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The escape problem and stochastic resonance in a bistable system driven by fractional Gaussian noise

Wei Wang, Zhi Yan, Xianbin Liu *

State Key Laboratory of Mechanics and Control of Mechanical Structures, College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

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ABSTRACT

In the present paper, for a symmetrical bistable system that is excited by a fractional Gaussian noise, via the examination upon the qualitative changes of the stationary probability densities, the phenomena of the noise induced transition and escape and the stochastic resonance, which is in the sense of that the particles oscillate between the double-well potential, are investigated. For a high noise intensity, the probability density function obtained by Monte Carlo method changes from bimodal to unimodal by decreasing the values of Hurst index H. However, in the low noise intensity regime the transition could not occur for all H. Based on numerical results we demonstrate the fact that the mean first passage time (MFPT) is dependent on the Hurst index H and the noise intensity D, and possesses an exponential form as $T(D,H) = k_1(H)\exp(k_2(H)/D)$. In particular, with a higher noise intensity, $T(D,H)$ represents itself as a monotonous increasing function of the MFPT with the increasing H, whereas it exhibits as a non-monotonic function of H in the low noise intensity regime. Finally, nonlinear response theory is applied to investigate the stochastic resonance induced by Hurst index H and noise intensity D, for which we find that only for low noise intensities the stochastic resonance exists via increasing Hurst index which means the effect of noise reduction. This phenomenon is quite different from the one of the classical case.

1. Introduction

In the last few decades, a great deal of mathematical and experimental efforts have been devoted to the study of stochastic dynamical systems in many fields like physics, chemistry, biology, engineering and finance [1–6]. The investigations of the effects of random fluctuations or noises become essential and significant since while the systems (especially the nonlinear systems) under consideration are driven by random forces, many new features can occur, such as noise-induced transport [7], noise-induced transition [8–10], escape problem (mean first passage time) [11–13], stochastic resonance [1,14,15], coherence resonance [16] and so on. The first passage is a generic concept for quantifying when a random quantity such as the position of a diffusing molecule or the value of a stock crosses a preset target for the first time [17]. The so called first passage time is central to describe the kinetics in a large variety of systems and has been studied intensively across many branches of science [18–20].

Brownian motion and Gaussian white noise are the important concepts in stochastic dynamics. Brownian motion is a Gaussian process with independent increments and Gaussian white noise, the derivative process of Brownian motion, has independent values at each instant time. Though Gaussian white noise is an idealized noise with zero memory which does not occur in nature, it is extremely useful to model rapidly random fluctuating when the memory of the noise is extremely short compared to that of systems [21]. Meanwhile, the response of the systems driven by Gaussian white noise is Markov diffusion process which is easy to deal with by efficient mathematic techniques such as Ito stochastic differential equation and Fokker–Plank equation [2,21,22]. However, in the real world, there exist abundant random fluctuations or noises with long time memory, which may be modeled by fractional Brownian motion (fBm) and fractional Gaussian noise (fGn) with Hurst index H defined in $0 < H < 1$ [23,24]. Similarly to the Gaussian white noise, fGn is the derivative process of fBm [25]. The concept of fBm which was first introduced by Kolmogorov and reintroduced by Mandelbrot and Ness is a one-parameter extension of the classical Brownian motion since for $H = 0.5$ the fBm reduces to the Brownian motion and the fGn reduces to Gaussian white noise [25]. For $H \neq 0.5$ fBm is a Gaussian process with de-

* Corresponding author. Fax: +86 25 84892106.
E-mail addresses: weiwang1990@nuaa.edu.cn (W. Wang), xbliu@nuaa.edu.cn (X. Liu).

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pendent increments and fGn whose autocorrelation function has the form of power-law with heavy tail has long time memory. The fBm provides a powerful physical model for anomalous diffusion whose mean–squared displacement satisfies the power-law type [26]:

$$\langle x^2(t)\rangle \sim t^{2H} \tag{1}$$

For $H < 0.5$ we observe subdiffusion while in the case $H > 0.5$ the motion is superdiffusion. The fBm is widely used in modeling a variety of process including monomer diffusion in a polymer chain [27], diffusion of biopolymers in the crowded environment inside biological cells [28], and Econophysics [29]. Despite its popularity the exact stochastic properties of fBm are not well understood. Fortunately, in the last few years a lot of effort has been devoted to the investigations of fBm. Grebenkov suggested an empirical approximation for the probability density which is numerically validated for fBm [30]. According to Deng and Barkai who investigated the time average mean square displacement for fBm, the convergence to ergodic behavior is slow and surprisingly the Hurst index $H = 3/4$ marks the critical point of the speed of convergence [31]. In terms of the first passage problem, Jeon and Metzler obtained the power-law type first passage time density scales in the long-time limit by studying the survival probability and the corresponding first passage time density of fBm confined to a two-dimensional open wedge domain with absorbing boundaries [32].

The motion of the systems governed by fGn can be described by a stochastic differential equations (SDE) in R:

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t} = f\big(t, X(t)\big) + \xi_H(t) \tag{2}$$

where f is a function: $[0, T] \times R \to R$ and $\xi_H(t)$ is fGn. Its equivalent form is

$$\mathrm{d}X(t) = f\big(t, X(t)\big)\mathrm{d}t + \mathrm{d}B_H(t) \tag{3}$$

The stochastic process $X(t)$ is a non-Markov diffusion process since the fGn has long time memory. If $f \equiv 0$ the stochastic process $X(t)$ is fGm. Analysis of the dynamics of Eqs. (2) and (3) leads to the description of the stochastic dynamical behaviors for the non-Markov diffusion. In recent works, F.G. Li and B.Q. Ai [10] introduced additive fGn into the anti-tumor model and discussed the noise-induced transition. O.Y. Sliusarenko et al. [13] studied the escape problem in a linear Langevin equation with fGn and found that the escape becomes faster for decreasing values of Hurst index. M.L. Deng and W.Q. Zhu obtained the sample solution, correlation function and mean-square value of the responses of linear and nonlinear oscillators to fGn with Hurst index between 0.5 and 1 [33]. Goychuk and Hanggi presented an analytic study for subdiffusive escape of overdamped particles out of a cusp-shaped parabolic potential well which are driven by thermal, fGn in the case when the fluctuation dissipation theorem applies and found that the escape is governed asymptotically by a power-law scale [34]. Since bistable systems with double-well potential are widely applied in many disciplines and the research findings can be conveniently extended to multi-stable or more complex systems, they are of significance in the investigation of stochastic nonlinear dynamics. However, the transition, escape problem and stochastic resonance induced by external fGn in bistable systems have not been reported in the previous literatures. We should remark that in this paper we focus on the bistable systems in the presence of external fluctuations which do not obey the fluctuation dissipation theorem.

The main purpose of this paper is to investigate the transition, escape problem, stochastic resonance phenomenon and their relevant mechanisms in the bistable systems in the presence of external fGn.

This paper is organized as follows. In section 2, some preliminaries such as the theory of fBm and fGn, methods to generate the fBm and a bistable system driven by fGn are introduced. In section 3, the noise-induced transitions via the examination upon the qualitative changes of the stationary probability densities obtained by Monte Carlo method, are discussed. In section 4, the dependence of the mean first passage time on noise intensity and Hurst index are analyzed. In section 5, the stochastic resonance induced by noise intensity and Hurst index is investigated respectively. In particular, the relevant mechanisms are discussed. The results are summarized in section 6 with conclusions drawn.

2. Preliminaries

A standard fBm $B_H(t)$ with Hurst index $0 < H < 1$ is a continuous and centered Gaussian process with autocorrelation function:

$$E\big[B_H(t)B_H(s)\big] = \frac{1}{2}\big(t^{2H} + s^{2H} - |t-s|^{2H}\big) \qquad t, s \ge 0 \tag{4}$$

which has the following properties:

(1) $B_H(0) = 0$ and $E[B_H(t)] = 0$, $t \ge 0$
(2) $B_H(t)$ has homogeneous increments, i.e., $B_H(t+s) - B_H(s)$ has the same law of $B_H(t)$, $t, s \ge 0$
(3) $B_H(t)$ is a Gaussian process and $E[B_H^2(t)] = t^{2H}$, $t \ge 0$
(4) $B_H(t)$ has continuous trajectories.

We consider the correlation between two increments of fBm. The covariance between $B_H(t+\delta t) - B_H(t)$ and $B_H(s+\delta t) - B_H(s)$ with $s - t = n\delta t$ and n positive integer is

$$\rho_H(n) = \frac{1}{2}(\delta t)^{2H}\big[(n+1)^{2H} + (n-1)^{2H} - 2n^{2H}\big] \tag{5}$$

For $H = 0.5$, we obtain $\rho_H(n) = 0$. The $B_H(t)$ is then a standard Brownian motion $B(t)$ and the increments of process are independent. For $H > 0.5$ the increments are positively correlated while they are negatively correlated for $H < 0.5$. More details of fBm can be found in Refs. [23,24].

The fGn is the derivative process of the fBm, i.e.,

$$\xi_H(t) = \frac{\mathrm{d}B_H(t)}{\mathrm{d}t} \tag{6}$$

The fGn is a stationary Gaussian process with long time memory whose mean is zero and the autocorrelation function has the form of power-law:

$$R(\tau) = E\big[\xi_H(t+\tau)\xi_H(t)\big] = H(2H-1)|\tau|^{2H-2} + 2H|\tau|^{2H-1}\delta(\tau) \tag{7}$$

For $H = 0.5$ the autocorrelation function exactly reduces to the Dirac function $\delta(\tau)$ for standard Gaussian white noise. In particular, for $H < 0.5$ Eq. (7) shows that the fGns are negatively correlated. It follows that a step in one direction is likely followed by a step in the other direction, whereas for $H > 0.5$ the fGns are positively correlated indicting that successive steps tend to point in the same direction.

There are mainly five different methods to generate the fBm: the method of Mandelbrot, that of Sellan, the Choleski method, the Levinson one and the method of Wood and Chan [35,36]. In this paper, to investigate the long time behaviors of the systems, we adopt the method of Wood and Chan to generate the fBm since it is fast even for a large value of time step by using the discrete Fourier method [37,38]. Fig. 1(a), (b) and (c) show the simulated simple paths of fBm for $H = 0.3$, $H = 0.5$ and $H = 0.8$ respectively which are generated by the method of Wood and Chan. As can be seen, with increasing the values of Hurst index H the sample

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