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Quantum information with even and odd states of orbital angular momentum of light

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ABSTRACT

We address the possibility of using even/odd states of orbital angular momentum (OAM) of photons for the quantum information tasks. Single photon qubit states and two photon entangled states in even/odd basis of OAM are considered. We present a method for the tomography and general projective measurement in even/odd basis. With the general projective measurement, we show the Bell violation and quantum cryptography with Bell's inequality. We also describe hyper and hybrid entanglement of even/odd OAM states along with polarization, which can be applied in the implementation of quantum protocols like super dense coding.

1. Introduction

Quantum information protocols mainly rely on the fact that particle can be in a complex superposition of states. Polarization state of photons is used extensively to implement many quantum protocols. The polarization of a photon spans in a two dimensional Hilbert space. So the polarization state of a photon is considered as a qubit. Also, one can generate photons entangled in polarization using spontaneous parametric down conversion (SPDC) of a laser beam. All four maximally entangled states, Bell states, can be achieved in the polarization degree of freedom (DOF).

Orbital angular momentum (OAM) is another degree of freedom of photon that can be used in quantum protocols along with polarization so that the information carried per photon can be increased [1]. OAM entanglement can also be achieved by SPDC and many quantum protocols were demonstrated using the same [2–9]. The basis states of OAM span an infinite dimensional Hilbert space. This higher dimensionality is very useful for the denser coding of information in single photons [10]. One can achieve OAM entanglement in higher dimensions that can be used for many quantum protocols. Also using fractional values of OAM [11], one can achieve higher dimensional spatial entanglement [12]. However, we often need to use two dimensional OAM states for the ease of measurements. Also for many protocols using hybrid-entanglement, entanglement between polarization and OAM of photons, we need the two dimensional OAM sub-space [13,14].

Experimentally the restriction of OAM states to 2D is done by post selection to two orthogonal OAM states using diffractive holograms and a single mode fibre [2]. This results in the loss of photons, which reduces efficiency of the protocol. We investigate the possibility of using a 2D OAM space without any photon loss. This is possible since any infinite set of integers can be grouped into two natural categories: even and odd. In the case of OAM, this becomes possible because of an effective even/odd OAM sorter, an optical set up designed to separate even and odd states of OAM. However, the even/odd states of OAM have not been extensively explored for quantum information tasks. For that, one has to develop projective measurement in even/odd basis. We propose simple interferometric method for the projective measurements.

We demonstrate the tomography of the even/odd states with projective measurement in Pauli's operator bases. We also describe hyper-entanglement and hybrid-entanglement with polarization as another DOF and propose interferometric set up for the spin orbit Bell state analysis (SOBA). Measurements for checking the Bell's inequality in even/odd OAM entanglement is discussed for the first time. This can be applied in entanglement based cryptographic protocols. It is theoretically impossible to distinguish all Bell states using local operations and classical communications (LOCC) [15]. However, with hyper-entanglement and SOBA, one can distinguish all the Bell states of polarization using LOCC. Using the same, we describe efficient super dense coding.

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2. From infinite dimensional OAM space to two dimensional even/odd OAM space

The general infinite dimensional OAM space is spanned by the OAM values from $-\infty, .. -1, 0, +1, ... +\infty$. A general state in this infinite dimensional basis can be written as

$$|\psi\rangle = \sum_{m=-\infty}^{+\infty} c_m |m\rangle \tag{1}$$

with $\sum_{m=-\infty}^{+\infty} |c_m|^2 = 1$. However, many quantum experiments were realized using the OAM qubits in the reduced Hilbert space $\{|m\rangle, |-m\rangle\}$. In such cases OAM encoding or measurements were performed using diffraction through holograms and the mode filtering. Basically, here one neglects the photons generated with OAM $l \neq m, -m$ which results in photon loss. Moreover, the efficiency of mode filtering is also a limiting factor for quantum experiments with OAM. Thus to make an equivalent qubit state, Eq. (1) can be re written as

$$|\psi\rangle = \sum_k \left(c_{2k}|2k\rangle + c_{2k+1}|2k+1\rangle\right). \tag{2}$$

This can be considered as an even/odd OAM qubit. We define the appropriate operators in order to perform the measurements in the even/odd basis. The general projection operator is

$$P(\theta,\phi) = \sum_k \Big(\cos(\theta)|2k\rangle + e^{i\phi}\sin(\theta)|2k+1\rangle\Big) \Big(\cos(\theta)\langle 2k| + e^{-i\phi}\sin(\theta)\langle 2k+1|\Big). \tag{3}$$

With these projective measurements we can consider the whole OAM state as a qubit state and use for quantum protocols.

In the case of OAM entanglement, when we work with $\{m, -m\}$ basis, photons corresponding to other modes are lost in the measurement. For example, when we pump a non-linear crystal for SPDC using a Gaussian beam, the signal and idler photons are entangled in OAM. The two photon state is given as

$$|\Psi\rangle_{12} = c_0|0\rangle|0\rangle + \sum_{m=1}^{+\infty} c_m \left(|m\rangle|-m\rangle + |-m\rangle|m\rangle\right) \tag{4}$$

with $\sum_{m=0}^{+\infty} |c_m|^2 = 1$. In many of the OAM entanglement experiments, this state is projected in $\{+1, -1\}$ basis for treating it as a two qubit entangled state. In such cases, the probability of getting photons entangled in $\{+1, -1\}$ OAM states is $|c_1|^2 \ll 1$. Thus most of the down converted photons remain unused.

For even/odd OAM entanglement, we consider the parametric down conversion of an optical vortex of order 1 and having vertical polarization in a type I second order non-linear crystal. The state corresponding to the pair of photons produced by SPDC of this beam is given by

$$|\Psi\rangle_{12} = \sum_{m=-\infty}^{+\infty} c_m |m\rangle_1 |1-m\rangle_2 \otimes |H\rangle_1 |H\rangle_2 \tag{5}$$

By grouping all even and odd OAM states, one can rewrite the expression for the OAM state in Eq. (5) as

$$\sum_{m=-\infty}^{+\infty} c_m(|m\rangle_1|1-m\rangle_2) = \sum_{k=-\infty}^{+\infty} c_{2k}(|2k\rangle_1|1-2k\rangle_2) + \sum_{k=-\infty}^{+\infty} c_{1-2k}(|1-2k\rangle_1|2k\rangle_2). \tag{6}$$

Thus

$$|\Psi\rangle_{12} = \left(\sum_{k=-\infty}^{+\infty} c_{2k}(|2k\rangle_1|1-2k\rangle_2) + \sum_{k=-\infty}^{+\infty} c_{1-2k}(|1-2k\rangle_1|2k\rangle_2)\right) \otimes |H\rangle_1|H\rangle_2 \tag{7}$$

From the conservation of OAM, we have

$$\sum_{k=-\infty}^{+\infty} |c_{2k}|^2 = \sum_{k=-\infty}^{+\infty} |c_{1-2k}|^2 = \frac{1}{2}\sum_{m=-\infty}^{+\infty} |c_m|^2 = \frac{1}{2}. \tag{8}$$

Thus one can arrive at an operational expression for even/odd OAM entanglement as

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|E\rangle_1|O\rangle_2 + |O\rangle_1|E\rangle_2\right) \otimes |H\rangle_1|H\rangle_2. \tag{9}$$

Here $|E\rangle$ and $|O\rangle$ correspond to the even/odd states on detection. Thus, we get a two qubit entanglement in OAM without loosing any photons.

3. State tomography for OAM states in even/odd basis

State of OAM in $\{|m\rangle, |-m\rangle\}$ Hilbert space can be represented using OAM Poincaré sphere [16,17]. For even/odd basis, we need to find the Stokes vector for the superposition state given in Eq. (2) by projective operators that are

$$\begin{aligned}
P_0 &= \sum_k (|2k\rangle\langle 2k| + |2k+1\rangle\langle 2k+1|) \\
P_1 &= \sum_k (|2k\rangle\langle 2k| - |2k+1\rangle\langle 2k+1|) \\
P_2 &= \sum_k (|2k\rangle\langle 2k+1| + |2k+1\rangle\langle 2k|) \\
P_3 &= \sum_k i(|2k\rangle\langle 2k+1| - |2k+1\rangle\langle 2k|)
\end{aligned} \tag{10}$$

Now, we define the Stokes parameters as

$$\begin{aligned}
s_0 &= \langle\psi|P_0|\psi\rangle \equiv \sum_k \left(c_{2k}c_{2k}^* + c_{2k+1}c_{2k+1}^*\right) \\
s_1 &= \langle\psi|P_1|\psi\rangle \equiv \sum_k \left(c_{2k}c_{2k}^* - c_{2k+1}c_{2k+1}^*\right) \\
s_2 &= \langle\psi|P_2|\psi\rangle \equiv \sum_k \left(c_{2k}^*c_{2k+1} + c_{2k}c_{2k+1}^*\right) \\
s_3 &= \langle\psi|P_3|\psi\rangle \equiv i\sum_k \left(c_{2k}^*c_{2k+1} - c_{2k}c_{2k+1}^*\right)
\end{aligned} \tag{11}$$

3.1. Measurements in linear even/odd basis for s_0 and s_1

We consider an OAM sorter [18–20] for the measurement of s_0 and s_1. There are other methods of OAM sorting where each and every OAM are transformed into transverse momentum states [21]. But for our description, we consider the even/odd sorter only. The setup is given in Fig. 1. Consider a general even/odd OAM superposition state given in Eq. (2). Applying beam splitter operation, the state evolves through two arms of the interferometer with a $\frac{\pi}{2}$ phase. In the reflected arm, a dove prism is inserted which is rotated by an angle $\frac{\alpha}{2}$. The dove prism angle can be calibrated using a Hermite Gaussian beam HG_{01} passing through it, as the rotation of the dove prism will result in rotation of the two lobes. Dove prism introduces an