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Trigonometric protocols for shortcuts to adiabatic transport of cold atoms in anharmonic traps

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ABSTRACT

Shortcuts to adiabaticity have been proposed to speed up the "slow" adiabatic transport of ultracold atoms. Their realizations, using inverse engineering protocols, provide families of trajectories with appropriate boundary conditions. These trajectories can be optimized with respect to the operation time and the energy input. In this paper we propose trigonometric protocols for fast and robust atomic transport, taking into account cubic or quartic anharmonicities of the trapping potential. Numerical analysis demonstrates that this choice of the trajectory minimizes the final residual energy efficiently, and shows extraordinary robustness against anharmonic parameters. These results might be of interest for the state-of-the-art experiments on ultracold atoms and ions.

1. Introduction

There is currently much interest in precise and rapid manipulation of ultracold neutral atoms or ions [1–7], with the applications ranging from basic science and metrology to quantum information processing. Several approaches, different from typically "slow" adiabatic driving, have been put forward to achieve fast non-adiabatic transport, for example, optimal control theory [8–11]. The reduced transport time makes the cold atom manipulation more practical and permits researchers to avoid decoherence processes.

Recently developed concept of "shortcuts to adiabaticity" (STA) [12] provides alternative high-fidelity techniques for fast transport [13–25]. Among them, the inverse engineering, combined with perturbation theory and optimal control, is considered as a versatile toolbox for designing the optimal protocols, according to different physical criteria or operational constraints [15,16]. In other words, among the family of shortcuts satisfying the initial and final conditions, the specific trap trajectory can be chosen by optimizing the operation time or transient excitation energy, with a restriction of the allowed transient frequencies [15]. Furthermore, fast transport can be optimized with respect to spring-constant (color) noise, position fluctuation [20], and spring-constant error [22].

Although for many cold atom or ion experiments, shortcuts to adiabatic transport are usually designed for perfectly harmonic traps, most confining traps, *i.e.*, magnetic quadrupole potential [10], gravitomagnetic potential [26], electrostatic potential [27] and optical dipole traps [28], are of course intrinsically anharmonic. Actually, the anharmonic effects are of paramount importance in actual traps [29], which implies the unwanted final excitation, or even loss of atoms. This sets the bottleneck for implementing the possible speed-up, due to the intermediate energy excitation [30]. In Ref. [31], the optimal "bang-singular-bang" control is designed to achieve fast transitionless expansion of ultracold neutral atoms or ions in Gaussian anharmonic trap, with minimizing the time-averaged perturbative energy. In fact, the anharmonicity is also one of significant problems on designing the fast non-adiabatic ion transport [17], in which the optimal strategy is strongly required to minimize the energy of excitation. Up to now, several works have been devoted to dealing with the atomic transport in anharmonic traps and overcoming their difficulty. (i) The trap trajectory of transport in general power-law traps including cubic or quartic anharmonicities has been calculated from the classical Newton equation, and the quantum case for an atomic wave packet has been checked later [23]. (ii) The counter-diabatic driving, suggesting the compensating force, has been proposed for nonharmonic traps [19], which was implemented for trapped-ion displacement in phase space [25]. However the trap frequency and size of the atomic cloud might be modified with the anharmonicity [24]. (iii) The combination of inverse engineering and optimal control theory is proposed, but the anharmonicites should be considered as perturbation [24].

In this article, we present the trigonometric protocols for shortcuts to adiabatic transport in anharmonic traps, including the cubic or quartic terms. Particularly, we try a simple but efficient cosine ansatz with additional boundary condition, to eliminate the anharmonic corrections by nullifying final residual energy. Such choice

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also works perfectly for cancelling the spring constant error for two-ion transport [32]. This protocol is also similar to but different from sine protocol implemented in the experiment of atomic transport, in which high efficiency [11], above 97%, has been reported. Furthermore, our numerical analysis has illustrated that the designed shortcuts, particularly using the cosine ansatz, is much more stable with respect to anharmonic parameters, and the corresponding final residual energy is smaller. These results presented here, being aimed at the recent transport experiments with ultracold neutral atoms [8,10,11], are applicable to trapped ions [6,7,25] as well.

2. Transport with anharmonic traps

2.1. cubic anharmonicity

First of all, we consider the transport of an atom of mass m confined in an anharmonic trap with the cubic term, resulting from an expansion around the minimum of the real tapping potential [8, 23,24,27]. The whole trapping potential is written as

$$V(x,t) = \frac{1}{2}m\omega_0^2\,[x - x_0(t)]^2 + \frac{1}{3}m\frac{\omega_0^2}{\xi}\,[x - x_0(t)]^3\,, \tag{1}$$

where $x_0(t)$ represents the trap trajectory to be determined and ξ quantifies the strength of the cubic anharmonicity. According to the Newton's law, the motion of an atom obeys

$$\ddot{x} + \omega_0^2\,[x - x_0(t)] + \frac{\omega_0^2}{\xi}\,[x - x_0(t)]^2 = 0, \tag{2}$$

from which, by introducing $u = \omega_0 t_f$, $\tilde{x}_0 = x_0(t)/d$, $s = t/t_f$, we have the dimensionless form (from now on dots are derivatives with respect to $\tau = \omega_0 t$)

$$\ddot{\tilde{x}} + u^2\left(\tilde{x} - \tilde{x}_0\right) + \frac{u^2 d}{\xi}\left(\tilde{x} - \tilde{x}_0\right)^2 = 0. \tag{3}$$

In this case, an exact trap trajectory,

$$\tilde{x}_0(s) = \tilde{x}(s) + \frac{\xi}{2d}\left(1 - \sqrt{1 - \frac{4\ddot{\tilde{x}}d}{\xi u^2}}\right), \tag{4}$$

can be worked out by choosing the appropriate $\tilde{x}(s)$ with the right boundary conditions, when the inverse engineering is exploited. Nevertheless, there are many possible options, which leaves a space for the optimization. To minimize the anharmonic effect, we rewrite Eq. (3) and solve it perturbatively, see Ref. [23],

$$\ddot{\tilde{x}} + u^2(\tilde{x} - \tilde{x}_0) = -\frac{u^2 d}{\xi}(\tilde{x} - \tilde{x}_0)^2 \simeq -\frac{d}{\xi}\frac{(\ddot{\tilde{x}}_1)^2}{u^2}, \tag{5}$$

where $\tilde{x}_1(s)$, the trajectory of the center of mass in harmonic trap, satisfies

$$\ddot{\tilde{x}}_1 + u^2(\tilde{x}_1 - \tilde{x}_0) = 0. \tag{6}$$

The perturbative solution to the first order is $\tilde{x}(s) \simeq \tilde{x}_1(s) + (d/\xi) f_1(s)$ with

$$f_1(s) = -\frac{1}{u^3}\int_0^s \dddot{\tilde{x}}_1^2(s')\sin[u(s-s')]ds'. \tag{7}$$

Obviously, when the anharmonicity correction is eliminated, $f_1(s) = 0$, we have $\tilde{x}(s) \simeq \tilde{x}_1(s)$.

Moreover, in order to guarantee the shortcut to adiabaticity, we have to nullify the initial and final residual energy (in the unit of $\hbar\omega_0$) [22–24],

$$\Delta E = \frac{m\omega_0 d^2}{\hbar}\left[\frac{\dot{\tilde{x}}_1^2}{2u^2} + \frac{(\tilde{x}_1 - \tilde{x}_0)^2}{2} + \frac{d}{3\xi}(\tilde{x}_1 - \tilde{x}_0)^3\right], \tag{8}$$

which implies the boundary conditions at the edges, that is, $\tilde{x}_1(0) = 0$, $\dot{\tilde{x}}_1(0) = \dot{\tilde{x}}_1(1) = 0$, $\ddot{\tilde{x}}_1(0) = \ddot{\tilde{x}}_1(1) = 0$, and $\tilde{x}_1(1) = 1$. These are similar to those imposed from the commutator relation between the Lewis–Riesenfeld dynamical invariant and Hamiltonian at the edges, see Refs. [14,15]. Here an additional condition, $f_1(s) = 0$, should be fulfilled to cancel the anharmonic effect, thus nullifying the excitation energy.

In this spirit, we try the trigonometric protocols for designing the shortcut with minimizing the anharmonic effect. By assuming the cosine ansatz, $\tilde{x}_1(s) = a_0 + \sum_{j=1}^{3} a_j \cos[(2j-1)\pi s]$, we solve the trajectory of the center of mass satisfying all six boundary conditions mentioned above, and the additional condition $f_1(s) = 0$. As a consequence, we obtain

$$\tilde{x}_1(s) = \frac{1}{2} + a_1\cos(\pi s) + a_2\cos(3\pi s) + a_3\cos(5\pi s), \tag{9}$$

with the numbers $a_1 = -0.579$, $a_2 = 0.08725$ and $a_3 = -0.00825$. In this situation, $\tilde{x}(s) \simeq \tilde{x}_1(s)$, the trajectory of the trap center, $\tilde{x}_0(s)$, can be solved from Eq. (6), and the final residual energy (8) is nothing but zero. However, a_j $(j = 1,2,3)$ require high accuracy in the experiments, since the final residual energy is sensitive to the parameter fluctuation.

For comparison, we also write down the simple sine ansatz,

$$\tilde{x}_1(s) = s - (1/2\pi)\sin(2\pi s)\,. \tag{10}$$

This sine ansatz is relevant to but slightly different from that used in the experiment [11], in which $\tilde{x}_0(s) = s - (1/2\pi)\sin(2\pi s)$ is assumed and thus $\tilde{x}_1(s) = s - (9/10\pi)\sin(2\pi s)$. As a matter of fact, the reason for achieving high fidelity is that the $\tilde{x}_1$ satisfies the boundary conditions $\tilde{x}_1(0) = \dot{\tilde{x}}_1(0) = \dot{\tilde{x}}_1(1) = 0$ and $\tilde{x}_1(1) = 1$. However, the boundary conditions $\ddot{\tilde{x}}_1(0) = \ddot{\tilde{x}}_1(1) = -0.8 \neq 0$ suggest that it is not perfect shortcut. Furthermore, an additional free parameter is required in sine ansatz to nullify $f_1(s)$, which results in

$$\tilde{x}_1(s) = s + a_1\sin(2\pi s) + a_2\sin(4\pi s)\,, \tag{11}$$

with $a_1 = 0.3135$ and $a_2 = -0.236348$. By interpolating such $\tilde{x}_1(s)$, one can calculate $\tilde{x}_0(s)$ accordingly from Eq. (6). Fig. 1 shows and compare all trigonometric protocols $\tilde{x}_1$, see Eqs. (9)–(11), and the designed trap trajectories $\tilde{x}_0$.

In order to check the validity of the approximation, we further apply the designed trajectory $\tilde{x}_0$ and calculate the actual trajectory of the center of mass $\tilde{x}(s)$ in anharmonic trap from Eq. (3), with the initial boundary conditions, $\tilde{x}(0) = 0$ and $\dot{\tilde{x}}(0) = 0$. Thus, the final residual energy (8) is obtained by replacing $\tilde{x}_1(s)$ with $\tilde{x}(s)$, and is rather than zero. In Fig. 2, the final residual energy is compared for different trigonometric protocols. We demonstrate that cosine ansatz is much smaller than the one for sine ansatz, since the orthogonality of trigonometric functions in Eq. (7). This suggests that the consine ansatz for fast transport is much more stable with respect with the cubic anharmonicity. Moreover, the final excitation energy in principle decreases when the anharmonic effect becomes weaker, with increasing ξ. Since the sine ansatz (11) with an additional parameter for nullifying $f_1(s)$ is only valid for small anharmonicity when $\xi/d > 32.36$.

Here we would like to emphasize the advantages of the cosine ansatz presented here. On one hand, such type of trigonometric ansatz with only four free parameters is much simpler than the conventional polynomial ansatz, where at least seven coefficients should be assumed and solved numerically [14,15]. On the other hand, the cosine ansatz is more efficient to cancel the anharmonic correction, induced from Eq. (7). The final residual energy can be reduced by about two or three orders of magnitude, as compared to the other sine ansatz, see Fig. 2.