Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

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Ultracold bosons in a one-dimensional optical lattice chain: Newton's cradle and Bose enhancement effect

Ji-Guo Wang, Shi-Jie Yang *

Department of Physics, Beijing Normal University, Beijing 100875, China

A R T I C L E I N F O

Article history:
Received 22 September 2016
Received in revised form 30 December 2016
Accepted 12 March 2017
Available online xxxx
Communicated by F. Porcelli

Keywords:
Tunneling
Atoms in optical lattices
Boson systems
Traversal time

A B S T R A C T

We study a model to realize the long-distance correlated tunneling of ultracold bosons in a one-dimensional optical lattice chain. The model reveals the behavior of a quantum Newton's cradle, which is the perfect transfer between two macroscopic quantum states. Due to the Bose enhancement effect, we find that the resonantly tunneling through a Mott domain is greatly enhanced.

1. Introduction

One of the most remarkable effects in the quantum physics in contrast to the classical physics is the quantum tunneling. It presents in a wide variety of phenomena from the Josephson effects, scanning tunneling microscopy to electron transport through quantum dots [1–5]. For interacting many-particle systems, the quantum tunneling can be greatly modified in the presence of other particles. Correlated tunneling plays an important role in the quantum magnetism, the high-temperature superconductivity, as well as the multi-particle entanglement [6–8]. When particles are placed in sufficiently deep periodic lattices which is described by the tight-binding models, the kinetic energy is exhausted and the inter-particle interaction dominates the ground states as well as the quantum dynamics. Correlated tunneling is induced via the virtual second-order quantum transitions [9,10]. On the other hand, particles transporting through a series of lattice sites involves higher orders of successive quantum transitions. The indistinguishability of the bosons gives rise to the Bose enhancement effect [11,12]. It may result in many interesting phenomena such as the Sauter–Schwinger effect in tilted Mott insulators (MIs) and the electron long-distance transport in molecular systems [13–15].

In the past decades, high precision techniques in the ultracold quantum gases have provided versatile means to investigate the static as well as dynamical properties of strongly correlated systems [16–21]. The effects of resonant tunneling [22–29], non-equilibrium dynamics [30–35] and quantum state transfer [36–38] had been widely studied in a variety of experiments. The experiment by Kinoshita et al. [39] attempted to create a quantum Newton's cradle in a Bose–Einstein condensate (BEC). In a recent publication [40], the authors proposed a scheme of the quantum Newton's cradle by using a BEC of two internal states in the strong Tonks–Girardeau (TG) regime [41,42]. The wave packet propagates along a one-dimensional (1D) optical lattice chain which is similar to the momentum propagation of the classical Newton's cradle (Fig. 1(c)). However, the conditions are too ideal to be realizable in experiments.

Recently, experimentalists directly observed resonantly enhanced long-range quantum tunneling in a 1D MI chain that is suddenly quenched by tilting the lattice sites [22]. Motivated by their works, we propose a model to simulate a perfect quantum Newton's cradle. The model can be achieved by adjusting the magnetic field gradient in the z direction to control the depths of the end lattice site. It consists of N lattice sites with $N-1$ strongly interactive bosons. We assume that the two terminal lattice sites have an equal potential energy $E1$ whereas the middle lattice sites have an equal potential energy $E2$. The system is initially set in the state $|L\rangle = |1, 1, \ldots, 1, 0\rangle$ (Fig. 1(a)), where each lattice site is occupied by one particle except that the rightmost lattice site is empty. The resonance quantum tunneling reveals a perfect transfer between the states $|L\rangle$ and $|R\rangle = |0, 1, \ldots, 1, 1\rangle$ (Fig. 1(b)), which simulates the realization of a quantum Newton's cradle. The reso-

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* Corresponding author.
E-mail address: yangshijie@tsinghua.org.cn (S.-J. Yang).

http://dx.doi.org/10.1016/j.physleta.2017.03.015

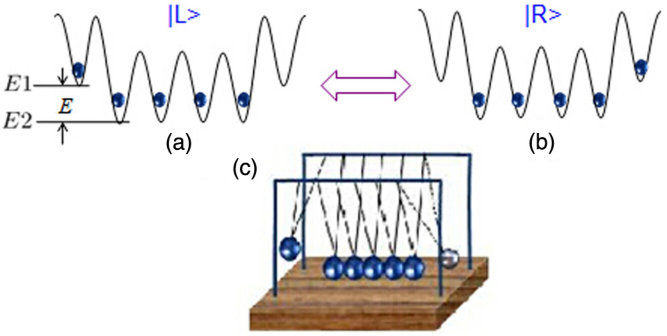

Fig. 1. (Color online.) Schematic illustration of the macroscopic quantum states transfer from the initial state $|L\rangle$ (a) to the state $|R\rangle$ (b) and vice versa. The potential energies of the two terminal lattice sites are higher than the middle lattice sites ($E > 0$). (c) A description of the classical Newton's cradle.

nantly tunneling through the MI domain is greatly enhanced gives rise to the Bose enhancement effect.

The paper is organized as follows. In Sec. 2, we describe the model and present the exact diagonalization (ED) results. In Sec. 3, we formulate the quantum perturbation theory to explain our numerical results and construct an effective model to illustrate the quantum dynamics. In Sec. 4, we examine the particle number quantum fluctuation and the center-of-mass motion of the system. Sec. 4 contains a brief summary.

2. Model and numerical results

We consider a N-site Bose–Hubbard Hamiltonian filled with $(N-1)$ Bose atoms [31,43],

$$\hat{H} = -J\sum_{\langle i,j\rangle}\hat{b}_i^{\dagger}\hat{b}_j + \frac{U}{2}\sum_{i=1}^{N}\hat{n}_i(\hat{n}_i-1) + \sum_{i=1}^{N}E_i\hat{n}_i, \tag{1}$$

where $\hat{b}_i^{\dagger}$ ($\hat{b}_i$) is the creation (annihilation) operator and $\hat{n}_i = \hat{b}_i^{\dagger}\hat{b}_i$ is the number operator. J is the nearest-neighbor tunneling coefficient and U is the Hubbard interaction. E_i is the on-site potential energy of the ith lattice site. The two terminal lattice sites energies are set to $E_1 = E_N = E1$ whereas the middle lattice sites potential energies are $E_i = E2$ $(1 < i < N)$ with the potential energy difference $E = E1 - E2 > 0$, as shown in Fig. 1(a, b).

We focus on the regime of strongly repulsive interaction $U/J \gg 1$. Each of the $N-2$ middle lattice sites is filled with one particle. Double occupancy is damped and the region forms a MI domain. The extra particle is put in the leftmost lattice site, leaving the rightmost lattice site empty. The initial state is described by the Fock state $|\psi(0)\rangle = |L\rangle$. We track the probability of temporal appearance of the state $|R\rangle$ by the quantity $P(t) = |\langle R|\psi(t)\rangle|^2$. At first sight, we may think the state transfer amplitude from $|L\rangle$ to $|R\rangle$ is small because the process is reachable only via higher orders of quantum transitions. However, the virtual intermediate processes greatly enhanced the tunneling amplitude due to Bose enhancement effect. As a comparison, we examine long-distance tunneling of the same lattice sites but only with a single particle (the single particle system), i.e., the oscillation between the states $|1, 0, \ldots, 0, 0\rangle$ and $|0, 0, \ldots, 0, 1\rangle$.

We fix the tunneling $J = 1$ and the Hubbard interaction $U = 40$. (The critical point between MI phase and superfluid (SF) phase has been estimated to be $(J/U)_c \simeq 0.29$ in 1D Bose–Hubbard model [44].) Fig. 2 shows the temporal evolution of the probabilities from the initial state $|L\rangle$ to the state $|R\rangle$ for different potential energy difference E in the MI domain system and the single particle system. In order to embody the Bose enhancement effect, we introduce the relative tunneling frequency (RTF) as the ratio of the oscillating frequencies ν_1 in the MI domain system and ν_2 in the single atom system. The ED results of the RTF increase significantly as E increases from 0 to U and reduce as E increases when $E > U$. The RTF reaches the maximum at $E = U$, it indicates that the resonance phenomenon takes place. We also find the RTF increases with the system size N for a fixed E, which shows the system size can affect the Bose enhancement effect. Although the potential energy difference E may be large enough to match the multiple occupations, the quantum processes simultaneously evolve higher orders of perturbations, the multiple occupations are damped, as shown in Fig. 2(f) of the green dashed line.

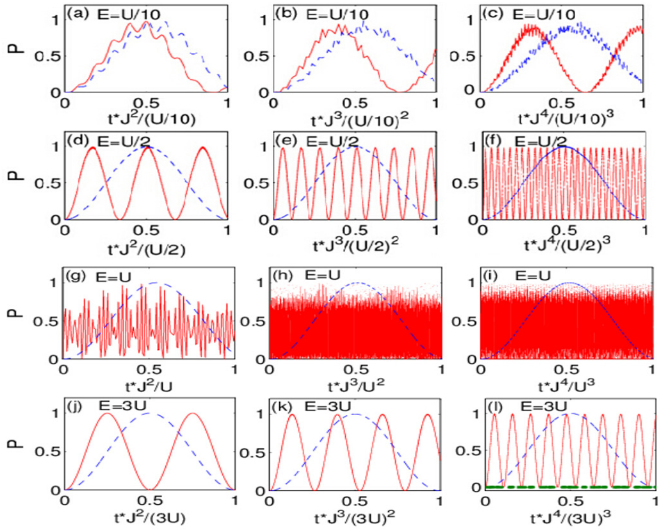

Fig. 2. (Color online.) Temporal evolution of the probabilities $P(t)$ from the initial state $|L\rangle$ to the state $|R\rangle$ for different E. From upper to lower: $E = U/10$, $U/2$, U and $3U$. The system sizes from left to right column are $N = 3$, 4 and 5. Red solid lines represent the MI domain system whereas blue dashed lines represent the single particle system. The green dashed line in (f) is the probability of the state $|0, 0, 3, 1, 0\rangle$.

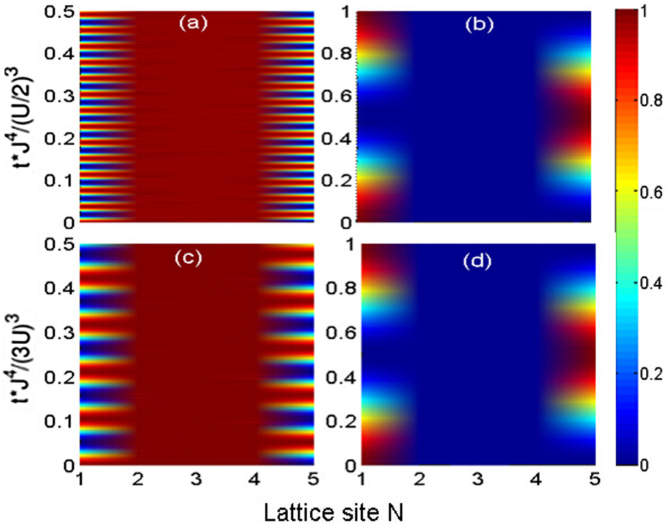

Fig. 3. (Color online.) Time evolution of the particle occupation numbers of the lattice sites in the MI domain system (a, c) and the single particle system (b, d) for the system size $N = 5$. The upper panel $E = U/2$ and the lower panel $E = 3U$. It shows that multiple occupations are damped.

We further show the time evolution of the particle occupation numbers of the lattice sites in the MI domain system (left column) and the single particle system (right column) for the system size $N = 5$ in Fig. 3. Obviously, the particle oscillates between the two terminal lattice sites without occupying the middle lattice sites.

Please cite this article in press as: J.-G. Wang, S.-J. Yang, Ultracold bosons in a one-dimensional optical lattice chain: Newton's cradle and Bose enhancement effect, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2017.03.015