Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

An improved continuum model for traffic flow considering driver's memory during a period of time and numerical tests

Cheng Rongjun [a,b,c], Ge Hongxia [a,b,c], Wang Jufeng [d,*]

[a] *Faculty of Maritime and Transportation, Ningbo University, Ningbo 315211, China*
[b] *Jiangsu Province Collaborative Innovation Center for Modern Urban Traffic Technologies, Nanjing 210096, China*
[c] *National Traffic Management Engineering and Technology Research Centre, Ningbo University Sub-centre, Ningbo 315211, China*
[d] *Ningbo Institute of Technology, Zhejiang University, Ningbo 315100, China*

ARTICLE INFO

Article history:
Received 18 May 2017
Received in revised form 27 June 2017
Accepted 27 June 2017
Available online xxxx
Communicated by C.R. Doering

Keywords:
Traffic flow
Extended continuum model
Driver's memory
Stability

ABSTRACT

Considering effect of driver's memory during a period of time, an improved continuum model for traffic flow is proposed in this paper. By means of linear stability theory, the improved model's linear stability with driver's memory is obtained, which demonstrates that driver's memory have significant influence stability of traffic flow. The KdV–Burgers equation is deduced to describe the propagating behavior of traffic density wave near the neutral stability line by nonlinear analysis. Numerical results show that driver's memory has negative impact on stability of traffic flow, which will lead to traffic congestion.

1. Introduction

With the development of social economy and the increase of urban population and cars, traffic problems such as traffic congestion, air pollution, parking, traffic accident, have increasingly become the focus of attention [1–13]. Therefore, more and more scholars are devoted to the study of traffic flow. For the better understanding of traffic flow, a variety of traffic flow models have been proposed with different perspectives, which can reveal kinds of interesting and important traffic phenomena such as density waves, local clusters, phantom traffic jam, phase transition, stop-and-go flows. Generally speaking, these existing models are divided into four categories, such as the car-following models [14–26], the cellular automaton models [27–30], the gas kinetic models [31–33], and the hydrodynamic models [34–36], which can figure out the complicated constitution behind the traffic congestion phenomenon.

The aim of car-following models is to describe the interactions between vehicles on the basis of the idea that a driver controls the vehicle to response the stimulus from the preceding cars. In more than 50 years, many car-following models have been built. Among this models, the optimal velocity model (OVM) [37] proposed by Bando is one of the most popular models, which can explain the qualitative characteristics of the actual traffic flow, such as the stop-and-go phenomenon, traffic instability and the congestion evolution and so on. Inspired by the idea of OVM, many new car-following models have been presented considering various aspects of driving behaviors [14–26]. However, the OVM has the drawbacks of high acceleration rate and unrealistic deceleration. To improve OVM, Jiang et al. [38] put forward a full velocity difference model (FVDM) with the consideration of full velocity difference.

By observing the driver's behavior in real traffic, Herman founded that a driver will remain the memory of historic information during the driving process [39]. Therefore, the effects of driver's memory has been taken into account in car-following models, which shown that the past information will enhance the traffic flow stability [40–47]. From the above models, we founded that the driver's memory effect is only considered at a time point. In real traffic, driving behavior is a continuous process, which indicates that a driver response correctly based on the corresponding received information not only at a time point but also during a period of time. So it is necessary and important to take the effect of driver's memory during a period of time into account in the traffic flow models. However, there is little research on the continuum models related to driver's memory during a period of time. According to this point, an improved continuum model will be presented to investigate driver's memory during a period of time on traffic flow.

* Corresponding author.
E-mail address: wjf@nit.zju.edu.cn (J. Wang).

http://dx.doi.org/10.1016/j.physleta.2017.06.047

In this paper, an improved continuum traffic flow model with the consideration of driver's memory during a period of time is presented through applying the usual connection method of macro–micro variables. The effect of driver's memory on the stability of traffic flow is studied. The linear and non-linear analysis for this new traffic model is obtained by means of linear and non-linear theory, respectively. The KdV–Burgers equation is derived and the soliton solution is given. Theoretic analysis and numerical simulations have been proposed to explore this complex phenomenon resulted. Numerical simulations demonstrate that the effect of driver's memory has negative impact on the stability of traffic flow, which leads to traffic congestion and occurrence of chaotic phenomena.

2. The improved continuum model

It is well known that the driver's memory plays an important effect in real traffic. Considering the effect of driver's memory during a period of time, we propose an improved car-following model as follows:

$$\frac{dv_n(t)}{dt} = a\left[V\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} \Delta x_n(\mu)d\mu\right) - v_n(t)\right] + \lambda\Delta v_n(t), \qquad (1)$$

where $\Delta v_n = v_{n+1} - v_n$ is the velocity difference between the preceding vehicle $n+1$ and the following vehicle n, $\Delta x_n = x_{n+1} - x_n$ represents the headway difference between car $n+1$ and car n, $V(\cdot)$ represents the optimal velocity function, a and λ are sensitivity parameters, τ_0 is the memory time length of the period. Equation (1) indicates that traffic acceleration is determined by both the optimal velocity and velocity difference and the optimal velocity with the consideration of driver's memory effect during a period of time $[t-\tau_0, t]$. When $\tau_0 \to 0$, Eq. (1) is reduced to $\frac{dv_n(t)}{dt} = a[V(\Delta x_n(t)) - v_n(t)] + \lambda\Delta v_n(t)$, which is FVDM. In this viewpoint, our improved model is an extended version of FVDM. The $V(\Delta x_n(t))$ used in this paper is adopted as below [37]:

$$V\big(\Delta x_n(t)\big) = v_{\max}\big[\tanh\big(\Delta x_n(t) - h_c\big) + \tanh(h_c)\big]/2, \qquad (2)$$

where h_c is the safety distance and $v_{\max}$ is the maximal velocity.

In order to deduce the continuum model from the car-following model, we convert the above micro variables into macro variables by the following transform:

$$\begin{aligned} &v_n(t) \to v(x,t), \quad v_{n+1}(t) \to v(x+h,t), \\ &V'\big(\Delta x_n(t)\big) \to \bar{V}'(h), \quad V\big(\Delta x_n(t)\big) \to V_e(\rho), \\ &V\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} \Delta x_n(\mu)d\mu\right) \to V\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} h(x,t)dt\right), \end{aligned} \qquad (3)$$

where h represents the mean distance between two adjacent vehicles, $\rho(x,t)$ and $v(x,t)$ represents the macro density and speed at place (x,t). Through the density ρ and the mean headway $h = \frac{1}{\rho}$, we define the equilibrium speed $V_e(\rho)$ and have $\bar{V}'(h) = -\rho^2 V_e'(\rho)$.

For the simplicity of analysis, we make the Taylor series expansion of $v(x+h,t)$ to the second-order term and neglect the high-order terms and derive

$$\Delta v_n(t) = v(x+h,t) - v(x,t) = v'(x,t)h + \frac{1}{2}v''(x,t)h^2. \qquad (4)$$

Putting Eqs. (3) and (4) into Eq. (1), we can derive

$$\begin{aligned} \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} &= a\left[V\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} h(x,t)dt\right) - v_n(t)\right] \\ &\quad + \lambda\left(v'(x,t)h + \frac{1}{2}v''(x,t)h^2\right). \end{aligned} \qquad (5)$$

Equation (5) can be rewritten as

$$\begin{aligned} &\frac{\partial v}{\partial t} + (v - \lambda h)\frac{\partial v}{\partial x} \\ &= a\left[V\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} h(x,t)dt\right) - v_n(t)\right] + \frac{\lambda h^2}{2}v''(x,t). \end{aligned} \qquad (6)$$

Combining the conservative equation with Eq. (6), an improved continuum model considering driver's memory during a period of time is derived as follows:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \rho\dfrac{\partial v}{\partial x} + v\dfrac{\partial \rho}{\partial x} = 0, \\ \dfrac{\partial v}{\partial t} + (v - \lambda h)\dfrac{\partial v}{\partial x} \\ \quad = a\left[V\left(\dfrac{1}{\tau_0}\displaystyle\int_{t-\tau_0}^{t} h(x,t)dt\right) - v_n(t)\right] + \dfrac{\lambda h^2}{2}v''(x,t). \end{cases} \qquad (7)$$

3. Stability analysis

In order to obtain the stability condition for the improved continuum traffic model (7) and clarify how to select the parameters to illustrate the phase transition, the linear stability theory is applied to analyze the traffic flow model.

First of all, the variable $V(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} h(x,t)dt)$ is processed by using mean value theorem for integrals as follows:

$$\begin{aligned} V\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} h(x,t)dt\right) &= V_e\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} \rho(x,t)dt\right) \\ &= V_e\big(\rho(x, t - \gamma\tau_0)\big), \end{aligned} \qquad (8)$$

where $t - \gamma\tau_0 \in [t-\tau_0, t]$ and $0 < \gamma < 1$.

Adopting of Taylor expansions of $\rho(x, t-\gamma\tau_0)$ and $V_e(\rho(x, t-\gamma\tau_0))$ by ignoring higher order terms as follows:

$$\rho(x, t-\gamma\tau_0) = \rho(x,t) - \gamma\tau_0\frac{\partial \rho}{\partial t}, \qquad (9)$$

$$V_e\big(\rho(x, t-\gamma\tau_0)\big) = V_e\big(\rho(x,t)\big) - \gamma\tau_0 V_e'(\rho)\frac{\partial \rho}{\partial t}. \qquad (10)$$

Substituting Eqs. (9) and (10) into Eq. (7), we will derive

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \rho\dfrac{\partial v}{\partial x} + v\dfrac{\partial \rho}{\partial x} = 0, \\ \dfrac{\partial v}{\partial t} + \big(v - \lambda h - a\gamma\tau_0\rho V_e'(\rho)\big)\dfrac{\partial v}{\partial x} \\ \quad = a\big[V_e(\rho) - v(t)\big] + a\gamma\tau_0 V_e'(\rho)v\dfrac{\partial \rho}{\partial x} + \dfrac{\lambda h^2}{2}v''(x,t). \end{cases} \qquad (11)$$

For the convenience of analysis, a vector form of Eq. (11) is written as follows:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{A}\frac{\partial \boldsymbol{U}}{\partial x} = \boldsymbol{E}, \qquad (12)$$

where