Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Local hidden-variable model for a recent experimental test of quantum nonlocality and local contextuality

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ARTICLE INFO

Article history:
Received 28 January 2017
Received in revised form 4 May 2017
Accepted 5 May 2017
Available online xxxx
Communicated by P.R. Holland

Keywords:
Quantum nonlocality
Contextuality
Entanglement
Quantum measurement

ABSTRACT

An experiment has recently been performed to demonstrate quantum nonlocality by establishing contextuality in one of a pair of photons encoding four qubits; however, low detection efficiencies and use of the fair-sampling hypothesis leave these results open to possible criticism due to the detection loophole. In this Letter, a physically motivated local hidden-variable model is considered as a possible mechanism for explaining the experimentally observed results. The model, though not intrinsically contextual, acquires this quality upon post-selection of coincident detections.

1. Introduction

Quantum nonlocality and quantum contextuality are intimately related in a manner that had not been appreciated for some time. Both are used in the construction of various no-go theorems for ruling out different classes of hidden variable models, yet the two properties are, in some sense, very much intertwined. The Bell–Kochen–Specker theorem demonstrates that quantum mechanics is fundamentally contextual in the sense that it is inconsistent with a hidden-variable model that does not exhibit contextuality [1–3]. These so-called noncontextual hidden-variable (NCHV) models may be characterized as having a probability distribution over the hidden variable space that is independent of the choice of measurement basis. In a similar manner, the Bell inequality is obeyed so long as the probability distribution over the hidden variable space is the same for all choices of measurement settings [4]. Thus, violations of the Bell inequality may be seen as a signature of contextuality [5]. If, furthermore, the invariance of the probability distribution can be justified on the grounds of local realism, then such violations may be seen as a signature of nonlocality, meaning that they are inconsistent with any local hidden-variable (LHV) theory [6].

The difficulty with these no-go theorems is that contextuality can arise in subtle ways that may have nothing to do with quantum mechanics. One of the best examples of this comes from post-selection. In experiments using entangled photons, one often post-selects on outcomes for which a coincident detection of both photons is achieved [7–11]. Doing so, however, creates a situation in which, from a hidden-variable perspective, different subensembles are downselected for each measurement setting. If one then adopts the fair-sampling hypothesis, then one is asserting, without independent justification, that these subensembles are in fact the same and, hence, that there is no contextuality. A subsequent violation of a Bell inequality, then, leaves open the question of whether the assumption of noncontextuality was indeed correct. This, of course, is the origin of the so-called detection loophole [12–14]. Although the detection loophole has been closed in some experiments [15–18], this is not true for many cases and, so, the matter of their interpretation is left open.

Recently, the connection between nonlocality and contextuality was studied experimentally using pairs of photons that were exquisitely prepared in a hyperentangled state involving both polarization and spatial modes [19]. The resulting quantum state may be thought of as a four-qubit system, with the first two qubits corresponding to the polarization and spatial modes of one photon and the second two qubits corresponding to those of the other photon. Using an experimental design developed by Cabello [20], the two photons were then each subjected to a set of two-qubit measurements chosen so that their outcomes would satisfy certain Bell-like inequalities whose violation would be indicative of contextuality. The authors conclude that “there are correlations in nature which cannot be explained by LHV theories *because* they

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http://dx.doi.org/10.1016/j.physleta.2017.05.010
0375-9601/

contain single-particle correlations which cannot be reproduced with NCHV theories" [19].

This conclusion appears premature given the experimenters' reliance on the fair-sampling assumption. An interesting and still open question is whether the fair-sampling assumption is, indeed, valid. This question is of general physical interest and quite independent of whether one has closed the detection loophole or not. Given the apparent reasonableness of this assumption, an investigation of specific LHV models may help to shed some light on whether it is, indeed, reasonable to suppose that detected photons are statistically identical to their undetected kin.

In this Letter, a previously described LHV model is used to reproduce the results of this experiment under similar experimental conditions [21]. This is made possible by virtue of the fact that, like the experimenters, we restrict consideration to coincident detections only, thus giving rise to contextuality as an emergent property of the post-selection process. Variations of this LHV model are described elsewhere and have been used to explain the appearance of contextuality and nonlocality in entangled photon experiments [22].

2. Description of the experiment

The experiment of Ref. [19] may be described in terms of a four-qubit system. Consider a sequence of four bits $x_1, x_2, x_3, x_4 \in \{0, 1\}$ used to index one of 16 basis states, each of which is written as

$$|x_1\rangle_1 \otimes |x_2\rangle_2 \otimes |x_3\rangle_3 \otimes |x_4\rangle_4 = |x_1x_2x_3x_4\rangle \,. \tag{1}$$

In the context of the experiment, qubits 1 and 2 correspond to the polarization and spatial modes, respectively, of the qubit measured by Alice, while qubits 3 and 4 correspond to those measured by Bob. A hyperentangled state is prepared that may be described as follows:

$$|\Psi_{1234}\rangle = \frac{1}{2}\Big[|0011\rangle - |0110\rangle - |1001\rangle + |1100\rangle\Big] \,. \tag{2}$$

The experiment now considers combinations of the following nine two-qubit observables:

$$A = \mathbf{Z} \otimes \mathbf{I}_2 \quad B = \mathbf{I}_2 \otimes \mathbf{Z} \quad C = \mathbf{Z} \otimes \mathbf{Z} \tag{3a}$$

$$a = \mathbf{I}_2 \otimes \mathbf{X} \quad b = \mathbf{X} \otimes \mathbf{I}_2 \quad c = \mathbf{X} \otimes \mathbf{X} \tag{3b}$$

$$\alpha = \mathbf{Z} \otimes \mathbf{X} \quad \beta = \mathbf{X} \otimes \mathbf{Z} \quad \gamma = \mathbf{Y} \otimes \mathbf{Y} \tag{3c}$$

where $\mathbf{I}_2$, $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ are the Pauli spin matrices and $\otimes$ denotes the Kronecker product between matrices. These observables form a Mermin–Peres magic square such that $AB = C$, $ab = c$, $Aa = \alpha$, $Bb = \beta$, but $\alpha\beta = \gamma = -Cc$. Each of the six rows and columns comprises a compatible set of observables and, hence, may be measured in a common basis. Such constructions have been used extensively to study quantum contextuality [23,24].

A measurement of A by Alice, who has local access only to qubits 1 and 2, is denoted by the 16×16 matrix $A \otimes \mathbf{I}_4$, where $\mathbf{I}_4$ is the 4×4 identity matrix. Similarly, a measurement of A by Bob, who has local access only to qubits 3 and 4, is denoted by $\mathbf{I}_4 \otimes A$. Thus, a measurement of A by Alice and A by Bob corresponds to the separable observable $(A \otimes \mathbf{I}_4)(\mathbf{I}_4 \otimes A) = A \otimes A$. (In Ref. [19], $A \otimes \mathbf{I}_4$ is denoted A, and $\mathbf{I}_4 \otimes A$ is denoted A', so the product of the two is there denoted AA'. Here we write the Kronecker product explicitly for clarity.) Each of these measurements can be performed in one of two experimental contexts, corresponding to the intersecting row or column in the magic square.

In the experiment, Alice measures all three observables in the chosen basis. Bob, however, measures only one, so his choice of basis is irrelevant. Following the notation of Ref. [19], the choice of basis for Alice will be denoted by one of CAB, cba, $\beta\gamma\alpha$ for the three rows and αAa, βbB, $c\gamma C$ for the three columns. Thus, six different sets of measurements are performed, each corresponding to one of the six basis choices for Alice. From these, two averaged quantities are measured, $\langle\chi\rangle$ and $\langle S\rangle$. These are combined to form a single quantity, $\langle\omega\rangle = \langle\chi\rangle + \langle S\rangle$, which, according to Cabello, satisfies the inequality $\langle\omega\rangle \le 16$ for any LHV model [20].

The quantity $\langle\chi\rangle$ is given solely in terms of Alice's observables and is defined as

$$\begin{aligned}\langle\chi\rangle = \langle CAB \otimes \mathbf{I}_4\rangle + \langle cba \otimes \mathbf{I}_4\rangle + \langle \beta\gamma\alpha \otimes \mathbf{I}_4\rangle \\ + \langle \alpha Aa \otimes \mathbf{I}_4\rangle + \langle \beta bB \otimes \mathbf{I}_4\rangle - \langle c\gamma C \otimes \mathbf{I}_4\rangle \,.\end{aligned} \tag{4}$$

For any quantum state, the ideal quantum predictions for the first five terms are each $+1$, while that for the last is -1, thereby yielding a maximal value of $\langle\chi\rangle = 6$. According to Cabello, if the measured system exhibits no contextuality then the inequality $\langle\chi\rangle \le 4$ must hold [5]. Thus, any observed violation of this latter inequality is an indication of contextuality. The experimentally measured value for $\langle\chi\rangle$ was 5.817 ± 0.011, thus showing a clear violation of this inequality.

The quantity $\langle S\rangle$ is given in terms of observables for both Alice and Bob and is defined as

$$\begin{aligned}\langle S\rangle = &-\langle A \otimes A\rangle_{CAB} - \langle B \otimes B\rangle_{CAB} \\ &- \langle b \otimes b\rangle_{cba} - \langle a \otimes a\rangle_{cba} \\ &+ \langle \gamma \otimes \gamma\rangle_{\beta\gamma\alpha} + \langle \alpha \otimes \alpha\rangle_{\beta\gamma\alpha} \\ &- \langle A \otimes A\rangle_{\alpha Aa} - \langle a \otimes a\rangle_{\alpha Aa} \\ &- \langle b \otimes b\rangle_{\beta bB} - \langle B \otimes B\rangle_{\beta bB} \\ &+ \langle \gamma \otimes \gamma\rangle_{c\gamma C} + \langle C \otimes C\rangle_{c\gamma C} \,.\end{aligned} \tag{5}$$

Note that the subscripts on each expectation value are simply a reminder of the measurement context; in truth, each uses the same quantum state $|\Psi\rangle$ given by Eqn. (2). For this state, the ideal quantum predictions for the twelve terms are $\langle A \otimes A\rangle_{CAB} = \langle A \otimes A\rangle_{\alpha Aa} = -1$, $\langle B \otimes B\rangle_{CAB} = \langle B \otimes B\rangle_{\beta bB} = -1$, $\langle a \otimes a\rangle_{cba} = \langle a \otimes a\rangle_{\alpha Aa} = -1$, $\langle b \otimes b\rangle_{cba} = \langle b \otimes b\rangle_{\beta bB} = -1$, $\langle \gamma \otimes \gamma\rangle_{\beta\gamma\alpha} = \langle \gamma \otimes \gamma\rangle_{c\gamma C} = +1$, $\langle \alpha \otimes \alpha\rangle_{\beta\gamma\alpha} = +1$, and $\langle C \otimes C\rangle_{c\gamma C} = +1$, yielding a maximal value of $\langle S\rangle = 12$. The experimentally measured value for $\langle S\rangle$ was 11.430 ± 0.016. Combined with the result for $\langle\chi\rangle$, this gives a value for $\langle\omega\rangle$ of 17.247 ± 0.019, in clear violation of the aforementioned inequality and, therefore, interpreted as a signature of quantum nonlocality.

These results, while statistically significant, were obtained under experimental conditions such that the overall detection efficiency was found to be only 3.3%. As the authors acknowledge, such low detection efficiency, combined with the fair-sampling assumption, opens up the detection loophole. They do note, however, that replacing the avalanche photodiode detectors used in the experiment with superconducting detectors, which can have efficiencies of over 95%, should suffice to close this loophole.

3. LHV model

For our LHV model, let $\boldsymbol{\lambda} \in \Lambda \subset \mathbb{C}^{16}$ be a 16×1 complex vector denoting the hidden variable state, each element of which may be indexed by a four-bit integer string $x_1x_2x_3x_4$ and defined such that

$$\lambda_{x_1x_2x_3x_4} = s(\sqrt{2} - 1)\langle x_1x_2x_3x_4|\Psi_{1234}\rangle + \nu_{x_1x_2x_3x_4} \,, \tag{6}$$

where $s > 0$ is a model tuning parameter and $\boldsymbol{\nu}$ is a normalized complex standard Gaussian random vector. The factor of $\sqrt{2} - 1$ ensures that, for $s \le 1$, we have $\|\boldsymbol{\lambda}\|^2 \le 2$, since $\|\boldsymbol{\nu}\| \le 1$. We shall denote by $\Pr[\cdot]$ the resulting probability distribution of the hidden variables.