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11 The embiguity of eigenligity in quantum and eleccical eigenlation $\frac{1}{12}$ The ambiguity of simplicity in quantum and classical simulation $\frac{1}{78}$

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¹⁸ ARTICLE INFO ABSTRACT ⁸⁴ 19

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30 96 Quantum information Information theory Stochastic process Hidden Markov model *-*-Machine

²⁰ Article history: **A system's perceived simplicity depends on whether it is represented classically or quantally. This is not ⁸⁶** ²¹ Received 27 September 2016 **Sourprising, as classical and quantum physics are descriptive frameworks built on different assumptions** $\frac{87}{2}$ 22 Received in revised form 19 December 2016 **that capture, emphasize, and express different** properties and mechanisms. What is surprising is that, as 88 23 89 we demonstrate, simplicity is ambiguous: the *relative* simplicity between two systems can *change sign* 24 Communicated by CP Docting **the contract of the moving between classical and quantum descriptions. Here, we examine the minimum required 90** 25 91 memory for simulation. We see that the notions of absolute physical simplicity at best form a partial, 26 92 not a total, order. This suggests that appeals to principles of physical simplicity, via Ockham's Razor 27 93 or to the "elegance" of competing theories, may be fundamentally subjective. Recent rapid progress in 28 Information theory
28 September 2014 12:00 Templetic of the ambiguity of simplicity will strongly ₉₄ 29 95 impact statistical inference and, in particular, model selection.

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³⁵ We are to admit no more causes of natural things than such as Fermi gas supports coexistence of both quantum mechanical states ¹⁰¹ We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.

> [Isaac Newton, 1687 Philosophiæ Naturalis Principia Mathematica, Book III, p. 398 [\[1\]\]](#page--1-0)

1. Introduction

⁴⁴ and pioneering work in optics, Newton engendered a critical ab-
 $\frac{110}{2}$ process is more random than another via comparing their temper-⁴⁵ stract transition that has resonated down through the centuries, $\frac{1}{4}$ atures or thermodynamic entropies. But how to compare them in ⁴⁶ guiding and even accelerating science's growth: Physics began to the erms of their structural simplicities? We make use of a well devel-⁴⁷ perceive the world as one subject to concise mathematical Laws.
 113 perceive the statistical the statistical subset of simplicity in stochastic processes – the statistical table Above, Newton suggests that these Laws are not only a correct per-
Complexity = a measure of internal memory [5] or the minimum ⁴⁹ ception but they are also *simple*. Consequently, one should aban-
required memory to simplete a process it provides a concrete and 50 don the Ptolemaic epicycles for Newton's elegant $F = ma$ and $\frac{1}{16}$ interpretable apexies to the question which process is structurally $\frac{1}{51}$ $\frac{1}{5}$ $\frac{1}{17}$ $\frac{1}{17}$ $\frac{1}{17}$ $\frac{1}{17}$ $\frac{1}{17}$ $\frac{1}{17}$ interpretable answer to the question, which process is structurally $\frac{117}{117}$ $F_g \propto m_1 m_2/r^2$.

⁵³ sider simplicity as a means for comparing alternative theories. Here, with recent process in quantum computation [7, 0] an inter

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³⁶ are both true and sufficient to explain their appearances. at its core and classical states on its periphery [\[3,4\].](#page--1-0) The overriding ¹⁰² ³⁷ 103 [Isaac Newton, 1687 impression is that now is an interesting time for the foundations¹⁰³ ¹⁰⁴ 10⁴ 104 Philosophiæ Naturalis Principia Mathematica, boriguantum mechanics. The following adds a new phenomenon to 39 105 these debates on the balance of classical and quantum theories, as ⁴⁰ concerns the simplicity of their descriptions.¹⁰⁶

41 **1. Introduction 107 10** 42 108 cesses. A theory, then, is a mathematical object capable of yield-⁴³ Beyond his theory of gravitation, development of the calculus, ing a process' probabilities. We can straightforwardly say that one 108 52 The desire for simplicity in a theory naturally leads us to con-
 $\frac{1}{2}$ from the simplex to the most complicated [6] ing a process' probabilities. We can straightforwardly say that one process is more random than another via comparing their temperatures or thermodynamic entropies. But how to compare them in terms of their structural simplicities? We make use of a well developed measure of simplicity in stochastic processes — the statistical complexity – a measure of internal memory $\lceil 5 \rceil$ or the minimum required memory to simulate a process. It provides a concrete and simpler? By applying this comparison, we may order all processes from the simplest to the most complicated $[6]$.

 54 we compare the parsimony of two descriptions of stochastic pro-
 $\frac{120}{\pi}$ action this technology about if we add guartum monitor to our technology of the stochastic pro- 55 cesses — one classical and one quantum. Classical versus quantum $\frac{121}{20}$ which college begins to that and the data can apply $\frac{121}{20}$ 56 comparisons have, of late, captured our attention both for rea-
 $\frac{122}{15}$ of the nouvelable supplies for example it was shown that 57 sons of principle and of experiment. *Quantum supremacy* holds that $\frac{0.0001 \text{ m}}{2}$ and $\frac{$ ⁵⁸ quantum systems behave in ways beyond those that can be ef-
 $\frac{1}{2}$ the light state and such a second finite state of the light state of t 59 ficiently simulated by classical computers [\[2\].](#page--1-0) A single cold 2D $\frac{10-14}{10-14}$ and even in some cases infinitely simplet [15,15]. $\frac{60}{200}$ is the set of the state $\frac{1}{2}$. The state $\frac{1}{2}$ Recently, this quantum advantage was verified experimentally [\[17\].](#page--1-0) 61 127 Proceeding with these methods, we discover what is most surpris-62 **128 E-mail addresses: caghamohammadi@ucdavis.edu (C. Aghamohammadi). ing: the** *relative simplicity* **of classical and quantum descriptions 128** 63 jrmahoney@ucdavis.edu (J.R. Mahoney), chaos@ucdavis.edu (J.P. Crutchfield). **Can change. Specifically, there are stochastic processes, A and B,** 129 With recent progress in quantum computation [\[7–9\],](#page--1-0) an interesting twist comes about if we add quantum mechanics to our modeling toolbox. Descriptions that act on a quantum substrate offer new and surprising options. For example, it was shown that a quantum mechanical description can lead to a simpler representation [\[10–14\]](#page--1-0) and even in some cases infinitely simpler [\[15,16\].](#page--1-0)

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66 and the contract of the con

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1 for which classical theory says *A* is simpler than *B*, but quan-² tum mechanics says *B* is simpler than *A*. What started out as a $\alpha_1, \alpha_2, \dots, \alpha_n$ 3 neat classical array is upended by a new quantum simplicity or-4 der. This means quantizing a simple classical model may not be $\uparrow : 1 - q$ 5 71 as simple as quantizing a more complicated classical model. As a quantum model class.

¹¹ We consider stationary, ergodic processes: each a bi-infinite sists of a set $\{|n_\nu(I)\rangle\}$ of pure signal states that are in one-to-one 77 12 sequence of random variables $X_{-\infty;\infty} = \ldots X_{-2}X_{-1}X_0X_1X_2\ldots$ correspondence with the classical causal states $\sigma_k \in S$. Each signal ⁷⁸ ¹³ where each random variable X_t takes some value x_t in a discrete state $|\eta_k(L)\rangle$ encodes the set of length-L words that may follow σ_k , τ_s 14 alphabet set A and where all probabilities $Pr(X_t, \ldots, X_{t+L})$ are as well as each corresponding conditional probability. Fixing L, we ⁸⁰ time-invariant.

16 82 *How is their degree of randomness quantified?* Information the-17 ory [\[18\]](#page--1-0) measures the uncertainty in a single observation X_0 via $|\eta_j(t)\rangle \equiv \sum_{\lambda} \sum_{\lambda} \langle Pr(w^L, \sigma_k | \sigma_j) |w^L \rangle |\sigma_k \rangle$, (2) 83 18 the Shannon entropy: $H[X_0] = -\sum_{x \in A} Pr(x) \log_2 Pr(x)$ and the ir-

the space of the subsequential proportion is the entropy rate [10]. 19 reducible uncertainty per observation via the *entropy rate* [\[19\]:](#page--1-0) **19.19 reducible 19.19 reducible uncertainty per observation via the** *entropy rate* **[19]:** 20 *h_μ* = lim_{*L*→∞} *H*[*X*_{0:*L*}]/*L*. If we interpret the left half *X*_{−∞:0} = where *w*^{*L*} denotes a length-*L* word and Pr(*w*^{*L*}, σ_{*k*}|σ_{*j*}) = Pr(*X*_{0:*L*} = 86 21 ... $X_{-2}X_{-1}$ as the "past" and the right half $X_{0:\infty} = X_0X_1X_2...$ w^L , $S_L = \sigma_k/S_0 = \sigma_j$). The resulting Hilbert space is the product as 22 as the "future", we see that the entropy rate is the average $\mathcal{H}_w \otimes \mathcal{H}_\sigma$. Factor space \mathcal{H}_σ is of size $|\mathcal{S}|$, the number of classical as 23 uncertainty in the next observable given the entire past: h_{μ} = causal states, with basis elements $|\sigma_k\rangle$. Factor space \mathcal{H}_w is of size as 24 $H[X_0|X_{-\infty:0}]$. Thus, as we take into account past correlations, the $|\mathcal{A}|^L$, with basis elements $|w^L\rangle = |x_0\rangle \cdots |x_{L-1}\rangle$. 90 25 anive uncertainty $H[X_0]$ reduces to h_{μ} . as the "future", we see that the entropy rate is the average uncertainty in the next observable given the entire past: h_{μ} = naive uncertainty $H[X_0]$ reduces to h_μ .

26 92 *How reducible is our uncertainty in the future X*0:∞ *knowing the past* 27 93 *X*−∞:0*?* The answer is given by the mutual information between 28 the past and the future — the *excess entropy* [\[20\]:](#page--1-0) $E = I[X_{-\infty:0} : \text{ } G = -\text{IF}(P \log P)$, (3) 94 29 X_{0:∞}]. With *h*_μ and **E**, we measure randomness and predictability, where $ρ = Σ$, π_i|*n*_i)/*n*_i|. This quantum analog of memory is generrespectively.

32 write a computer code that follows an algorithm and allocate the \sim cal and quantum informational sizes are equal exactly when both 98 33 memory the algorithm needs. For a given process *computational* $\frac{1}{\text{model}}$ are "maximally simple"; $\mathbf{F} - \mathbf{C} - \mathbf{C}$ 34 100 *mechanics* [\[21\]](#page--1-0) identifies the optimal algorithm — the process' ³⁵ ϵ -machine. This is a *unifilar hidden Markov model* [\[22\]](#page--1-0) that uses **3 Ising chain simplicity 101** 36 102 only the minimum required memory for simulation. We view a 37 process' *∈*-machine as the "theory" of a process in that it speci-
¹⁰³ 38 104 fies a mechanism that exactly simulates a process' behaviors. In 39 this way, computational mechanics supplements **E** and h_μ with a $H = -\sum_i (fs_is_j + bs_i)$, (4) (4) 40 Improvement of structure — the minimum required amount memory to $\frac{1}{2}$ is 41 Simulate the given process. The contract of the contract of

43 109 equivalence relation ∼ that groups histories, say *x*−∞:*^t* and *x*−∞:*t* , 44 that lead to the same future predictions $Pr(X_{t:\infty}|\cdot)$: *x*_{−∞}: \sim ternal magnetic field. $\qquad \qquad$ 110 45 111 *x*−∞:*t* ⇐⇒ Pr*(Xt*:∞|*x*−∞:*t)* = Pr*(Xt*:∞|*x*−∞:*t)*. From this, one con-46 cludes that a process' ϵ -machine is, in a well defined sense, its fines a stationary stochastic process which has been analyzed using 112 47 simplest predictive theory. The same state of the computational mechanics [\[25\].](#page--1-0) Importantly, spins obey a condisimplest predictive theory.

49 we ask: What is the minimum memory necessary to implement opti- "future" spins (right half) depend not on the entire past (left half) 115 50 mal prediction? The answer is the historical information stored in but only on the most recent spin x₀. The conclusion (see Supp. Ma-
 51 the ϵ -machine. Quantitatively, this is the Shannon entropy of the terials) is that the two-state Markov chain process is minimally 117 Translating this notion of simplicity into a measurable quantity, we ask: *What is the minimum memory necessary to* implement *opti-*

$$
C_{\mu} = H[\mathcal{S}] = -\sum_{\sigma \in \mathcal{S}} \pi_{\sigma} \log_2 \pi_{\sigma},
$$
\n(1)
$$
Fig. 2 \text{ shows that } C_{\mu} \text{ is a monotonically increasing function of } P \text{ and } q.
$$
\n11.
$$
Fig. 2 \text{ shows that } C_{\mu} \text{ is a monotonically increasing function of } P \text{ and } q.
$$
\n12.
$$
T: 1 - C_{\mu} \propto T^{-2} \text{ at high } T.
$$
\nIn particular, for the three

56 It is well known that the excess entropy is a lower-bound on processes chosen at temperatures $T_\alpha < T_\gamma < T_\delta$, $C_\mu^\alpha < C_\mu^\gamma < C_\mu^\delta$. ⁵⁷ this structural measure: **E** \leq *C_μ*. In fact, this relation is only rarely consider now the quantum representation of these spin config-
⁵⁷ ⁵⁸ an equality [\[23\].](#page--1-0) And so, while **E** quantifies the amount to which urations. Each causal state is mapped to a pure quantum state that ¹²⁴ ⁵⁹ a process is subject to explanation by its *∈*-machine "theory", this resides in a spin one-half space [13]: ¹²⁵ 60 126 simplest theory is typically larger, informationally speaking (*Cμ*), ⁶¹ than the predictability benefit it confers. That said, the ϵ -machine $|\sigma_1\rangle = \sqrt{p}|\uparrow\rangle + \sqrt{1-p}|\downarrow\rangle$ 127 62 is the best (simplest) theory. Thus, we use C_{μ} to define our no-
 $(\sigma_2) = \sqrt{1 - a} |t\rangle + \sqrt{a} |1\rangle$ (5) ¹²⁸ 63 tion of classical simplicity. It provides an interpretable ordering of 128 129 129 129 It is well known that the excess entropy is a lower-bound on processes — process *A* is simpler than process *B* when $C^A_\mu < C^B_\mu$.

 65 We may also consider the recently proposed quantum-machine the quantum machine notation of Eq. (2).) Intuitively, the quan- 131 66 representation of processes [\[10–12\].](#page--1-0) The quantum-machine con-
 66 representation of processes [10–12]. The quantum-machine con-
tum overlap accounts for the fact that the conditional predictions of 132 We may also consider the recently proposed quantum-machine

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 6 consequence model selection is complicated by the addition of a Fig. 1. The ϵ -machine for the nearest-neighbor Ising spin chain has two causal states 72 and the addition of a *σ*₁ and *σ*₂. If the last observed spin *x*₀ is up (*s*₀ = +1) the current state is *σ*₁ and *τ*₃ is up (*s*₀ = +1) the current state is *σ*₁ and *τ*₃ 8 74 spin observed is up and, if the current state is *σ*2, with probability *q* the next spin 9 75 **2. Classical and quantum simplicity** Fig. 1. The ϵ -machine for the nearest-neighbor Ising spin chain has two causal states if it's down $(s_0 = -1)$ is σ_2 . If the current state is σ_1 , with probability *p* the next observed is down.

15 81 construct quantum states: sists of a set $\{\eta_k(L)\}$ of pure *signal states* that are in one-to-one correspondence with the classical causal states $\sigma_k \in \mathcal{S}$. Each signal state $|\eta_k(L)\rangle$ encodes the set of length-*L* words that may follow σ_k , as well as each corresponding conditional probability. Fixing *L*, we

$$
|\eta_j(L)\rangle \equiv \sum_{w^L \in \mathcal{A}^L} \sum_{\sigma_k \in \mathcal{S}} \sqrt{\Pr(w^L, \sigma_k | \sigma_j)} |w^L\rangle |\sigma_k\rangle , \qquad (2)
$$

where w^L denotes a length-*L* word and $Pr(w^L, \sigma_k | \sigma_i) = Pr(X_{0:L} =$ w^L , $S_L = \sigma_k | S_0 = \sigma_i$). The resulting Hilbert space is the product $|A|^L$, with basis elements $|w^L\rangle = |x_0\rangle \cdots |x_{L-1}\rangle$.

of the stationary state:

$$
C_q = -\text{Tr}(\rho \log \rho) \tag{3}
$$

30 respectively.
20 **1.1** *in***tegrates** that the classical: $C_q \le C_\mu$. Also, due to the Holevo 36 31 Let's say we want to simulate a given process. To do this we bound [10.24] $\mathbf{F} < C$. Though $\frac{1}{2}$ in process space, the classiwhere $\rho = \sum_i \pi_i |\eta_i\rangle\langle\eta_i|$. This quantum analog of memory is gener-bound [\[10,24\],](#page--1-0) $\mathbf{E} \leq C_a$. Though rare in process space, the classical and quantum informational sizes are equal exactly when both models are "maximally simple": $\mathbf{E} = C_q = C_\mu$.

3. Ising chain simplicity

The Ising spin-chain Hamiltonian is given by:

$$
H = -\sum_{\langle i,j \rangle} (Js_i s_j + b s_i) \,, \tag{4}
$$

42 The *ε*-machine consists of *causal states σ* ∈ *S* defined by an where *s_i*, the spin at site *i*, takes values $\{-1, +1\}$, *J* is the nearest-
108 where s_i , the spin at site *i*, takes values $\{-1, +1\}$, *J* is the nearestneighbor spin coupling constant, and *b* is the strength of the external magnetic field.

48 114 tional independence: Pr*(X*0:∞|*x*−∞:0*)* = Pr*(X*0:∞|*x*0*)*. That is, the 52 causal-state stationary distribution { $π_σ$ }, the *statistical complexity*: represented by the ϵ -machine in Fig. 1. Using Eq. (1), the statis-118 53 119 tical complexity is directly calculated as a function of *p* and *q*. 55 $\sigma \in \mathcal{S}$ $\sigma \in \mathcal{S}$ at high *T*. In particular, for the three 121 In equilibrium the bi-infinite chain of spin random variables defines a stationary stochastic process which has been analyzed using but only on the most recent spin x_0 . The conclusion (see Supp. Materials) is that the two-state Markov chain process is minimally represented by the ϵ -machine in Fig. 1. Using Eq. (1), the statis-

> Consider now the quantum representation of these spin configurations. Each causal state is mapped to a pure quantum state that resides in a spin one-half space [\[13\]:](#page--1-0)

$$
|\sigma_1\rangle = \sqrt{p}|\uparrow\rangle + \sqrt{1-p}|\downarrow\rangle
$$

$$
|\sigma_2\rangle = \sqrt{1-q}|\uparrow\rangle + \sqrt{q}|\downarrow\rangle. \tag{5}
$$

⁶⁴ processes – process A is simpler than process B when $C_{\mu}^A < C_{\mu}^B$. (We use a more compact spin up/down notation, rather than ¹³⁰ tum overlap accounts for the fact that the conditional predictions

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