Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Cohering power of quantum operations

Kaifeng Bu [a], Asutosh Kumar [b,c], Lin Zhang [d], Junde Wu [a,*]

[a] School of Mathematical Sciences, Zhejiang University, Hangzhou 310027, PR China
[b] Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211019, India
[c] Homi Bhabha National Institute, Anushaktinagar, Mumbai 400094, India
[d] Institute of Mathematics, Hangzhou Dianzi University, Hangzhou 310018, PR China

ARTICLE INFO

Article history:
Received 4 February 2017
Received in revised form 14 March 2017
Accepted 16 March 2017
Available online xxxx
Communicated by P.R. Holland

Keywords:
Quantum coherence
Quantum operations
Cohering power

ABSTRACT

Quantum coherence and entanglement, which play a crucial role in quantum information processing tasks, are usually fragile under decoherence. Therefore, the production of quantum coherence by quantum operations is important to preserve quantum correlations including entanglement. In this paper, we study cohering power–the ability of quantum operations to produce coherence. First, we provide an operational interpretation of cohering power. Then, we decompose a generic quantum operation into three basic operations, namely, unitary, appending and dismissal operations, and show that the cohering power of any quantum operation is upper bounded by the corresponding unitary operation. Furthermore, we compare cohering power and generalized cohering power of quantum operations for different measures of coherence.

1. Introduction

Quantum superposition, arising from the linearity of quantum mechanics, is the most fundamental feature of quantum mechanics. It is one of the characteristic distinguishing properties between classical and quantum systems. It is responsible for almost all the intriguing quantum phenomena such as interference of microscopic particles. Quantum coherence [1], which is identified by the presence of off-diagonal terms in the quantum states, is a direct consequence of the superposition principle. It builds the foundation of quantum theory. Moreover, it should be noted that quantum coherence is a prerequisite for various quantum correlations including quantum entanglement [2], which form an important physical resource in quantum information processing tasks [3]. Entangled states have vast applications in as many fields as quantum communication and computation, in quantum metrology [4]. However, unlike entanglement and quantum correlations, quantum coherence is a basis-dependent quantity. That is, coherence of a given quantum state can be quite different within different reference frameworks. For example, while the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$ has coherence in σ_z-basis $\{|0\rangle, |1\rangle\}$, it has zero coherence in σ_x-basis $\{|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}\}$. On the other hand, $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle)$, is both coherent and entangled in both the above bases. The dependence of quantum coherence on the choice of basis–the reference basis–can sound disturbing at first, but it is naturally determined by the experimental situation at hand. Quantum coherence has been widely applied in fields like quantum thermodynamics [5–9] and quantum biology [10–12]. These suggest coherence to be a useful resource at the nanoscale. It has also application in quantum parallelism [3]. These important advancements in quantum domain suggest that there should be a quantitative framework for coherence. Like entanglement, a rigorous framework for the quantification of quantum coherence, from a resource-theoretic point of view, has been developed recently in Ref. [1]. In any resource theory, there are two basic components: free (allowed) states and free (allowed) operations. The resource theory of quantum coherence, likewise, is based on the set of "incoherent operations" as the free operations and the set of "incoherent states" as the set of free states. As remarked earlier, the set of incoherent states and the set of incoherent operations depend critically on the choice of basis. Recently, a significant effort has been devoted towards quantifying quantum superposition, and hence quantum coherence, from a resource theoretic perspective [1,13–40]. In order to exploit quantum coherence, we need to quantify coherence in a given state. Along this line, several kinds of coherence measures such as l_1-norm of coherence, relative entropy of coherence and skew-information of coherence have been introduced in [1,14]. That quantum coherence can be measured with entanglement was shown in Ref. [16].

* Corresponding author.
E-mail addresses: bkf@zju.edu.cn (K. Bu), asukumar@hri.res.in (A. Kumar), linyz@zju.edu.cn (L. Zhang), wjd@zju.edu.cn (J. Wu).

http://dx.doi.org/10.1016/j.physleta.2017.03.022

It turns out that a proper measure of coherence should satisfy following properties: (i) (*Nullity*) Incoherent states have zero coherence, (ii) (*Monotonicity*) Incoherent completely positive and trace preserving maps cannot increase coherence, and/or the average coherence should not increase under selective measurements, and (iii) (*Convexity*) Non-increasing of coherence under the mixing of quantum states. Besides, the relationship of coherence with other quantities like mixedness and quantum discord were revealed in [17,22,25,27]. Several statistical properties of quantum coherence have also been obtained in Refs. [41,42] in parallel to entanglement. Furthermore, the operational resource theory of quantum coherence has been developed in [18], and the transformation processes like coherence distillation have been studied to give physical interpretations to the coherence measures mentioned above.

Quantum systems are notoriously different from classical systems, and can outperform classical systems in many information-processing tasks, including quantum communication and computation [3]. Therefore, quantum technology has significant importance to information technology. However, one of the major bottlenecks of quantum technology is the quantum decoherence effect [3]. In the phenomenon of decoherence, a quantum system inevitably interacts with its surroundings and loses quantum coherence. Quantum correlations, especially entanglement, have been found to be fragile under decoherence [43,44]. The decoherence effect inherently leads to the dissipation from quantum systems to classical systems. However, we need to avoid the phenomenon of decoherence when we implement quantum techniques in quantum information and computation [3]. Due to immense importance of quantum correlations and entanglement–being indispensable resources–in quantum information processing tasks, maintaining coherence and/or quantum correlations in quantum systems is a challenging assignment. Here, the cohering power of quantum operations turns up as a savior. The cohering power of a quantum operation quantifies the ability to produce coherence. Authors in [31] have calculated cohering power of some special qubit operations exactly. In our work, we investigate two different types of cohering power of generic quantum operations. First, we give an operational interpretation of cohering power. Moreover, we address the problem of estimating cohering power of a generic quantum operation in terms of simple operations. We calculate the cohering power of three basic quantum operations, namely, unitary operation, appending operation and dismissal operation. Then, by dividing a generic quantum operation into these three quantum operations, we obtain an upper bound on the cohering power of this generic operation in terms of cohering power of these basic operations. More importantly, we compare two different kinds of cohering powers ($\mathscr{C}_\mathcal{C}$ and $\widehat{\mathscr{C}}_\mathcal{C}$) for unitary operations. For l_1-norm measure, they coincide in single qubit case only, while they are different for any number of qubits for relative entropy of coherence.

The paper is organized as follows. Introductory material about coherence is presented in Sect. 2. We give an information-theoretic interpretation of cohering power in Sec. 3. In Sec. 4, we calculate cohering power of three basic quantum operations, and obtain an upper bound on the cohering power of a generic quantum operation in terms of that of these basic quantum operations. Sec. 5 is devoted to comparison of cohering powers, $\mathscr{C}_\mathcal{C}$ and $\widehat{\mathscr{C}}_\mathcal{C}$, for different measures of coherence. Finally, we conclude in Sec. 6.

2. Preliminary and notations

Quantum operations. – A quantum operation Φ is defined as a linear completely positive and trace preserving (CPTP) map. In Kraus representation, a linear map Φ is a CPTP map if and only if it can be expressed by a set of Kraus operators $\{K_\mu\}_{\mu=1}^N$ as following,

$$\Phi(\rho) := \sum_{\mu=1}^{N} K_\mu \rho K_\mu^\dagger, \tag{1}$$

with $\sum_\mu K_\mu^\dagger K_\mu = \mathbb{I}$.

Resource theory of quantum coherence. – For a d dimensional quantum system with a fixed reference basis $\{|i\rangle\}$, any state which is diagonal in the reference basis is called an *incoherent state*. And the set of all incoherent states is denoted by $\mathcal{I}$. Then, a quantum operation Φ is said to be *incoherent operation* if there exists a set of Kraus operators $\{K_\mu\}$ of Φ such that for each K_μ, $K_\mu \mathcal{I} K_\mu^\dagger \subset \mathcal{I}$ (up to a normalization) [1]. To quantify the coherence of any quantum state ρ, we consider the following two proper measures of coherence [1],

(i) l_1-norm of coherence, $\mathcal{C}_{l_1}$, is defined by

$$\mathcal{C}_{l_1}(\rho) = \sum_{i\neq j} |\rho_{ij}|, \tag{2}$$

where $\rho_{ij} = \langle i|\rho|j\rangle$.

(ii) Relative entropy of coherence, $\mathcal{C}_r$, is defined by

$$\mathcal{C}_r(\rho) := \min_{\sigma\in\mathcal{I}} S(\rho \,\|\, \sigma) = S(\rho_{\text{diag}}) - S(\rho), \tag{3}$$

where $S(\rho \,\|\, \sigma) = \text{Tr}\rho(\log\rho - \log\sigma)$ is the relative entropy between ρ and σ, $S(\rho) = -\text{Tr}\rho\log\rho$ is the von Neumann entropy of ρ, and ρ_{diag} is the diagonal part of ρ in the reference basis $\{|i\rangle\}$.

It follows from the definition of coherence measures that:

$$\mathcal{C}_{l_1}(\rho\otimes\sigma) + 1 = (\mathcal{C}_{l_1}(\rho) + 1)(\mathcal{C}_{l_1}(\sigma) + 1), \tag{4}$$

and

$$\mathcal{C}_r(\rho\otimes\sigma) = \mathcal{C}_r(\rho) + \mathcal{C}_r(\sigma). \tag{5}$$

Let $\mathcal{C}$ denote the coherence measure $\mathcal{C}_{l_1}$ or $\mathcal{C}_r$. Recall that the cohering power and the generalized cohering power of a quantum operation are defined respectively by [22]:

$$\begin{aligned}\mathscr{C}_\mathcal{C}(\Phi) &:= \max\{\mathcal{C}(\Phi(\delta)) : \delta \in \mathcal{I}\}\\ &= \max\{\mathcal{C}(\Phi(\delta)) : \delta = |k\rangle\langle k|, k\in[d]\},\end{aligned} \tag{6}$$

$$\widehat{\mathscr{C}}_\mathcal{C}(\Phi) := \max\{\mathcal{C}(\Phi(\rho)) - \mathcal{C}(\rho) : \rho \in D(\mathcal{H})\}, \tag{7}$$

where $[d]$ denotes the set $\{1,\ldots,d\}$, and Equation (6) follows from the convexity of the measures of coherence [1].

Properties of matrix norm $\|\cdot\|_{1\to1}$. – For a matrix $A \in \mathbb{C}^{d\times d}$, we define its matrix norm $\|A\|_{1\to1}$ by [45]

$$\begin{aligned}\|A\|_{1\to1} &:= \max\{\|Ax\|_1, \|x\|_1 = 1\}\\ &= \max\left\{\sum_{i=1}^{d} |A_{ij}| : j = 1,\ldots,d\right\},\end{aligned}$$

where $x = (x_1, x_2, \ldots, x_d)^T$ and $\|x\|_1 = \sum_{i=1}^d |x_i|$. It turns out that

$$\left\|\prod_j A_j\right\|_{1\to1} \leqslant \prod_j \|A_j\|_{1\to1}, \tag{8}$$

$$\left\|\bigotimes_j A_j\right\|_{1\to1} = \prod_j \|A_j\|_{1\to1}. \tag{9}$$

3. Operational interpretation of cohering power of quantum operations

By definition, the cohering power of a quantum operation Φ can be used to measure the maximal amount of coherence generated by Φ. Besides, based on an idea in the entanglement theory