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Collapse and revival for a slightly anharmonic Hamiltonian

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The effect of quantum collapse and revival is a fascinating interference phenomenon. In this paper the phenomenon is studied analytically and numerically for a simple system, a slightly anharmonic oscillator. The initial wave-function corresponds to a displaced ground state of a harmonic oscillator. Possible experimental realizations for cold atoms are discussed in detail.

1. Introduction

Collapse and revival phenomena are fascinating and are encountered in a variety of physical situations, that were explored experimentally and theoretically. In the present paper a simple example that can be analyzed analytically, and may be realized experimentally, is presented.

The first collapse and revival phenomenon that was observed and explained theoretically is the Talbot effect [1,2], where the amplitude of an optical signal collapses and then revives partially and completely. A quantum phenomenon of this type is the quantum carpet [3–5]. Some revivals discussed here are of different nature than the ones found for the Talbot effect and the quantum carpet. Collapses and revivals, as well as fractional revivals and superrevival structures were observed for wave packets in Rydberg atoms [6–13]. Also, this phenomenon was observed for interacting bosons [14,15], and model systems [16–20]. Similar phenomenon was found for chaotic systems [21]. For the two site Bose Hubbard model, the difference in populations of the two sites exhibits collapses and revivals [22].

A simple model where the phenomenon of collapses and revivals is found is for noninteracting bosons in a weakly anharmonic trap [23]. In this specific situation the particles are prepared in the ground state of the trap and then the potential is instantaneously shifted by some distance. In a harmonic trap, the wave packet oscillates with the frequency of the trap and so do the various observables, for example, the position x and the momentum p. As a result of the anharmonicity, these oscillations are superimposed by an envelope exhibiting collapses and revivals. Some of our numerical results were presented in [23] that focused on a different issue, namely echoes resulting of the interplay between two displacements. In the present work, analytic formulas for the evolution of the observables are derived.

One should remember that within the model we explore the evolution is coherent and information is not lost even during the collapse. This coherence enables the revivals.

The model presented here is simple and the evolution of the observables is described in a straightforward manner. The simplicity is of great value if used to explore more complex situations. For example, effects of interparticle interactions will result in deviations from our predictions. These can be detected in experiments. The significance of these effects can be tuned by the particle density.

The analytical method we use in the present paper is the semiclassical approximation assuming large quantum numbers.

When we expand a symmetric potential around its minimum, the leading order is harmonic and the first correction is a term of the form βx^4. Therefore, we study a model described by the Hamiltonian

$$H' = \frac{p'^2}{2m'} + \frac{1}{2}m'\omega_0'^2 x'^2 + \frac{\beta'}{4}x'^4, \quad (1)$$

In dimensionless units

$$x = x'/\sqrt{\hbar/m'\omega_0'} \quad (2)$$

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$$t = \omega_0' t' \tag{3}$$

$$H = \frac{H'}{\hbar \omega_0'} \tag{4}$$

$$p = p' \sqrt{\frac{1}{\hbar m' \omega_0'}}, \tag{5}$$

where prime denotes the corresponding values in physical units, and the Hamiltonian takes the form

$$H = \frac{p^2}{2} + \frac{1}{2} x^2 + \frac{1}{4} \beta x^4, \tag{6}$$

where

$$\beta = \beta' \left(\frac{\hbar}{m'^2 \omega_0'^3} \right). \tag{7}$$

In the present work we assume $\beta \ll 1$. The Schrödinger equation is

$$i \frac{\partial}{\partial t} \psi = H \psi. \tag{8}$$

The model (6) is an idealized model that will be shown to exhibit collapse and revival behavior. This is a very general phenomenon, and therefore it is relevant for a large variety of models. More generally, let us expand eigenenergies around a level $\bar{n}$ [24, 25] in the form

$$E_n \simeq E_{\bar{n}} + E_{\bar{n}}' \delta + \frac{1}{2} E_{\bar{n}}'' \delta^2 + \frac{1}{6} E_{\bar{n}}''' \delta^3 \tag{9}$$

where δ is defined by $\delta = n - \bar{n}$, $\bar{n}$ is the state with maximal probability for a displaced ground state of a harmonic oscillator, $E_{\bar{n}}'$, $E_{\bar{n}}''$ and $E_{\bar{n}}'''$ are derivatives of the energy with respect to the quantum number n, calculated at $\bar{n}$. We consider a situation where the derivatives decrease rapidly with the order, so that

$$E_{\bar{n}}' \gg E_{\bar{n}}'' \gg E_{\bar{n}}'''. \tag{10}$$

This is the case for high energy levels n if the energy is to a good approximation proportional to a power of the quantum number n [8]. From condition (10) follows the separation of time scales in the system dynamics.

In Section 2 the dynamics of observables is calculated for the simple model (6) while in Section 3 possible experimental realizations in the field of cold atoms are presented. The results are summarized and discussed in Section 4.

2. Collapse and revival of the expectation values of position and momentum

In this section we study the expectation values of the position and momentum operators for an initial Gaussian wavepacket displaced by some distance d from the minimum of the potential. For small β the system exhibits collapses and revivals [23]. This phenomenon is explained in terms of the semiclassical approximation using a method similar to the one used in [22]. The phenomenon was found numerically and semianalytically for the system (1) in [23]. Since this phenomenon is expected to take place for high energy levels, where the energy is much larger than the level spacing, we use the semiclassical approximation in the leading order.

It is important to remember that second order in β of the semiclassical approximation is more accurate than second order in perturbation theory for high energy levels (see detailed discussion in [22]). Using the semiclassical spectrum the evolution of $\langle \hat{x}(t) \rangle$ and $\langle \hat{p}(t) \rangle$ is calculated.

2.1. Energy spectrum calculation using the WKB (Wentzel, Kramers and Brillouin) approximation

In order to use the WKB approximation, we calculate the action integral as

$$I = \frac{1}{\pi} \int_{-\tilde{a}}^{\tilde{a}} p dx = \frac{1}{\pi} \int_{-\tilde{a}}^{\tilde{a}} \sqrt{2 \left(E - \frac{1}{2} x^2 - \frac{\beta}{4} x^4 \right)} dx \tag{11}$$

where $\pm \tilde{a}$ are the turning points of the path related to the energy by

$$E = \frac{1}{2} \tilde{a}^2 + \frac{\beta}{4} \tilde{a}^4. \tag{12}$$

The integral is calculated to the second order in β and is found to be

$$I = E - \frac{3}{8} \beta E^2 + \frac{35}{64} \beta^2 E^3. \tag{13}$$

This is the value of the action for a fixed value of energy and β. Solving for the energy as function of the action to the second order in β results in

$$E = I + \frac{3}{8} \beta I^2 - \frac{17}{64} \beta^2 I^3. \tag{14}$$

The action is quantized as

$$I_n = n + \frac{1}{2}. \tag{15}$$

Substituting (15) in (14), the energy spectrum of the Hamiltonian to second power of β yields

$$E_n = \left(n + \frac{1}{2} \right) + \frac{3\beta}{8} \left(n^2 + n + \frac{1}{4} \right) - \beta^2 \left(\frac{17}{64} n^3 + \frac{51}{128} n^2 + \frac{51}{256} n + \frac{17}{512} \right). \tag{16}$$

A slightly different spectrum is obtained by using quantum perturbation theory. Comparison to the exact result obtained by numerical diagonalization of the Hamiltonian (6) shows that the WKB approximation gives a more accurate energy spectrum, for high levels. For $\beta = 1 \cdot 10^{-4}$ (the value used in Fig. 1) the WKB method gives a more accurate result for $n > 4$.

2.2. Explicit calculation of $\langle \hat{x}(t) \rangle$ and $\langle \hat{p}(t) \rangle$

The initial wavefunction corresponds to the ground state of the harmonic oscillator $|n = 0\rangle$, displaced by d at time $t = 0$. The displacement operator is $T(d) = e^{-i\hat{p}d}$. The displacement of the harmonic ground state satisfies [23]

$$\langle m | T(d) | 0 \rangle = e^{-\frac{\gamma^2}{2}} \frac{\gamma^m}{\sqrt{m!}} = e^{-\frac{\gamma^2}{2}} C_m(\gamma), \tag{17}$$

where

$$\gamma = \frac{d}{\sqrt{2}} \tag{18}$$

and

$$C_n(\gamma) = \frac{\gamma^n}{\sqrt{n!}}. \tag{19}$$

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