frequency while Hegerfeldt's theorem [19] tells us that this restriction leads to instantaneous spreading, and the Newton–Wigner (NW) position eigenvectors [20] are localized in the sense that they are orthogonal but their relationship to the physical fields and to current sources is nonlocal in configuration space. We will argue here that both of these sources of nonlocality are nonphysical: In biorthogonal QM [21] the nonlocal transformation to the NW basis is not required [22] and a scalar product exists [23] that does not require separation of the fields into their nonlocal [5] positive and negative frequency parts [24]. As a consequence real fields are allowed and the paradoxical observer dependence of particle density on acceleration [7,25] can be avoided. In the manifestly covariant formalism derived here we identify the position eigenvalues of relativistic QM with the spatial parameters in the canonical quantum field theory (QFT) operators and the localized states as derivatives of Green functions that describe an array of emitting and absorbing devices localized in spacetime. This unifies the physical interpretation of the position coordinate in classical electromagnetism, relativistic QM and QFT.

The conventional scalar product [7] is a difference of particle and antiparticle terms so it is indefinite unless the field is limited to positive frequencies. The scalar product derived in [23,26] is positive definite for both positive and negative frequency fields. If this scalar product is used, inclusion of negative frequency states becomes mathematically straightforward. First quantized fields de-

* Corresponding authors.
E-mail addresses: margaret.hawton@lakeheadu.ca (M. Hawton), debierrev@mpi-hd.mpg.de (V. Debierre).

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scribing neutral KG particles and photons are real so inclusion of negative frequency fields is not only reasonable, it is essential. For photons the first quantized theory derived in [23] restricted to real fields is essentially classical electrodynamics with a rule for calculating number density. However, QFT is required for description of multiphoton states and entanglement. It is a consequence of the Reeh–Schlieder (RS) theorem [27] of algebraic QFT (AQFT) [28] that there are no local creation or annihilation operators and the vacuum is entangled across spacelike separated regions [3–5,29]. Any number operator that counts a particle by annihilating it with the positive frequency part of a field operator and then recreating it is nonlocal. Localization is a finite region requires summation over positive and negative frequencies [30]. A particle's energy must be bounded from below to prevent it from acting as an infinite energy source [4] so it is essential to make a distinction between energy and frequency. In AQFT, also called local QFT, causality is enforced by the microcausality assumption that field operators defined across space-like separated regions commute [3]. In Sections 3 and 4 both positive and negative frequency states will be defined and in Section 6 their interpretation in terms of absorbed and emitted positive energy particles will be elaborated on. We find no conflict of our localized bases with AQFT; rather the RS theorem and microcausality support our proposal that, in a covariant formulation, both absorbed and emitted neutral bosons should be counted.

The wave functions derived in [23,26] are projections of the field onto NW position eigenvectors [20]. NW found a position operator for KG particles, but they concluded that the only photon position operator is the Pryce operator whose vector components do not commute [31], making the simultaneous determination of photon position in all three directions of space impossible. They had assumed spherically symmetrical position eigenstates for photons, while photon position eigenvectors have an axis of symmetry like twisted light [32]. The photon Poincaré group is discussed in [33,34]. Following the NW method with omission of the spherical symmetry axiom, a photon position operator with commuting components and cylindrically symmetrical eigenvectors can be constructed [33]. Since spin and orbital angular momentum are not separately observable [35], its eigenvectors have only definite total angular momentum along some fixed but arbitrary axis [34]. A generalization of this cylindrically symmetric NW position operator was derived independently in [23]. Here we retain this symmetry but omit the NW similarity transformation that leads to nonlocality in configuration space.

The similarity transformation to the NW basis preserves scalar products but its nonlocal relationship to the physical fields has been interpreted as a nonlocal relationship between number density and energy density [14]. The nontrivial metric factor in the NW basis is not physically observable [36], and this suggests that nonlocality of the NW position eigenvectors is also not physically observable. Here we will work in the formalism of biorthogonal QM [21] that does not require transformation to the NW basis. Biorthogonal QM in a finite dimensional Hilbert space is summarized here as follows: The eigenvectors of a quasi-Hermitian [37] operator $\hat{O}$ and its adjoint $\hat{O}^\dagger$ are not orthogonal, as is the case for conventional Hermitian operators, but biorthogonal. This means that, given the eigenvector equations

$$\hat{O}\,|\omega_i\rangle = \omega_i|\omega_i\rangle, \tag{1}$$

$$\hat{O}^\dagger|\tilde{\omega}_j\rangle = \omega_j|\tilde{\omega}_j\rangle \tag{2}$$

we have $\langle\tilde{\omega}_j|\omega_i\rangle = \delta_{ji}\langle\tilde{\omega}_i|\omega_i\rangle$ and the completeness relation $\widehat{1} = \sum_i |\omega_i\rangle\langle\widetilde{\omega}_i|/\langle\tilde{\omega}_i|\omega_i\rangle$. An arbitrary state $|\psi\rangle$ has an associated state $|\widetilde{\psi}\rangle$. If an arbitrary state vector is expanded as $|\psi\rangle = \sum_i c_i\,|\omega_i\rangle$ in the Hilbert space $\mathcal{H}$ then in biorthogonal QM its associated state is $|\widetilde{\psi}\rangle = \sum_i c_i\,|\widetilde{\omega}_i\rangle \in \mathcal{H}^*$ where $c_i = \langle\widetilde{\omega}_i|\psi\rangle = \langle\omega_i|\widetilde{\psi}\rangle$. Using these expansions it is straightforward to verify that $\langle\widetilde{\psi}_1|\psi_2\rangle = \langle\psi_1|\widetilde{\psi}_2\rangle$. The probability for a transition from a quantum state $|\psi\rangle$ to an eigenvector $|\widetilde{\omega}_i\rangle$ of $\hat{O}^\dagger$ is

$$p_i = \frac{|\langle\widetilde{\omega}_i|\psi\rangle|^2}{\langle\widetilde{\psi}|\psi\rangle\langle\widetilde{\omega}_i|\omega_i\rangle}. \tag{3}$$

A generic operator can be written in the form

$$\widehat{F} = \sum_{i,j} f_{ij}\,|\omega_i\rangle\langle\widetilde{\omega}_j| \tag{4}$$

where $f_{ij} = \langle\widetilde{\omega}_i|\widehat{F}|\omega_j\rangle$ can be viewed as a matrix [21]. An equivalent bottom up approach is to start with a set of linearly independent not necessarily orthogonal vectors and obtain a biorthogonal basis and operators describing observables [36]. In Section 3 we will apply this formalism to the biorthogonal position eigenvectors $|\phi(x)\rangle = \widehat{\phi}(x)|0\rangle$ and $|\widetilde{\phi}(x)\rangle = |\pi(x)\rangle \propto \widehat{\pi}(x)|0\rangle$ where $x^\mu = (ct, \mathbf{x})$, $\widehat{\phi}(x)$ is a field operator, $\widehat{\pi}(x)$ is its conjugate momentum operator, and $|0\rangle$ is the global vacuum state. Extension of biorthogonal QM to this infinite-dimensional Hilbert space is not rigorous; for example completeness could fail as discussed in [21].

The rest of this paper is organized as follows: In Section 2 KG wave mechanics, with the field rescaled here to facilitate application to particles with zero mass, is reviewed. In Section 3 the covariant position operator and positive definite probability density are derived. In Section 4 the KG position observable discussed in Sections 2 and 3 is extended to photons. In Section 5 the wave function of the photon emitted by an atom is discussed, in Section 6 inclusion of negative frequency states, causality and localized states are examined, and in Section 7 we conclude.

2. Klein–Gordon wave mechanics

We will start with a review of the KG position observable problem. The KG equation

$$\partial_\mu\partial^\mu\phi(x) + \frac{m^2c^2}{\hbar^2}\phi(x) = 0 \tag{5}$$

describes charged and neutral particles with zero spin (pions). Here covariant notation and the mostly minus convention are used in which $x^\mu = x = (ct, \mathbf{x})$, $\partial_\mu = (\partial_{ct}, \nabla)$, m is the mass of the KG particle, c is the speed of light, $2\pi\hbar$ is Planck's constant and $f_1 \overleftrightarrow{\partial}_\mu f_2 \equiv f_1(\partial_\mu f_2) - (\partial_\mu f_1) f_2$. The function $\phi(x)$ is any scalar field that satisfies the KG equation (5). The four-density

$$J^\mu_{KG}(x) = \mathrm{i}g\phi(x)^* \overleftrightarrow{\partial}^\mu\phi(x), \tag{6}$$

satisfies a continuity equation. Plane wave normal mode solutions to (5) proportional to $\exp(-\mathrm{i}\omega t)$ are referred to as positive frequency solutions, while those proportional to $\exp(\mathrm{i}\omega t)$ are negative frequency. Completeness requires that both positive and negative frequency modes be included. Their contributions to $J^0_{KG}(x)$ are of opposite sign, so $J^0_{KG}(x)$ is interpreted as charge density and the quantity g in (6) is set equal to $qc/\hbar$ for particles of charge q.

If only particles, as opposed to both particles and antiparticles, are to be considered, then the KG field can be restricted to positive frequencies and the scalar product [7]

$$(\phi_1, \phi_2)_{KG} = \frac{\mathrm{i}}{\hbar}\int_t d\mathbf{x}\,\phi_1(x)^* \overleftrightarrow{\partial}_t\phi_2(x) \tag{7}$$

is positive definite. Here t denotes a spacelike hyperplane of simultaneity at instant t. The integrand of (7) looks like a particle density but this is misleading since $J^0_{KG}(x)$ can still be negative if components with two or more different frequencies are added