Physics Letters A 381 (2017) 931–934

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Three-dimensional ultrashort optical Airy beams in an inhomogeneous medium with carbon nanotubes

Alexander V. Zhukov [a,*], Roland Bouffanais [a], Mikhail B. Belonenko [b,c], Ilya S. Dvuzhilov [c]

[a] Singapore University of Technology & Design, 8 Somapah Road, 487372 Singapore
[b] Laboratory of Nanotechnology, Volgograd Institute of Business, 400048 Volgograd, Russia
[c] Volgograd State University, 400062 Volgograd, Russia

ARTICLE INFO

Article history:
Received 7 November 2016
Accepted 6 January 2017
Available online 11 January 2017
Communicated by R. Wu

Keywords:
Carbon nanotubes
Ultrashort pulses
Airy beams

ABSTRACT

In this Letter, we consider the problem of the dynamics of propagation of three-dimensional optical pulses (a.k.a. light bullets) with an Airy profile through a heterogeneous environment of carbon nanotubes. We show numerically that such beams exhibit sustained and stable propagation. Moreover, we demonstrate that by varying the density modulation period of the carbon nanotubes one can indirectly control the pulse velocity, which is a particularly valuable feature for the design and manufacturing of novel pulse delay devices.

1. Introduction

Localized electromagnetic wave packet tends to spread in both space and time under the combined effects of dispersion and diffraction, which are always present in any medium. Over the past two decades, significant research activity has been dedicated to devising new ways to overcome these universal broadening effects in order to generate sustained localized wave packets [1]. Such localized wave packets that are localized in space and that can travel through a medium while retaining their spatiotemporal shape—in spite of diffraction and dispersion effects—are referred to as light bullets. When propagating through a nonlinear medium, three-dimensional (3D) light bullets tend to vanish as a consequence of a host of instabilities [2]. However, recent advances in the development of new media with atypical electronic properties have opened new avenues in the generation of sustained propagation of light bullets. In turn, this has generated particular interest in relation with the peculiar nature and dynamics of propagation of these ultrashort optical pulses [3]. Light bullets have also gained significant attention in the field of nonlinear optics owing to potential game-changing applications in modern optoelectronics.

In 2007, Airy optical beams have been achieved by the use of a spatial light modulator [4]. The latter propagates in free space retaining its form at a certain interval, and the trajectory of the main peak is bent recalling the rainbow. It is known that Airy beams have infinite energy (i.e. they are physically unrealizable, but in practice they are approximately generated to some extent) and retain their intensity during the propagation. Thus, Airy beams propagate with an apparent lack of diffractive spreading effects. Moreover, there is increased resistance to amplitude and phase distortions. Other improvements have also been obtained with Airy–Bessel wave packets producing linear light bullets [5]. These unique properties of such optical beams had been captured by earlier pioneering studies based on theoretical analysis [6].

Carbon nanotubes (CNTs) have been used to generate media with unique features as a result of their nonlinear optical properties. CNTs have generated tremendous interest in the research community owing to the simplicity of their structure and their unique properties, which in turn contributed significantly to both the development in optical pulses propagation studies, as well as the development of optical devices based on them. Probably one of the most important feature of CNTs is the ability to use them as a medium for the formation of light bullets [7–11]. Usually the propagation of optical pulses are considered in a uniform CNT environment that does not allow to control the pulse velocity. However, if the propagation velocity of light bullets is determined by the refractive index of the medium and can vary within a wide range, one can perform a further modulation in the refractive index. In turn, this favors the formation of media with a modulated refractive index, thereby enabling the control of the propagation velocity of light bullets as well as the delay time.

As a consequence, various models of propagation of extremely short pulses in a heterogeneous environment have been proposed,

* Corresponding author.
E-mail address: alex.zhukov@outlook.sg (A.V. Zhukov).

http://dx.doi.org/10.1016/j.physleta.2017.01.008
0375-9601/

especially given that such an environment makes it possible to control not only the propagation velocity, but also, e.g., the transverse structure of the pulse [12–14]. The most straightforward way to create such a heterogeneous environment—with CNTs and a spatially modulated refractive index—is to get a nonuniform distribution of CNTs. This leads to a change in the propagation velocity of the optical pulse, and therefore one will be able to control the pulse delay time in such an environment.

Given these recent independent developments in terms of: *(i)* Airy wave packets as light bullets, and *(ii)* novel CNT-based inhomogeneous media, it appears timely to investigate the details of the propagation of 3D ultrashort Airy beams in such inhomogeneous media based on CNTs. The present Letter presents the first such study based on a combination of theoretical and numerical analyses.

2. Fundamental equations

Consider the propagation of extremely short optical pulses in an environment of carbon nanotubes, where the electric field is directed along the axis of the nanotubes. The Hamiltonian of the electron subsystem reads

$$\mathcal{H} = \gamma \sum_{j\sigma} a^{\dagger}_{j\sigma} a_{j\sigma} + \text{c.c.}, \tag{1}$$

where $a^{\dagger}_{j\sigma}$ and $a_{j\sigma}$ are the creation and annihilation operators respectively, for electron with spin σ located at the jth node, γ is the hopping integral determined by the overlap of the electron wave functions on the neighboring nodes. The abbreviation 'c.c.' in Eq. (1) stands for the complex conjugate term. Using the Fourier transform

$$a^{\dagger}_{n\sigma} = \frac{1}{\sqrt{N}} \sum_j a^{\dagger}_{j\sigma} \exp(ijn),$$

$$a_{n\sigma} = \frac{1}{\sqrt{N}} \sum_j a_{j\sigma} \exp(-ijn), \tag{2}$$

one can easily diagonalize the Hamiltonian by applying a Bogoliubov transformation, thereby yielding the electron spectrum $\epsilon_s(\mathbf{p})$, which describes the properties of the electronic subsystem in the absence of Coulomb repulsion. For carbon nanotubes of the zigzag type, namely $(m, 0)$, the dispersion relation for the energy of conduction electrons reads [15–17]:

$$\epsilon_s(\mathbf{p}) = \pm\gamma \left\{1 + 4\cos(a\mathbf{p})\cos\left(\pi\frac{s}{m}\right) + 4\cos^2\left(\pi\frac{s}{m}\right)\right\}^{1/2}. \tag{3}$$

Here, $s = 1, 2 \ldots m$, $\gamma \approx 2.7$ eV, $a = 3b/2\hbar$, $b = 0.142$ nm is the distance between adjacent carbon atoms.

Maxwell's equations in a cylindrical coordinates system can be written as

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \mathbf{E}}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{4\pi}{c}\frac{\partial \mathbf{j}}{\partial t} = 0, \tag{4}$$

where $\mathbf{E}$ is the electric field of the light wave, $\mathbf{j}$ is the electron current density, t is the time, and c is the light velocity in the medium. Let us modify Eq. (4) given our particular choice of the Coulomb gauge, $\mathbf{E} = -\frac{1}{c}\partial\mathbf{A}/\partial t$. Integrating Eq. (4) over time once, we obtain its generalization for a nonlinear medium as follows

$$\frac{\partial^2 \mathbf{A}}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \mathbf{A}}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{4\pi}{c}\mathbf{j} = 0. \tag{5}$$

The vector potential $\mathbf{A}$ and the current density $\mathbf{j}$ are assumed to have the following form $\mathbf{A} = \{0, 0, A(z,r,t)\}$ and $\mathbf{j} = \{0, 0, j(z,r,t)\}$, respectively. By solving Eq. (5) for the vector potential $\mathbf{A}$, one can deduce the current density

$$j = en(z,r)\sum_{ps} v_s\left(p - \frac{e}{c}A(t)\right)\langle a^{\dagger}_{ps}a_{ps}\rangle, \tag{6}$$

where $v_s(p) = \partial\epsilon_s(p)/\partial p$ is the electron group velocity, $n(z,r)$ is the electron density of CNTs system with possible variations along the radial coordinate r and the axial coordinate z. Angle brackets denote an average with the nonequilibrium density matrix $\rho(t)$:

$$\langle\Theta\rangle = \mathrm{Tr}\left[\Theta(0)\rho(t)\right], \tag{7}$$

where Θ is the arbitrary dynamic quantity, and Tr denotes the trace of a matrix. With account for the conservation law, $\left[a^{\dagger}_{ps}a_{ps}, \mathcal{H}\right] = 0$, the equation of motion for the density matrix gives us the relation $\langle a^{\dagger}_{ps}a_{ps}\rangle = \langle a^{\dagger}_{ps}a_{ps}\rangle_0$, where $\langle\Theta\rangle_0 = \mathrm{Tr}\left[\Theta(0)\rho(0)\right]$. Note that $\rho_0 = \exp\left(-\mathcal{H}/k_BT\right)/\mathrm{Tr}\left[\exp\left(-\mathcal{H}/k_BT\right)\right]$, where $\rho_0 \equiv \rho(0)$, k_B is the Boltzmann constant, and T is the temperature. Expanding $v_s(p)$ in a Fourier series, we have

$$v_s\left(p - \frac{e}{c}A(t)\right) = \sum_k A_{ks}\left\{\sin(kp)\cos\left(\frac{ke}{c}A(t)\right) - \cos(kp)\sin\left(\frac{ke}{c}A(t)\right)\right\},$$

where

$$A_{ks} = \int_{-\pi/a}^{\pi/a} v_s(p)\sin(kp)dp$$

are the coefficients of expansion which decrease with increasing k.

Given that the distribution function ρ_0 is an even function of the quasi-momentum p, the averaging of $\sin(kp)$ vanishes, so that

$$v_s\left(p - \frac{e}{c}A(t)\right) = -\sum_k A_{ks}\cos(kp)\sin\left(\frac{ke}{c}A(t)\right). \tag{8}$$

Substituting Eq. (8) into Eq. (6) and performing the summation over s and p, we come to

$$j = -en(z,r)\sum_k B_k \sin\left(\frac{ke}{c}A(t)\right), \tag{9}$$

$$B_k = \sum_{s=1}^{m}\int_{-\pi/a}^{\pi/a} dp\, A_{ks}\cos(kp)\frac{\exp(-\beta\epsilon_s(p))}{1 + \exp(-\beta\epsilon_s(p))},$$

where n_0 is the equilibrium electron concentration, $\beta = 1/k_BT$. Note that the current density given by Eq. (9) explicitly contains the nonuniform electron density $n(z,r)$. Further, in the numerical calculations, this distribution will be given the simple periodic form

$$n(z,r) = 1 + \alpha\cos\left(\frac{2\pi z}{\chi}\right),$$

where α is the nonlinearity modulation depth, χ stands for the modulation period. Note that in this paper we only consider modulations along the z-axis.

It is worth noting that due to the field inhomogeneity along certain directions (e.g., the field is directed and nonuniform along the z-axis), the current is also not uniform. The heterogeneity of the current causes an accumulation of charges in some areas that can be assessed from the charge conservation law: