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Duality of boson and fermion: New intermediate-statistics

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ABSTRACT

In this work, we propose a new model for describing an intermediate-statistics particles system. Starting with a deformed grand partition function, we investigate several thermodynamical and statistical properties of a gas model of two-parameter deformed particles. We specifically focus on the low-temperature behavior of the model and the conditions under which either boson condensation or fermion condensation would occur in such a model are discussed. Our results obtained in this study reveal that the present deformed gas model exhibits duality of boson and fermion, and can be useful for approaching the thermostatistics of condensation characteristics in quantum systems.

1. Introduction

Non-linear behavior observed in complex systems is often required to consider non-trivial models beyond the standard approaches in several subfields of research in physics and mathematics. In the past years, deformed oscillator systems [1–3] have been one of the effective ways to approximate such non-linear behavior including many-body quantum interactions. For instance, they were used to understand higher-order effects in the many-body interactions in nuclei [4], and were also used to discuss the dynamical mass generated for quarks as well as the pure nuclear pairing force version of the Bardeen–Cooper–Schrieffer (BCS) many-body formalism [5]. Beside other applications in nuclear and particle physics [6], they were used to analyze some of the entanglement characteristics within a composite particle system [7,8] in the framework of quantum information theory [9].

On the other hand, deformed oscillator systems have been used to obtain some new formulations for constructing generalized statistical mechanics. In this context, we should mention some of the earlier studies in the literature conducted by one- and two-parameter deformed boson and fermion systems with or without quantum group symmetry structures [10–18]. As parallel to these investigations, statistical and thermodynamical consequences of studying q- and p,q-deformed boson and fermion oscillators have also been investigated [19–38].

Moreover, deformed structures have importance for constructing possible forms of intermediate-statistics, which shows an interpolation between the standard quantum statistics, namely the Bose–Einstein (BE) and Fermi–Dirac (FD) statistics. Although, some of the earlier versions of non-standard quantum statistics have been proposed in the literature such as Gentile [39], Green [40] and others [41–49], representation structures and possible physical applications for intermediate-statistics behavior are still being an active research area.

In this Letter, we first introduce a model for describing intermediate-statistics behavior by means of the (p,q)-deformed statistics, where p and q are real positive deformation parameters. By deforming the statistical weight, we introduce a new grand partition function for the model in terms of the deformation parameters p and q. We then discuss general thermostatistical properties of the model and in particular, we focus on the conditions under which either boson condensation or fermion condensation would occur in such a model. As is different from the other studies in the literature that we mentioned above, the present deformed model contains particles possessing both bosonic and fermionic features, and its particles exhibit essentially quasi-particle character having duality of boson and fermion. It reveals an effective approach to deal with the thermostatistics of quantum systems including condensation characteristics.

We organize the Letter as follows: In Section 2, we introduce the fundamental aspects of our model. In Section 3, we investigate general thermostatistical properties of a gas of deformed bosons and fermions in the thermodynamical limit, and derive many of the important thermodynamical functions in terms of the defor-

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mation parameters p and q. In the last section, we give our concluding remarks.

2. Physical basis of the model

As is known that the grand canonical partition function for bosons and fermions are given by the following relation [50–52]:

$$\Xi(T,V,\mu)=\sum_{\{n_k\}} g\{n_k\}\exp\left\{-\beta\sum_{k=1}^{\infty} n_k(\varepsilon_k-\mu)\right\}, \tag{1}$$

where μ is the chemical potential and $\beta=1/kT$, where k is the Boltzmann constant and T is the temperature of the system. The statistical weight of a set of occupation numbers $\{n_1,n_2,\ldots\}$ is defined as

$$g\{n_k\}=1 \quad \text{for}\ n_k=0,1,2,3,\ldots, \tag{2}$$

for bosons, and

$$g\{n_k\}=\begin{cases}1, & n_k=0,1,\\ 0, & n_k=2,3,4,\ldots,\end{cases} \tag{3}$$

for fermions, respectively. Hence, we know that different statistical weight leads to different quantum statistics. From this fact, we can find new quantum statistics interpolating the BE and FD statistics by deforming the statistical weight in a more general form.

In this work, we will consider two-parameter deformation of the statistical weight, which unifies the Bose and Fermi statistics. Our starting point is to generalize the statistical weight so that it may possess two-parameter denoted by p and q, which can have real positive values. The new statistical weight should also correspond to the standard BE and FD statistics in the special limiting cases. For this purpose, we can consider the following statistical weight:

$$\begin{aligned}&g\{n_k=0\}=1,\\&g\{n_k=1\}=p+q,\\&g\{n_k=2\}=p(p+q),\\&g\{n_k=3\}=p^2(p+q),\\&g\{n_k=4\}=p^3(p+q),\\&\ldots.\end{aligned} \tag{4}$$

This can be written as

$$g\{n_k\}=\begin{cases}1, & n_k=0,\\ p^{n_k-1}(p+q), & n_k=1,2,3,\ldots.\end{cases} \tag{5}$$

This statistical weight reduces to the BE statistics when $p=1$, $q=0$, while it reduces to the FD statistics when $p=0$, $q=1$. Note that the derivation of Eq. (5) is given in Appendix. Inserting Eq. (5) into Eq. (1), we obtain

$$\Xi(T,V,\mu,p,q)=\prod_{k=1}^{\infty}\left[1+\sum_{n_k=1}^{\infty}p^{n_k-1}(p+q)\exp\bigl(-\beta n_k(\varepsilon_k-\mu)\bigr)\right], \tag{6}$$

which leads to the following expression for the logarithm of the grand partition function:

$$\ln\Xi(T,V,\mu,p,q)=\sum_{k=1}^{\infty}\ln\left(\frac{1+qze^{-\beta\varepsilon_k}}{1-pze^{-\beta\varepsilon_k}}\right), \tag{7}$$

where $z=\exp(\beta\mu)$ is actually a deformed fugacity of the model and it will be defined below. If the smallest energy level is zero as in the case of an ideal gas, then $\varepsilon_{k=0}=0$, which in turn implies $0\le z\le(1/p)$.

In the next section, we will search for general thermostatistical consequences of the model represented via Eq. (7).

3. General thermostatistical properties of the model

As in the usual procedure in statistical mechanics [50–52], assuming a large volume and a large number of particles, we can replace the summations by integrals as

$$\sum_k \to \int_0^{\infty}\rho_{p,q}(\varepsilon)d\varepsilon, \tag{8}$$

where the one-particle density of states is defined by

$$\rho_{p,q}(\varepsilon)=g_{p,q}\frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}, \tag{9}$$

where $g_{p,q}$ is an additional degeneracy factor due to the spins of particles, and we have also used $\varepsilon=(\hbar^2\vec{k}^2/2m)$, which is valid for non-relativistic particles. Since we have $(2s+1)$ different spin orientations, it can be deduced some link between the additional degeneracy factor $g_{p,q}$ and the model deformation parameters (p,q) as $g_{p,q}=p+2q=2s+1$, which accordingly reduces to the standard results $g_{1,0}=1$ for usual bosons and $g_{0,1}=2$ for usual fermions, respectively. In this regard, we can also add an interpretation that intermediate values of p and q for the degeneracy factor $g_{p,q}$ in the model allow us to approximate some composite (or quasi)particle systems with fractional statistics.

However, the integral is not a good approximation near $\varepsilon=0$, where the discreteness of the energy levels is important. Due to the fact that when $\varepsilon=0$, $\ln\Xi$ in Eq. (7) diverges as $z\to(1/p)$, we have to take into account this term separately in the equation of state $(PV/kT=\ln\Xi)$ as follows:

$$\frac{PV}{kT}=\int_0^{\infty}\rho_{p,q}(\varepsilon)\ln\left(\frac{1+qze^{-\beta\varepsilon}}{1-pze^{-\beta\varepsilon}}\right)d\varepsilon+\ln\left(\frac{1+qz}{1-pz}\right). \tag{10}$$

On the other hand, the total number of particles in our model can be found from the relation $<n_k>=(-1/\beta)(\partial\ln\Xi/\partial\varepsilon_k)$ with the constraint $N=\sum_k<n_k>$. Accordingly, we obtain

$$N=\sum_{k=1}^{\infty}\frac{(p+q)ze^{-\beta\varepsilon_k}}{(1+qze^{-\beta\varepsilon_k})(1-pze^{-\beta\varepsilon_k})}+\frac{(p+q)z}{(1+qz)(1-pz)}, \tag{11}$$

where the mean occupation number of a single deformed particle state with energy ε_k can be deduced as

$$<n_k>=\frac{(p+q)ze^{-\beta\varepsilon_k}}{(1+qze^{-\beta\varepsilon_k})(1-pze^{-\beta\varepsilon_k})}. \tag{12}$$

Applying the same procedure given by Eqs. (8) and (9), the total number of particles for our model can be written as

$$N=\int_0^{\infty}\rho_{p,q}(\varepsilon)\frac{(p+q)ze^{-\beta\varepsilon}}{(1+qze^{-\beta\varepsilon})(1-pze^{-\beta\varepsilon})}d\varepsilon+\frac{(p+q)z}{(1+qz)(1-pz)}. \tag{13}$$

The last term in this equation is indeed the expected number of particles in the ground state. From Eqs. (10) and (13), we also have the following results:

$$\frac{P}{kT}=\left(\frac{g_{p,q}}{\lambda^3}\right)\bigl[f_{5/2}(z,q)+g_{5/2}(z,p)\bigr]+\frac{1}{V}\ln\left(\frac{1+qz}{1-pz}\right), \tag{14}$$

$$\frac{1}{\upsilon}=\frac{N}{V}=\left(\frac{g_{p,q}}{\lambda^3}\right)\bigl[f_{3/2}(z,q)+g_{3/2}(z,p)\bigr]+\frac{1}{V}\frac{(p+q)z}{(1+qz)(1-pz)}, \tag{15}$$

where $\lambda=h/\sqrt{2\pi mkT}$ is the thermal wavelength, and the deformed functions $f_n(z,q)$ and $g_n(z,p)$ are defined as