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Unified quantum density matrix description of coherence and polarization

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ABSTRACT

The properties of coherence and polarization of light has been the subject of intense investigations and form the basis of many technological applications. These concepts which historically have been treated independently can now be formulated under a single classical theory. Here, we derive a quantum counterpart for this theory, with basis on a density matrix formulation, which describes jointly the coherence and polarization properties of an ensemble of photons. The method is used to show how the degree of polarization of a specific class of mixed states changes on propagation in free space, and how an interacting environment can suppress the coherence and polarization degrees of a general state. This last application can be particularly useful in the analysis of decoherence effects in optical quantum information implementations.

1. Introduction

Coherence and polarization are undoubtedly two of the most important properties of light. In general terms, the coherence of an optical field can be understood as the ability to produce interference, as remarkably demonstrated by Young in his famous double-slit experiment, and theoretically developed by the works of Fresnel in the context of waves [1]. Another important development in the coherence theory was the one made by Glauber and Sudarshan, which established the connections about the coherence properties of light with the concept of photon statistics in a quantum mechanical scenario [2–4]. Conversely, the modern study of the polarization properties was introduced by Stokes, who proposed a set of parameters to completely describe the polarization state of a random electromagnetic wave; the so-called Stokes parameters [5,6], that can also be extended to the quantum realm [7]. Together, these two concepts form the basis of numerous applications of light in microscopy [8], cryptography [9,10], metrology [11], astronomy [12,13], as well as in future quantum information technologies [14,15].

Although the importance of the theories of coherence and polarization, their theoretical descriptions have historically been developed independently [16–18]. However, since the last decade the study of these two apparently distinct properties could be established into a single formulation through the unified theory of coherence and polarization introduced by Wolf [19]. In this seminal work, it was shown that both coherence and polarization of a random electromagnetic beam could be understood as manifestations of the correlations between fluctuations of the optical field. In this respect, coherence manifests itself from correlations between fluctuations of the electric field of a light beam at two or more points in space, whereas polarization arises from the correlations of the optical field components at a single point in space [20].

Since the publication of the unified theory, many other advances have been made towards a complete understanding of this problem. For example, the introduction of the generalized Stokes parameters [21], the description of the polarization change of partially coherent electromagnetic beam upon propagation in free space [22,23], and in the turbulent atmosphere [24,25], just to mention a few. Nevertheless, almost all these works have been limited to the scope of the classical electromagnetic theory [26,27]. In fact, there have been some recent works extending the classical unification theory to the realm of quantum mechanics by direct quantization of the electromagnetic field [28,29]. So far, this extension did not provide a significant clarification of the problem, when compared to the classical counterpart, maybe because the state of the field is characterized in the Fock space, which sometimes makes the physical intuition less precise and, depending on the environment in which the system is inserted, it is difficult to

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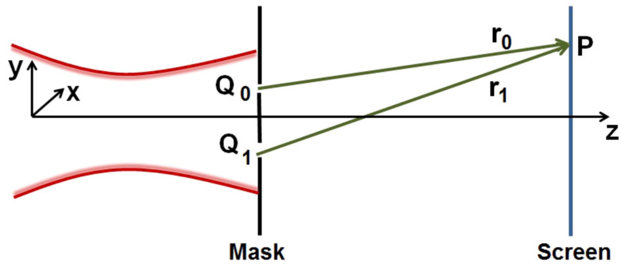

Fig. 1. (Color online.) Scheme of the double-slit experiment. An ensemble of photons impinges a mask containing two slits, Q_0 and Q_1, rendering two possible paths to each of them, which are afterwards detected on the screen.

write an appropriate Hamiltonian to account for the time evolution of the system [30].

In this work, we derive a unified quantum mechanical description of coherence and polarization from first principles, that is to say, without direct reference to the classical theory. As we shall see, the central element in this formalism is the density matrix of the system written directly in terms of the position and polarization Hilbert spaces. This last point is the responsible for making the method relatively simple when describing the behavior of a general ensemble of photons on propagation in free space, as well as under the action of an interacting environment. Indeed, we provide some applications of the model to demonstrate how a partially coherent ensemble of photons change the degree of polarization when propagating in free space, and how decoherence and depolarization take place when photons are subjected to an environment whose constituents can be refractive and birefringent. Since all these examples are presented by means of simple quantum-mechanical arguments, the present description can be particularly valuable in the study of environmental disturbance in optical quantum information processes, in which the properties of coherence and polarization play a fundamental role.

2. Theory

To start with, we derive an expression for the degree of spatial coherence of light in a context similar to the one used to derive the classical theory [19]. In doing so, let us consider a Young's double-slit experiment which consists in an ensemble of photons propagating close to the z-axis which are mostly blocked by a mask with two small openings on it. After this stage, the positions of the photons that passed through the slits are permanently registered by a distant detection screen, as shown in Fig. 1. Let $|0\rangle$ and $|1\rangle$ denote the quantum states of the photons which passed through the slits Q_0 and Q_1, respectively, and $|H\rangle$ and $|V\rangle$ the states of the photons linearly polarized along the horizontal (x-axis) and vertical (y-axis) directions, respectively. In this scenario, we can write the general quantum state of the photons in the form

$$|\psi\rangle = a|H,0\rangle + b|H,1\rangle + c|V,0\rangle + d|V,1\rangle, \tag{1}$$

with $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$, in order to specify simultaneously both the slit in which the photon passes through and the state of polarization. Also, we can write the density matrix for this system as $\hat{\rho} = |\psi\rangle\langle\psi|$, which provides a 4×4 matrix in the following format:

$$\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix} = \begin{pmatrix} |a|^2 & ab^* & ac^* & ad^* \\ ba^* & |b|^2 & bc^* & bd^* \\ ca^* & cb^* & |c|^2 & cd^* \\ da^* & db^* & dc^* & |d|^2 \end{pmatrix}, \tag{2}$$

where the asterisk denotes complex conjugation.

Now, if we are interested in computing the probability density $\rho(P)$ to find a photon at a point P on the detection screen, keeping in mind that horizontally polarized photons do not interfere with vertically polarized ones, we have that

$$\rho(P) = \langle H, P|\rho|H, P\rangle + \langle V, P|\rho|V, P\rangle, \tag{3}$$

where $|H, P\rangle$ and $|V, P\rangle$ represent the states of photons localized at P with horizontal and vertical polarizations, respectively. Assuming that the size of the slits is much smaller than the wavelength of the photons, we can consider that after passing through a given slit the wavefunction of the photons are spherical waves. Therefore, the probability amplitudes of finding a photon at P with horizontal (vertical) polarization which passed through the slit Q_0 (Q_1) are given, respectively, by

$$\psi^{(P)}_{H,V}(r_0) = \langle H, P|H, 0\rangle = \langle V, P|V, 0\rangle = \frac{e^{ikr_0}}{r_0} \tag{4}$$

and

$$\psi^{(P)}_{H,V}(r_1) = \langle H, P|H, 1\rangle = \langle V, P|V, 1\rangle = \frac{e^{ikr_1}}{r_1}, \tag{5}$$

with i and k being the imaginary unity and the wavenumber, respectively. The parameters r_0 and r_1 are the distances from the slits Q_0 and Q_1 to the point P, respectively. By substitution of Eqs. (4) and (5) into Eq. (3), and cancelling out terms with inner products between horizontal and vertical polarization states, we obtain that

$$\rho(P) = \frac{\rho_{11} + \rho_{33}}{r_0^2} + \frac{\rho_{22} + \rho_{44}}{r_1^2} + \frac{2Re[(\rho_{21} + \rho_{34})e^{ik(r_0 - r_1)}]}{r_0 r_1}, \tag{6}$$

where we used the fact that ρ is Hermitian, $\rho_{mn} = \rho^*_{nm}$, and Re denotes the real part.

Let us visualize Eq. (6) under a different perspective. Observe that if the slit Q_1 is closed, the amplitudes b and d are null in Eq. (1), therefore, Eq. (6) reduces to

$$\rho_0(P) = \frac{\rho_{11} + \rho_{33}}{r_0^2}, \tag{7}$$

which represents the probability density of finding a photon that emerged exclusively from Q_0 at P. Similarly, the probability density of finding a photon that emerged from Q_1 at P is given by

$$\rho_1(P) = \frac{\rho_{22} + \rho_{44}}{r_1^2}. \tag{8}$$

In this context, Eq. (6) can be rewritten as

$$\rho(P) = \rho_0(P) + \rho_1(P) + 2\sqrt{\rho_0(P)}\sqrt{\rho_1(P)}Re[\mu e^{ik(r_0 - r_1)}], \tag{9}$$

where the parameter μ is given by

$$\mu = \frac{\rho_{12} + \rho_{34}}{\sqrt{\rho_{11} + \rho_{33}}\sqrt{\rho_{22} + \rho_{44}}}. \tag{10}$$

The first two terms in Eq. (9) correspond to the sum of the individual probability densities of the photons which passed through each slit, and the last term is responsible for the interference pattern on the detection screen. Note that the parameter that dictates the prominence of the interference pattern in this system is μ, which we define as the *degree of coherence*. Therefore, this parameter can be measured by detecting the patterns due to the photons which emerge from each slit separately, and the pattern formed when both slits are open, by means of Eq. (9).

By substitution of the amplitudes a, b, c and d in Eq. (10), and using the Cauchy-Schwarz inequality, it is easy to show that $0 \leq |\mu| \leq 1$. Since the interference term is maximum when $|\mu| = 1$,

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