Physics Letters A ••• (••••) •••–•••

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Physics Letters A

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Analysis of whisker growth on a surface of revolution

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ARTICLE INFO

Article history:
Received 9 May 2017
Received in revised form 21 June 2017
Accepted 28 June 2017
Available online xxxx
Communicated by M. Wu

Keywords:
Whisker growth
Grain boundary diffusion
Growth rate
Stress relaxation

ABSTRACT

A general mass transport equation for diffusion-controlled whisker growth on a surface of revolution is formulated. Two limiting cases for the whisker growth of a circular cylinder-like whisker are analyzed; one surface is planner, and the other one is spherical. The growth rate is proportional to the concentration difference for the growth of the whisker on both the planar surface and the spherical surface. Using the relation between mechanical work and chemical energy, the growth rate is found to be proportional to the pressure difference. Assuming the whisker growth as the mechanism for stress relaxation of a thin film, the whisker length and the stress relaxation associated with the whisker growth are found to be exponential functions of time.

1. Introduction

Tin (Sn) whiskers, grown on the surface of Sn-plated films and Sn-based alloys, have imposed a great challenge to the reliability of micro- and nanoelectronics due to the possibility of short circuit induced by Sn whiskers. The random-like formation of Sn whiskers has complicated the understanding of the fundamental mechanisms controlling the nucleation and growth of Sn whiskers and the efforts to possibly mitigate the formation and growth of Sn whiskers. In general, the nucleation and growth of Sn whiskers can be considered as a local mechanism/process for stress relaxation, which competes with other mechanisms/processes, such as dislocation motion and grain boundary sliding, for stress relaxation. It is the energy gradient (stress gradient or concentration gradient) which determines the growth of Sn-whiskers.

Considering the confinement effect of a surface oxide layer on the stress relaxation in a Sn film, Tu [1] proposed that Sn whiskers initiate at sites with the fracture of the surface oxide layer and atomic migration is driven by a long-range stress gradient. Galyon and Palmer [2] pointed out the importance of the mass transport through grain boundary network in regulating the growth of Sn whiskers. Boettinger et al. [3] suggested that localized creep of columnar grain structures to relieve the compressive stress in thin films contributes to the growth of Sn whiskers. Yang [4] studied diffusion-limited growth of whiskers from the framework of lattice diffusion. Based on the concept of grain boundary fluid flow [5,6], Tu and Li [7] derived the growth rate of a whisker as a function of viscosity and the pressure gradient, which reduces to the one controlled by grain boundary diffusion [1] when the Einstein relation is used. Buchovecky et al. [8] suggested that Sn whiskers initiate from soft Sn-grains in a Sn surface coating and considered the contribution of both time-independent plastic deformation and grain boundary diffusion in numerical simulation of the growth of Sn-whiskers. Chason and co-workers [9] discussed the role of intermetallic growth, stress evolution, and plastic deformation in the whisker formation. Considering the mass transport through grain boundary network, Li and co-workers [10–12] analyzed the growth of whiskers and hillocks controlled by interfacial flow. Pei et al. [13] assumed that the change rate of average volume is proportional to the stress above a critical value and analyzed the stress relaxation during the growth of Sn whiskers. Wang et al. [14] developed a whisker pinch-off model to analyze the morphological changes of Sn whiskers and the decrease in the whisker density during thermal cycling. Recently, Yang [15] proposed a nonlinear viscous model for the growth of Sn whiskers and obtained the evolution of the whisker length as a function of time controlled by the stress relaxation during the whisker growth. All of the analyses have been based on the condition that the surface of Sn films is planar. No analysis has been focused on curved surfaces and from the theory of diffusion.

In the study of indentation-induced growth of Sn whiskers, Williams et al. [16] sketched a possible path for stress-assisted diffusion of atoms to the root of a whisker during the whisker growth, in which the atomic diffusion likely starts from the surface

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http://dx.doi.org/10.1016/j.physleta.2017.06.050

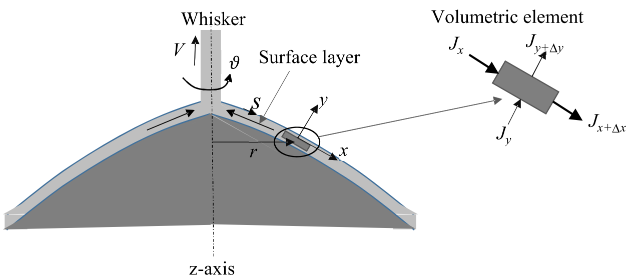

Fig. 1. Schematic diagram of the growth of a whisker on a surface of revolution and a volumetric element (J_x and J_y are the components of atomic flux in the x- and y-direction, respectively).

of a hemisphere instead of a planar surface. Considering the observation of the growth of Sn whiskers from pure Sn-plated relay terminal as shown in NASA website (https://nepp.nasa.gov/whisker/) and possible stress-assisted diffusion of atoms from the surface of a hemisphere for the indentation-induced whisker growth, we analyze the whisker growth on a surface of revolution from the framework of grain boundary/interface diffusion in contrast to the use of a long-range stress gradient as the driving force by Tu [1] and the use of grain boundary fluid flow by Tu and Li [7] and Yang [15]. The site for the growth of a whisker is assumed to be at the intersection between axisymmetric axis and the surface of revolution. A general equation for the diffusion on the surface of revolution is derived. Two limiting cases are analyzed for the whisker growth; one surface is spherical, and the other is planar.

2. Problem formulation

Yang and Li [17] pointed out that the stress-driven diffusion of atoms (vacancies) from high pressure to low pressure may not be complete without knowing the mechanism of diffusion since there still exists the diffusion of atoms (vacancies) driven by the gradient of concentration even under the condition of uniform pressure. The appropriate approach is to analyze the diffusion of atoms (vacancies) driven by the gradient of concentration and assume triple junctions, surfaces, and grain boundaries as the sinks and sources of vacancies. Here, we consider the diffusion in a grain boundary, which is driven by the gradient of concentration, for the analysis of the whisker growth.

Fig. 1 shows schematically the growth of a whisker on a surface of revolution with the axis of revolution being z axis and a volumetric element for the diffusion of atoms in the film abutted to an impermeable substrate. The surface of revolution is defined by a function $r(x)$, representing the radius of the median of the film, and an intrinsic curvilinear, orthogonal coordinate system (x, ϑ, y) is also depicted in Fig. 1. Due to axisymmetric characteristic of the problem, the non-zero components of atomic flux are J_x and J_y in the x- and y-direction, respectively. The change of the resultant amount of atoms in the volumetric element per unit time can be calculated as

$$\frac{\Delta n}{\Delta t} = (J_{x+\Delta x}\Delta l_{x+\Delta x} - J_x \Delta l_x)\Delta y + (J_{y+\Delta y} - J_y)\Delta l \Delta x \tag{1}$$

where Δn is the change of the resultant amount of atoms in the volumetric element in the time increment of Δt, Δl_x and $\Delta l_{x+\Delta x}$ are the chord lengths of the element at x and $x+\Delta x$, respectively, Δl is the average chord length of the element, Δx is the increment of x (which is equal to the incremental arc length measured along the median of the film from the axis of revolution, i.e. Δs), and Δy is the increment of y. For the volumetric element with both Δl_x and $\Delta l_{x+\Delta x}$ subtending the same angle, Eq. (1) can be simplified to the first order of approximation as

$$\begin{aligned}\frac{\Delta n}{\Delta t} &= (J_{x+\Delta x} r_{x+\Delta x} - J_x r_x)\Delta y \Delta\vartheta + (J_{y+\Delta y} - J_y)\Delta l \Delta x \\ &= \Delta(J_x r)\Delta y \Delta\vartheta + r\Delta J_y \Delta\vartheta \Delta x\end{aligned} \tag{2}$$

The change of the amount of atoms in the volumetric element can be calculated from the concentration, c, as $r\Delta c\Delta x\Delta y\Delta\vartheta$. Using the concentration, c, and letting $\Delta t \to 0$, one obtains a differential equation from Eq. (2) as

$$\frac{\partial c}{\partial t} = \frac{1}{r}\frac{\partial(J_x r)}{\partial s} + \frac{\partial J_y}{\partial y} \tag{3}$$

for the diffusion of atoms in a thin film on a surface of revolution with axisymmetric characteristic. The relationships between the flux components and the concentration of atoms in the curvilinear coordinate system are

$$J_x = -D\frac{\partial c}{\partial s} \quad \text{and} \quad J_y = -D\frac{\partial c}{\partial y} \tag{4}$$

with D being the diffusion coefficient of atoms in the film.

For the diffusion of atoms in a thin film with impermeable substrate and a surface confined by an ultrathin oxide layer, the boundary conditions for the flux component of J_y are

$$J_y\big|_{y=0} = J_y\big|_{y=h} = 0 \tag{5}$$

with h being the film thickness. Integrating Eq. (3) with respect to y from 0 to h yields

$$\frac{\partial}{\partial t}\int_0^h c\,dy = \frac{1}{r}\frac{\partial}{\partial s}\int_0^h J_x r\,dy \tag{6}$$

For quasi-steady state diffusion, Eq. (6) gives

$$\frac{1}{r}\frac{\partial}{\partial s}\int_0^h J_x r\,dy = 0, \quad \text{i.e.} \int_0^h J_x r\,dy = \text{const.} \tag{7}$$

Define the average concentration of atoms, $<c>$, as

$$<c> = \frac{1}{h}\int_0^h c\,dy \tag{8}$$

Substituting Eqs. (4) and (8) in Eq. (7), one obtains

$$r\frac{\partial <c>}{\partial s} = \text{const.} \tag{9}$$

which is the differential equation describing the diffusion of atoms in a thin film abutted to a surface of revolution under the condition that the diffusion is quasi-static and axisymmetric about the axisymmetric axis.

Using Eq. (9) and boundary conditions, one can calculate the concentration distribution of atoms in the thin film and the flux of atoms into the root of a whisker. From the mass balance, the growth rate of the whisker then can be determined. In the following, only two limiting cases are discussed; one surface is spherical, and the other is planar.

3. Limiting cases

Here, we assume that the whisker growth is controlled by grain boundary diffusion driven by the gradient of concentration. There are $h = \delta$ (the thickness of grain boundary) and $D = D_{gb}$ (the diffusivity of grain boundary).

Please cite this article in press as: F. Yang, Y. Shi, Analysis of whisker growth on a surface of revolution, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2017.06.050