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Trapped electrons and solitary waves in non-uniform mixture of ionized gases

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ABSTRACT

Nonlinear electrostatic structures are investigated in a non-uniform mixture of two ionized gases which account for the effects of electron trapping, nonextensivity and field-aligned shear flow of one ion species relative to the other. Since electron trapping, a prerequisite for structure formation, introduces a stronger nonlinearity therefore stationary descriptions are represented by a Schamel rather than a Korteweg–de Vries equation. For a given solitary pulse width, the amplitude is hence consequently larger. The shear flow also enhances the amplitude while increased population of the energetic electrons is destructive for the solitary structures. This theoretical model is a general one and here it is applied to a mixture of oxygen–hydrogen plasma of the *F*-region ionosphere for illustration.

Nonlinear electrostatic solitary structures formed by the perturbations of an arbitrary amplitude have been investigated using energy integral formalism in fully ionized gases (or plasmas) [1–6]. In the small amplitude limit, the perturbation expansion method has been employed to investigate ion acoustic solitary structures in an unmagnetized and magnetized plasmas [7–9]. The same approaches have also been used by many authors to investigate the soliton formation in the dusty plasmas [10,11] and in fluids of electrons, positrons and ions [6]. Ion acoustic wave is a fundamental low frequency mode of homogeneous plasmas which couples with the low frequency drift wave [12,13] in non-uniform plasmas. The nonlinear electrostatic structures have been observed in both Laboratory [14,15] as well as Space plasmas [16,17]. It is well-known that the nonlinear ion acoustic waves (IAWs) form KdV solitons in homogeneous plasmas.

Long ago [18], it was shown that electrons which have lesser kinetic energy are trapped in the nonlinear wave potential. This leads to the confinement of electrons by the wave potential to a region of the phase space where they oscillate. The amplitude, thickness and speed of the solitary wave depend crucially upon the population of the trapped electrons. The particle trapping has been observed in both Laboratory and Space plasmas [18–23]. Schamel developed the pseudo-potential method to construct the equilibrium solutions which is considered as a breakthrough in the theory of holes or phase space vortices [18,24–26]. Several authors have investigated the effects of the trapped electrons on solitons in electron–ion as well as in dusty plasmas [27–29].

Schamel et al. [30] presented a detailed analysis of the electron trapping nonlinearity in relation with the existence of coherent stationary electro-static structures in a current-carrying homogeneous plasma. They could show that such structures are nonlinear in character represented by a continuum of modes which are governed by a nonlinear dispersion relation. The latter resembles a van Kampen continuum but it is in fact determined by the trapping nonlinearity. This implies that the well-known linear modes, as coherent single mode structures, must be interpreted in the nonlinear sense. As discrete modes they are imbedded in the nonlinear continuum. Hence trapping and coherency, our main topic, are inextricably linked and can therefore not be treated as separate issues. This is often overlooked in conventional wave analyses which rest on perturbative nonlinear models in which linear wave theory provides the lowest order approximations.

In case of nonuniformity of plasma, the drift kinetic equation is a useful tool to study the low-frequency ($\omega \ll \Omega_i$) drift waves and their coupling with the ion acoustic wave. Eliasson and Shukla [31] presented a comprehensive review about the formation and properties of the coherent structures in the presence of the electron trapping. In a single ion component plasma, they obtained stationary solution of nonlinear drift waves using quasi-neutrality in the limit $|\partial_t| \ll \Omega_i$, $k_\perp \rho_e \ll k_\perp \rho_i \ll 1$ and $c_s \ll \omega/k_\parallel$. Several authors used drift kinetic equation for electrons to investigate the

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coherent structures defined by Schamel-type modified Kadomtsev–Petviashvili equation [32] to incorporate the electron trapping nonlinearity and the perpendicular space dependencies [31,33] and references therein. Several investigations have appeared in the literature on the nonlinear coherent structures and electron trapping in different limits [18,27,28,31,33–35].

The particle trapping is a source of free energy in the system. The sheared flow is an additional source of free energy in plasmas and D'Angelo [36] pointed out that it can produce purely growing electrostatic instability. In this case, the IAW does not appear in the magnetized plasma. However if the shear flow is negative then the IAW can propagate in plasma and its linear dispersion relation is modified [37]. Nonlinear dynamics of the shear flow modified the IAW has also been investigated [8].

In a mixture of ionized gases, if one of the species has sheared flow such as in the upper ionosphere, the oscillatory instabilities can appear instead of a purely growing D'Angelo mode. In a plasma of two gases, four modes of the ion acoustic waves will couple due to the shear flow; one corresponding to the lighter ions and the other corresponding to the heavier ions. If the heavier ions have field-aligned shear flow, then one of the branches of the ion acoustic wave can become unstable under a certain condition.

In this work, we investigate the effects of the trapped electrons and field-aligned shear flow of the heavier ions into the non-uniform stationary plasma of lighter ions. This complex nonlinear system of the magneto-fluids is analyzed by using the small amplitude limit and local approximation. The perturbed densities of the ions of the two charged gases are estimated by using the continuity equations up to ϵ^2 order, where $\epsilon \ll 1$ is the smallness parameter which determines the strength of the nonlinearity. The four coupled linear ion acoustic modes are also analyzed briefly.

The aim of present study is to investigate the linear and nonlinear dynamics of the electrostatic IAW in a system of two mixed ionized gases in the presence of trapped electrons and field-aligned shear flow of one of the ion species. The plasma of two mixed gases is magnetized with constant field $\mathbf{B}_0 = B_{0z}\mathbf{z}$. We label lighter ions with subscript "a" and heavier ions with subscript "b". It is assumed that the heavier ions fluid has field-aligned shear flow such that $\mathbf{v}_{b0}=\mathbf{v}_0(x)\mathbf{z}=\mathbf{v}_0$. The subscript naught (0) denotes the zero order quantities.

The continuity equation for the jth ions species ($j = a, b$) is given by:

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0, \tag{1}$$

and the equation of motion for ions can be expressed as,

$$(\frac{\partial}{\partial t} + \mathbf{v}_j \cdot \nabla)\mathbf{v}_j = \frac{e}{m_j}(\mathbf{E} + \frac{\mathbf{v}_j \times \mathbf{B}}{c}), \tag{2}$$

assuming $v_{ti}k_z \ll |\partial_t|$, ω, which is valid if the shear flow is in the negative direction and $T_i < T_e$ holds [8,37].

The equation for the perpendicular component of ions' velocity under the drift approximation $|\partial/\partial t| \ll \Omega_j$ ($\Omega_j = eB_0/m_j c$ represents the ions' gyro-frequency) can be written as,

$$\mathbf{v}_{j\perp} = \frac{c}{B_0}\hat{\mathbf{z}} \times \nabla_\perp \varphi - \frac{c}{\Omega_j B_0}\left[\frac{\partial}{\partial t} + \mathbf{v}_j \cdot \nabla_\perp\right]\nabla_\perp \varphi. \tag{3}$$

Here $\mathbf{v}_E = c/B_0(\hat{\mathbf{z}} \times \nabla_\perp \varphi)$ and $\mathbf{v}_{pj} = -\frac{c}{\Omega_j B_0}[\frac{\partial}{\partial t} + \mathbf{v}_j \cdot \nabla_\perp]\nabla_\perp \varphi$ represent the electric and polarization drift of the positive ions. The parallel velocity component of the ions "a" is:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla\right)\mathbf{v}_{a\parallel} \cdot \hat{\mathbf{z}} = -\frac{e}{m_a}\nabla\varphi \cdot \hat{\mathbf{z}}. \tag{4}$$

The shear flow of the lighter ions parallel to $\mathbf{B}_0$ leads to the following parallel velocity component of the fluid "b":

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla + v_0\partial_\parallel\right)\mathbf{v}_{b\parallel} \cdot \hat{\mathbf{z}} = -\frac{e}{m_b}\nabla\varphi \cdot \hat{\mathbf{z}} + \frac{c}{B_0}\nabla_\perp\varphi\frac{d\mathbf{v}_0}{dx} \cdot \hat{\mathbf{z}}. \tag{5}$$

The continuity equations for ions "a" and "b" can be written, respectively, as,

$$\frac{dn_a}{dt} - \frac{e}{T_{e0}}\rho_{sa}^2\frac{d}{dt}\nabla_\perp^2\varphi + \frac{cn_{a0}}{B_0}\nabla_\perp\varphi(\kappa_{na}) + \partial_\parallel\left(n_a v_{a\parallel}\right) = 0, \tag{6}$$

and

$$\frac{dn_b}{dt} - \frac{e}{T_{e0}}\rho_{sb}^2\frac{d}{dt}\left(\nabla_\perp^2\varphi\right) + \frac{cn_{b0}}{B_0}\nabla_\perp\varphi(\kappa_{nb}) + \partial_\parallel\left(n_b v_{b\parallel}\right) = 0, \tag{7}$$

where $d/dt = (\partial/\partial t + \mathbf{v}_E \cdot \nabla)$ is the total time derivative, $\rho_{sj} = c_{sj}/\Omega_j$ and $c_{sj} = (T_e/m_j)^{1/2}$ represent the ions' gyroradius and acoustic speed, respectively and $\kappa_{nj} = \frac{1}{n_{j0}}\left|dn_{j0}/dx\right|$ is the inverse of the density gradient scale length. In the magnetized plasmas, $\lambda_{De}^2 < \rho_s^2$ holds therefore, we use the following quasi-neutrality condition

$$n_a + n_b = n_e. \tag{8}$$

The electron distribution function splits into two parts; one for the free electrons f_{ef} and the other for the trapped electrons f_{et} as [29,38],

$$f_e(x, v) = f_{ef}(x, v) + f_{et}(x, v) \tag{9}$$

The free electrons are those having kinetic energies greater than the wave potential and are described by the following distribution,

$$f_{ef}(x, v) = C_q\left[1 - (q-1)\left(\frac{m_e v^2}{2K_B T_{ef}} - \frac{e\varphi}{K_B T_{ef}}\right)\right]^{\frac{1}{q-1}} \quad \text{for } v > \sqrt{2e\varphi/m_e}. \tag{10}$$

The trapped electrons with their kinetic energies less than the wave potential energy bounce back and forth in the trough of the potential well and follow the trapped velocity distribution which is given by [29,38]:

$$f_{et}(x, v) = C_q\left[1 - \beta(q-1)\left\{\frac{m_e v^2}{2K_B T_{et}} - \frac{e\varphi}{K_B T_{et}}\right\}\right]^{\frac{1}{q-1}} \quad \text{for } v \le \sqrt{2e\varphi/m_e}, \tag{11}$$

where the constant of normalization is $C_q = n_{e0}\frac{\Gamma[1/(1-q)]}{\Gamma[1/(1-q)-1/2]} \times \sqrt{m_e(1-q)/2\pi K_B T_e}$ for $-1 < q < 1$ and $C_q = n_{e0}(\frac{1+q}{2}) \times \frac{\Gamma[1/(1-q)+1/2]}{\Gamma[1/(1-q)]}\sqrt{m_e(q-1)/2\pi K_B T_e}$ for $q > 1$. The parameter q is called the entropic index which determines the strength of the nonextensivity. It can be noted that T_{et} and T_{ef} in Eqs. (10)–(11) correspond to the kinetic temperature of the free and trapped electrons in the absence of the nonextensive effects; i.e., $q \to 1$. The parameter $\beta = T_{ef}/T_{et}$ represents the free to trapped electrons temperature ratio which measures the trapped electrons in the system. In the limits $q > 1$ or $-1 < q < 1$, the distribution function in (9) exhibits the thermal cutoffs and for the latter case that is for $-1 < q < +1$, the hot electron number density including trapped electrons can be written as [38]:

$$n_e = n_{e0}\{1 + \frac{1+q}{2}\Phi - \frac{4(1-\beta)(1-q)^{1/2}}{3\sqrt{\pi}}\frac{\Gamma[1/(1-q)]}{\Gamma[(1+q)/(2-2q)]}\Phi^{3/2}\}. \tag{12}$$

In equation (12) the quantity n_{e0} is the equilibrium number density of the electrons, Γ is the standard gamma function and $\Phi = e\varphi/K_B T_{ef}$ is the normalized potential.