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Optical bistability and four-wave mixing in a hybrid optomechanical system

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A B S T R A C T

We explore theoretically the optical bistability and four-wave mixing (FWM) in a hybrid optomechanical system, where the mechanical resonator is simultaneously coupled to a cavity field and a two-level system (qubit). We can use a strong control field driving the cavity to control the bistable behavior of the steady-state photon number, phonon number, and the population inversion. The impact of qubit-resonator coupling strength on the bistable behavior is discussed. Furthermore, the two-level system can significantly modify the output fields of the cavity, leading to double optomechanically induced transparency (OMIT) and the enhancement of the FWM intensity. We find that the distance between the two peaks in the FWM spectrum can be controlled by the qubit-resonator coupling strength, and the peak value of the FWM intensity can be adjusted by the Rabi frequency of the control field.

1. Introduction

Hybrid quantum systems involving nanomechanical resonators have been under extensive investigation in the past few decades [1]. Work in these systems is motivated by studying the quantum behavior in macroscopic objects, and potential applications in quantum information science and nanoscale sensing. Various hybrid systems have been demonstrated by coupling mechanical resonators to other quantum objects, including optical cavities [2–4], superconducting transmission-line microwave cavity [5–7], and two-level systems in atoms, quantum dots, qubits as well as defects [8–14]. The interaction between mechanical and optical degrees of freedom via radiation pressure is the research topic of the rapidly developing field of cavity optomechanics [2–7]. Recently, remarkable progress has been made in this field, including ground state cooling of the nanomechanical resonator [15,16], optomechanically induced transparency (OMIT) [17–19], and ultrasensitive sensing [20,21]. Moreover, hybrid optomechanical systems with a two-level system were theoretically proposed [22–28] and experimentally demonstrated [29–31].

In the field of cavity optomechanics, optical bistability can be controlled by a strong laser field and was first observed by Dorsel et al. [32]. Recently, optical bistability in optomechanical systems with a quantum well [33], ultracold atoms [34–36], and a Bose–Einstein condensate (BEC) [37,38] has been extensively studied. Xiong et al. [39] and Dalafi et al. [40] theoretically discussed the impact of the cross-Kerr (CK) effect on the steady-state behavior of the mean photon number. In our previous work, we have investigated the optical bistability and dynamics in a hybrid optomechanical system, where the cavity field is coupled to the mechanical resonator via the radiation pressure and to the two-level system via the JC coupling [41]. Furthermore, in the presence of a strong control field, optical response of the optomechanical system to a weak probe field can be modified, leading to the phenomenon of OMIT [17–19] and double OMIT [23]. OMIT is the analog of electromagnetically induced transparency (EIT) [42,43], and EIT can be used to greatly enhance the four-wave mixing (FWM) [44,45].

Motivated by these developments, we study the optical bistability and four-wave mixing (FWM) in a hybrid optomechanical system, where the mechanical resonator is coupled to the cavity field via the radiation pressure and to the two-level system (qubit) via the Jaynes–Cummings (JC) coupling [22–24]. We find that the bistable behavior of the steady-state photon number and the phonon number can be effectively adjusted by the Rabi frequency of the control field and the detuning between the cavity and control field. When the qubit-resonator coupling strength is smaller than the transition frequency of the two-level system, the effect of the two-level system on the bistable behavior of the steady-steady state phonon number is negligible. However, the presence of the two-level system can lead to the appearance of

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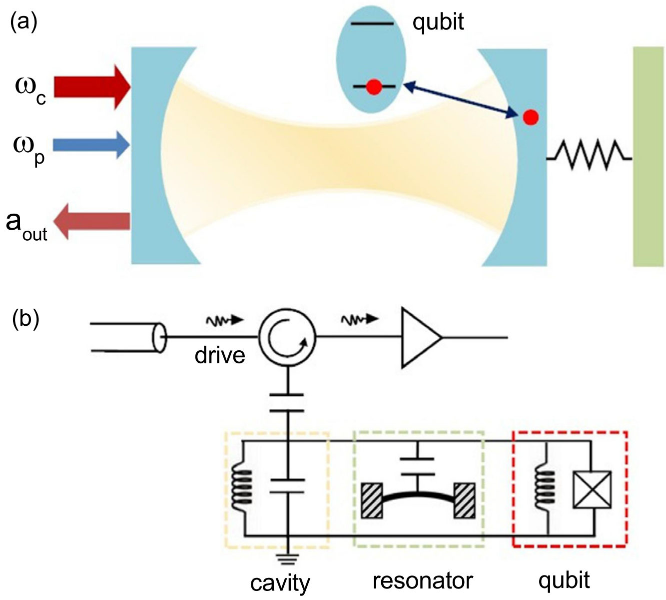

Fig. 1. Schematic diagram of the hybrid optomechanical system. (a) One mirror of the optomechanical cavity is fixed and the other is vibrating, and the cavity field is coupled to the mechanical resonator (vibrating mirror) via the radiation pressure. The mechanical resonator is also coupled to a two-level system (qubit) via the Jaynes–Cummings interactions. A strong control field with frequency ω_c and a weak probe field with frequency ω_p drive the cavity simultaneously, and a_{out} is the amplitude of the output fields. (b) Equivalent circuit diagram.

double OMIT in this hybrid system, therefore the FWM intensity can be doubly enhanced within the transparency windows.

2. Model and theory

The system under consideration is shown in Fig. 1. The mechanical resonator is coupled to a single-mode cavity field via the radiation pressure and to a two-level system via the Jaynes–Cummings (JC) coupling. There is no direct interaction between the cavity field and the two-level system. The Hamiltonian of this hybrid system can be written as

$$H_0 = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \frac{\hbar}{2}\omega_q\sigma_z - \hbar\chi a^\dagger a(b^\dagger + b) + \hbar g(b^\dagger\sigma_- + b\sigma_+), \quad (1)$$

where ω_a is the resonance frequency of the cavity field with the creation (annihilation) operator $a^\dagger(a)$; ω_b is the resonance frequency of the mechanical mode with the creation (annihilation) operator $b^\dagger(b)$; ω_q is the transition frequency between the ground state $|g\rangle$ and the excited state $|e\rangle$ of the two-level system, $\sigma_z \equiv |e\rangle\langle e| - |g\rangle\langle g|$ is the Pauli operator, and σ_+ (σ_-) is the raising (lowering) operator of the two-level system; χ is the coupling strength between the cavity field and the mechanical resonator, and g is the coupling strength between the mechanical resonator and the two-level system.

In order to investigate the optical response of the hybrid system, we assume that the cavity field is driven simultaneously by a strong control field with frequency ω_c and a weak probe field with frequency ω_p. In the rotating frame at the frequency ω_c, the Hamiltonian of the hybrid optomechanical system reads [23]

$$H = \hbar\Delta_a a^\dagger a + \hbar\omega_b b^\dagger b + \frac{\hbar}{2}\omega_q\sigma_z - \hbar\chi a^\dagger a(b^\dagger + b) + \hbar g(b^\dagger\sigma_- + b\sigma_+) + i\hbar(\Omega a^\dagger - \Omega^* a) + i\hbar(\varepsilon e^{-i\Delta t}a^\dagger - \varepsilon^* e^{i\Delta t}a), \quad (2)$$

where $\Delta_a = \omega_a - \omega_c$ is the detuning between the cavity field and the control field, and $\Delta = \omega_p - \omega_c$ is the detuning between the probe field and the control field; Ω and ε represent the Rabi frequencies of the strong control field and the weak probe field, respectively.

According to the Heisenberg equations of motion and introducing the corresponding damping and noise terms, the Heisenberg–Langevin equations can be written as:

$$\dot{a} = -(\gamma_a + i\Delta_a)a + i\chi a(b^\dagger + b) + \Omega + \varepsilon e^{-i\Delta t} + \sqrt{2\gamma_a}a_{in}(t), \quad (3)$$

$$\dot{b} = -(\gamma_b + i\omega_b)b + i\chi a^\dagger a - ig\sigma_- + \sqrt{2\gamma_b}b_{in}(t), \quad (4)$$

$$\dot{\sigma_-} = -(\frac{\gamma_q}{2} + i\omega_q)\sigma_- + igb\sigma_z + \sqrt{\gamma_q}\Gamma_-(t), \quad (5)$$

$$\dot{\sigma_z} = -\gamma_q(\sigma_z + 1) - 2ig(b\sigma_+ - b^\dagger\sigma_-) + \sqrt{\gamma_q}\Gamma_z(t), \quad (6)$$

where γ_a, γ_b, and γ_q denote the decay rates of the cavity field, mechanical mode, and two-level system, respectively. The operators $a_{in}(t), b_{in}(t), \Gamma_-(t)$, and $\Gamma_z(t)$ describe the corresponding environmental noises with zero mean values, i.e., $\langle a_{in}(t)\rangle = \langle b_{in}(t)\rangle = \langle\Gamma_-(t)\rangle = \langle\Gamma_z(t)\rangle = 0$. Using the mean-field approximation by factorizing the averages, one can obtain the following expectation value equations:

$$\langle\dot{a}\rangle = -(\gamma_a + i\Delta_a)\langle a\rangle + i\chi\langle a\rangle(\langle b^\dagger\rangle + \langle b\rangle) + \Omega + \varepsilon e^{-i\Delta t}, \quad (7)$$

$$\langle\dot{b}\rangle = -(\gamma_b + i\omega_b)\langle b\rangle + i\chi\langle a^\dagger\rangle\langle a\rangle - ig\langle\sigma_-\rangle, \quad (8)$$

$$\langle\dot{\sigma_-}\rangle = -(\frac{\gamma_q}{2} + i\omega_q)\langle\sigma_-\rangle + ig\langle b\rangle\langle\sigma_z\rangle, \quad (9)$$

$$\langle\dot{\sigma_z}\rangle = -\gamma_q(\langle\sigma_z\rangle + 1) - 2ig(\langle b\rangle\langle\sigma_+\rangle - \langle b^\dagger\rangle\langle\sigma_-\rangle). \quad (10)$$

In order to solve the nonlinear equations (7)–(10), we use the following ansatz [46]:

$$\langle a(t)\rangle = A_0 + A_+e^{i\Delta t} + A_-e^{-i\Delta t}, \quad (11)$$

$$\langle b(t)\rangle = B_0 + B_+e^{i\Delta t} + B_-e^{-i\Delta t}, \quad (12)$$

$$\langle\sigma_-(t)\rangle = L_0 + L_+e^{i\Delta t} + L_-e^{-i\Delta t}, \quad (13)$$

$$\langle\sigma_z(t)\rangle = Z_0 + Z_+e^{i\Delta t} + Z_-e^{-i\Delta t}, \quad (14)$$

where A_0, B_0, L_0, and Z_0 are, respectively, the steady-state solutions of a, b, σ_-, and σ_z in the absence of the probe field. For a weak probe field, $A_\pm, B_\pm, L_\pm$, and $Z_\pm$ are much smaller than the corresponding steady-state values. Upon substituting Eqs. (11)–(14) into Eqs. (7)–(10) and upon working to the lowest order in ε but to all orders in Ω, we obtain the following steady-state solutions:

$$A_0 = \frac{\Omega}{\gamma_a + i\Delta_a - i\chi(B_0 + B_0^*)}, \; B_0 = \frac{\chi|A_0|^2 - gL_0}{\omega_b - i\gamma_b},$$
$$L_0 = \frac{2gB_0Z_0}{2\omega_q - i\gamma_q}, \; Z_0 = -\frac{\gamma_q^2 + 4\omega_q^2}{\gamma_q^2 + 4\omega_q^2 + 8g^2|B_0|^2}, \quad (15)$$

$$A_- = \frac{\alpha_9\varepsilon}{\alpha_8\alpha_9 - \alpha_{10}}, \; A_+ = \frac{i\chi(\alpha_6 + \alpha_7^*)A_0^2\varepsilon^*}{\alpha_8^*\alpha_9^* - \alpha_{10}^*}, \quad (16)$$

where