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Casimir interaction of strained graphene

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ABSTRACT

We calculate the Casimir interaction of two freestanding graphene samples under uniaxial strain. Our approach fully takes retardation and dispersion into account and is based on quantum field theoretical expressions for conductivities in terms of the polarization operator. Contrary to some recent results the force shows a rather weak dependence on the realistic values of strain, changing just by a few percent in its maximum as compared to the non-strained case.

1. Introduction

The Casimir effect is an important tool for studying new materials [1]. Probably the best studied area is the Casimir interaction of graphene. The literature on this topic is already too large to be mentioned in a short paper like the present one, so that we again refer to the review [1] and to the mini-review [2].

One of the interesting facets of the graphene properties is concerned with its mechanical deformations which called for active research in the recent years, see a comprehensive review [3]. In particular, the possibilities of the strain engineering were considered in [4].

The van der Waals and Casimir interactions of the strained graphene layers were also considered. The van der Waals (non-retarded) interaction was computed in [5]. It was found that the force variation is rather small for the strain modulus within the elastic limits, but these variations become strong for an extremely large strain. The Casimir (i.e. fully retarded) interaction of two sheets of strained graphene was first calculated in [6] predicting once again quite a strong dependence of the interaction energy on the strain modulus already for moderate values of the strain. However, we have to disagree with these latter results, see Discussions.

We also like to mention a related calculation of the van der Waals force between an atom and strained graphene [7].

In this letter we report the calculation of the Casimir interaction between free-standing strained graphene which shows that the presence of the strain is hardly of any practical significance giving effect of the order of 6% of the already quite small Casimir interaction between graphene samples.

We shall use the approach based on the polarization tensor and (some) Quantum Field Theory methods [8–10]. This approach fully takes into account the retardation as well as the momentum dependence of conductivities (dispersion). The main advantage of this approach is that it is consistent with the only experiment on the Casimir interaction of graphene [11], as has been shown in Ref. [12].

We like to stress again that Casimir interactions of new materials have been a very active area of theoretical studies over the recent years. Taking into account possible technological applications and possible experimental activities we find important to clarify the (currently contradictory) issue of Casimir interaction of strained graphene by using the most reliable methods imposing as few assumptions as possible.

This paper is organized as follows. In the next section we derive the polarization operator for strained graphene basing on the modifications of the graphene microscopical Hamiltonian due to the in-plane strain. In the third section we present the reflection coefficients for a planar anisotropic conducting surface. We conclude the letter with presenting our numerical results for the Casimir interaction energy and discussing its properties, as well as discrepancies with previous research.

Throughout this paper we shall use the following notations. Latin letters from the beginning of alphabet will correspond to the directions along the surface of graphene, $a, b, c, \ldots = 1, 2$. The same directions plus the time coordinate will be denoted by indices from the middle of alphabet, $i, j, k, l, \ldots = 0, 1, 2$. We shall use the natural units $\hbar = c = 1$.

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2. Polarization tensor and conductivities

As was demonstrated in [13,14], a uniform planar strain of the graphene surface leads to the following modified Dirac action for quasiparticles

$$S_D = \int d^3x\, \bar{\psi}\rho_j^k \gamma^j (i\partial_k - eA_k)\psi\,, \tag{1}$$

where γ^j are $2+1$-dimensional 8×8 gamma matrices (a direct product of four 2×2 irreducible ones), A_l is the electromagnetic potential, and

$$\rho_0^0 = 1\,,\quad \rho_0^a = \rho_a^0 = 0\,,\quad \rho_a^b = v_{ab}\,. \tag{2}$$

The matrix v_{ab} can be interpreted as a tensorial Fermi velocity, see [14]

$$v_{ab} = v_F\left[\delta_{ab} - \frac{\beta}{4}\left(2u_{ab} + \delta_{ab}u_{cc}\right)\right], \tag{3}$$

that replaces the usual scalar Fermi velocity $v_F \simeq 1/300$. For graphene, $\beta \simeq 2$. u_{ab} is the strain tensor. Taking the direction of uniaxial uniform planar strain to have the angle θ with the x-direction, one gets [15]

$$u = \epsilon\begin{pmatrix} \cos^2\theta - \sigma\sin^2\theta & (1+\sigma)\sin\theta\cos\theta \\ (1+\sigma)\sin\theta\cos\theta & \sin^2\theta - \sigma\cos^2\theta \end{pmatrix}. \tag{4}$$

Here ϵ is the strain modulus that may reach 0.2 for elastic deformations. σ is the Poisson ratio. Possible values of σ are discussed in [16]. We take $\sigma = 0.14$. One can easily check, that conductivities and thus the Casimir energy are not sensitive to small variations of σ.

At zero temperature, the polarization tensor for the Dirac theory (1) can be written as

$$\begin{aligned}\tilde{\Pi}^{jk}(p) &= ie^2 \int \frac{d^3k}{(2\pi)^3}\,\mathrm{tr}\left(\rho_l^j \gamma^l \hat{S}(k)\rho_m^k \gamma^m \hat{S}(k-p)\right) \\ &= ie^2\rho_l^j\rho_m^k \int \frac{d^3k}{(2\pi)^3}\,\mathrm{tr}\left(\gamma^l \hat{S}_0(\rho k)\gamma^m \hat{S}_0(\rho(k-p))\right)\end{aligned}$$

where $\hat{S}$ is the Greens' function of the Dirac operator $\not{D} = i\rho_k^j\gamma^k\partial_j$, while $\hat{S}_0$ is the Greens' function of the same operator without rescaling $\not{D}_0 = i\gamma^j\partial_j$. By making the change of integration variables $k \to \rho k$ one arrives at the identity

$$\tilde{\Pi}^{jk}(p) = (\det\rho)^{-1}\rho_l^j\rho_m^k\Pi^{lm}(\rho p)\,. \tag{5}$$

This formula relates the polarization tensor $\tilde{\Pi}$ for the strained graphene to another polarization tensor, Π, which is calculated without strain and for the unit Fermi velocity. One can easily prove that the relation (5) holds (and has the form exactly as written above) also for a non-zero temperature and in the presence of a mass gap and of a chemical potential.

The expressions for Π^{ij} can be found in Refs. [8–10,17]. One should remember to put $v_F = 1$ in those expressions.[1] Here we present a short summary.

Due to the symmetry properties, the parity-even part of the polarization tensor depends on two functions A and B of the momenta

$$\Pi^{ji} = \Pi_0^{ji} A(p_0, p_a) + p_0^2 \Pi_u^{ji} B(p_0, p_a)\,, \tag{6}$$

where

$$\Pi_0^{ji} = g^{ji} - \frac{p^j p^i}{p^2}, \tag{7}$$

$$\Pi_h^{ji} = \frac{p^j p^i}{p^2} - \frac{p^j h^i + h^j p^i}{(ph)} + \frac{h^j h^i}{(ph)^2}p^2.$$

In the medium rest reference frame $h = (1,0,0)$. It is convenient to use instead of A and B other two independent functions,

$$\Pi_{00} \quad \text{and} \quad \Pi_{\rm str} = \Pi_{11} + \Pi_{22}\,. \tag{8}$$

The polarization tensor may be separated into a "vacuum" part $\Pi^{(vac)}$ corresponding to $T = \mu = 0$ and the rest, denoted by $\Delta\Pi$:

$$\Pi_{\rm xx}(p;\mu,T) = \Pi_{\rm xx}^{(vac)}(p) + \Delta\Pi_{\rm xx}(p;\mu,T)\,, \tag{9}$$

where xx stands for either 'str' or '00'.

The Casimir energy is defined by the Lifshitz formula (see eq. (32) below) as an integral and a sum over Euclidean momenta. Therefore, we perform now the Wick rotation $p_0 \to p_4 = ip_0$ and stay in Euclidean momentum space till the end of this section. Let us define

$$\mathbf{p}_\parallel = (p_1, p_2) \qquad p_\parallel = |\mathbf{p}_\parallel|\,, \qquad p^2 = p_4^2 + p_\parallel^2\,. \tag{10}$$

Then,

$$\begin{aligned}\Pi_{00}^{(\rm vac)}(p) &= \frac{\alpha p_\parallel^2 \Phi(p)}{p^2}\,, \\ \Pi_{\rm str}^{(\rm vac)}(p) &= -\frac{\alpha(2p_4^2 + p_\parallel^2)\Phi(p)}{p^2}\,,\end{aligned} \tag{11}$$

where

$$\Phi = 4\left[m + \frac{p^2 - 4m^2}{2p}\arctan\left(\frac{p}{2m}\right)\right]. \tag{12}$$

For the parts of the polarization tensor that depend on μ and T, we have [10]

$$\begin{aligned}&\Delta\Pi_{00}(p) = \\ &\quad = 8\alpha\int_m^\infty d\varkappa\left(1 + \mathrm{Re}\,\frac{-p^2 + 4ip_4\varkappa + 4\varkappa^2}{\sqrt{Q^2 - 4p_\parallel^2(\varkappa^2 - m^2)}}\right)\Xi(\varkappa)\end{aligned} \tag{13}$$

and

$$\begin{aligned}&\Delta\Pi_{\rm str}(p) = \\ &\quad = 8\alpha\int_m^\infty d\varkappa\,\mathrm{Re}\,\frac{\left(4ip_4\varkappa + 4\varkappa^2 - 4m^2\right)\Xi(\varkappa)}{\sqrt{Q^2 - 4p_\parallel^2(\varkappa^2 - m^2)}},\end{aligned} \tag{14}$$

we have used the notation $Q = p^2 - 2ip_4\varkappa$,

$$\Xi \equiv (e^{(\varkappa+\mu)/T} + 1)^{-1} + (e^{(\varkappa-\mu)/T} + 1)^{-1} \tag{15}$$

is the distribution function. α is the fine structure constant. With our conventions $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$.

Note that (11)–(14) for the polarization tensor take into account all species of quasiparticles in graphene which consist of $N = 4$ copies of a 2-component fermion.

Other calculations of the optical conductivity of graphene were performed in [18] using, however, a different strain model and disregarding the spatial dispersion. The optical conductivity of graphene under some other geometrical distortions of the crystal lattice was considered in [19] (random lattice deformations) and in [20] (out-of-plane deformations).

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[1] We stress that the tensor Π^{ij} plays an auxiliary role. For actual computations of the Casimir energy we shall use the tensor $\tilde{\Pi}^{ij}$ where the correct dependence on the Fermi velocity and on the strain is recovered through Eq. (5), see also Eqs. (2) and (3).