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#### $^{11}$  Unified quantum density matrix description of coherence  $\frac{11}{12}$  Unified quantum density matrix description of coherence  $\frac{77}{78}$  $13$  and polarization  $79$  $14$  80

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#### 20 and the contract of the con zi ARTICLE INFO ABSTRACT 87

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Polarization of light Coherence properties Entanglement

 $\frac{32}{2}$  by the set of the set o Decoherence

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#### $35$

23 Article history:<br>23 Bestimble 1992 The properties of coherence and polarization of light has been the subject of intense investigations ag  $^{24}$  Received in revised form 29 March 2017 treated independently can now be formulated under a single classical theory. Here, we derive a quantum 25 Accepted 10 May 2017<br>counterpart for this theory, with basis on a density matrix formulation, which describes jointly the 26 Bould Bould extra<br>Communicated by A. Eighd 27 Communicated by A. Elsicial expansion of the degree of polarization of a specific class of mixed states changes on propagation in free space, and 93 <sup>28</sup> *Kevwords:* environment can suppress the coherence and polarization degrees of a general state. <sup>94</sup> <sup>29</sup> Density matrix **the canalisat of the Constant Constant** This last application can be particularly useful in the analysis of decoherence effects in optical quantum <sup>95</sup> 30 96 information implementations. and form the basis of many technological applications. These concepts which historically have been

 $_{31}$  Coherence properties examples to the control of t

#### **1. Introduction**

39 105 Coherence and polarization are undoubtedly two of the most 40 important properties of light. In general terms, the coherence of and polarization introduced by Wolf [19]. In this seminal and 41 an optical field can be understood as the ability to produce in- Work, it was shown that both conerence and polarization of a ran-  $_{107}$ 42 108 terference, as remarkably demonstrated by Young in his famous 43 double-slit experiment, and theoretically developed by the works of the correlations between fluctuations of the optical field. In this 109 44 110 of Fresnel in the context of waves [\[1\].](#page--1-0) Another important devel-45 opment in the coherence theory was the one made by Glauber but the effect the controlled of a fight beam at two or more points  $\frac{1}{111}$ 46 and Sudarshan, which established the connections about the coher- The Space, whereas polarization arises from the correlations of the 112 47 ence properties of light with the concept of photon statistics in a  $\frac{1}{2}$  optical field components at a single point in space [20]. 48 quantum mechanical scenario [\[2–4\].](#page--1-0) Conversely, the modern study since the publication of the unified theory, many other ad-<br> 49 of the polarization properties was introduced by Stokes, who pro-<br>19 National Diversion made towards a complete understanding of this complete understanding of this interest. 50 116 posed a set of parameters to completely describe the polarization  $^{51}$  state of a random electromagnetic wave; the so-called Stokes pa- parameters [21], the description of the polarization change of par- 117  $^{52}$  sameters [\[5,6\],](#page--1-0) that can also be extended to the quantum realm [\[7\].](#page--1-0) Utally conerent electromagnetic beam upon propagation in free 118 53 Together, these two concepts form the basis of numerous appli-<br> $\frac{1}{22,23}$ , and in the turbulent atmosphere [24,25], just to 119  $54$  cations of light in microscopy [\[8\],](#page--1-0) cryptography [\[9,10\],](#page--1-0) metrology mention a rew. Nevertheless, almost all these works have been lim-<br>  $55$  [\[11\],](#page--1-0) astronomy [\[12,13\],](#page--1-0) as well as in future quantum information the two the scope of the classical electromagnetic theory [26,27]. In the 121 56 122 fact, there have been some recent works extending the classical technologies [\[14,15\].](#page--1-0)

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 $37$  **1. Introduction 1. Introduction 103 1. 103** 38 38 **104** 104 **104** 104 **104** 104 **104** 104 **104** 104 **104** 104 **104** 104 **104** 104 **104** 104 **104** 104 lished into a single formulation through the unified theory of co-herence and polarization introduced by Wolf [\[19\].](#page--1-0) In this seminal work, it was shown that both coherence and polarization of a random electromagnetic beam could be understood as manifestations of the correlations between fluctuations of the optical field. In this respect, coherence manifests itself from correlations between fluctuations of the electric field of a light beam at two or more points in space, whereas polarization arises from the correlations of the optical field components at a single point in space [\[20\].](#page--1-0)

57 Although the importance of the theories of coherence and po-<br>123  $58$  larization, their theoretical descriptions have historically been de-<br> $94$  quantization of the electromagnetic neighborhood  $28,29$ . So far, this ex-<br> $124$ 59 125 tension did not provide a significant clarification of the problem, 60 126 when compared to the classical counterpart, maybe because the  $\overline{\phantom{a}}$   $\overline{\phantom{a}}$  Correspondence to: Departamento de Física Universidade Federal de Pernam-<br>State of the field is characterized in the Fock space, which some-<br>127 62 128 times makes the physical intuition less precise and, depending on 63 63 **63** *E-mail address: bertulio.fisica@gmail.com.* The environment in which the system is inserted, it is difficult to 129 Since the publication of the unified theory, many other advances have been made towards a complete understanding of this problem. For example, the introduction of the generalized Stokes parameters [\[21\],](#page--1-0) the description of the polarization change of partially coherent electromagnetic beam upon propagation in free space  $[22,23]$ , and in the turbulent atmosphere  $[24,25]$ , just to mention a few. Nevertheless, almost all these works have been limited to the scope of the classical electromagnetic theory [\[26,27\].](#page--1-0) In unification theory to the realm of quantum mechanics by direct quantization of the electromagnetic field [\[28,29\].](#page--1-0) So far, this ex-

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#### 2 *B. de Lima Bernardo / Physics Letters A* ••• *(*••••*)* •••*–*•••



13 each of them, which are afterwards detected on the screen.

15 write an appropriate Hamiltonian to account for the time evolution  $\psi_{H,V}(r_0) = \langle H, P|H, \psi \rangle = \langle V, P|V, \psi \rangle = \frac{r_0}{r_0}$  (4)  $\psi_{H,V}(r_0) = \frac{r_0}{r_0}$  $16$  of the system [50]. of the system [\[30\].](#page--1-0)

 $_{17}$  In this work, we derive a unified quantum mechanical descrip-  $\frac{1}{2}$  and  $\frac{1}{2}$  an 18 84 tion of coherence and polarization from first principles, that is to 19 say, without direct reference to the classical theory. As we shall  $\psi_{H,V}^{(r)}(r_1) = \langle H, P|H, 1 \rangle = \langle V, P|V, 1 \rangle = \frac{1}{r_1}$  (5) as <sub>20</sub> see, the central element in this formalism is the density matrix  $^{11}$  and  $^{11}$  a <sub>21</sub> of the system written directly in terms of the position and polar- with *i* and *k* being the imaginary unity and the wavenumber, re- 87  $_{22}$  ization Hilbert spaces. This last point is the responsible for mak- spectively. The parameters  $r_0$  and  $r_1$  are the distances from the se  $_{23}$  ing the method relatively simple when describing the behavior of slits  $Q_0$  and  $Q_1$  to the point P, respectively. By substitution of as  $_{24}$  a general ensemble of photons on propagation in free space, as  $\;$  Eqs. (4) and (5) into Eq. (3), and cancelling out terms with inner age <sub>25</sub> well as under the action of an interacting environment. Indeed, products between horizontal and vertical polarization states, we sa  $_{26}$  we provide some applications of the model to demonstrate how a solution that the model of  $_{92}$  $_{27}$  partially coherent ensemble of photons change the degree of po-<br> $_{27}$   $_{23}$   $_{22}$   $_{23}$   $_{23}$   $_{24}$   $_{23}$   $_{23}$   $_{24}$   $_{25}$   $_{26}$   $_{27}$   $_{28}$   $_{29}$   $_{29}$   $_{20}$   $_{21}$   $_{22}$   $_{23}$   $_{20}$   $_{22}$   $_{2$ 28 larization when propagating in free space, and how decoherence  $\rho(P) = \frac{P(1-P-1)}{P} + \frac{P(2-P-1)}{P}$ <sub>29</sub> and depolarization take place when photons are subjected to an  $r_0$  and  $r_1$  and  $r_1$  and  $r_2$  and  $r_3$ 30 environment whose constituents can be refractive and birefrin-<br>30  $2Re[(\rho_{21}+\rho_{34})e^{ik(r_0-r_1)}]$  $31$  gent. Since all these examples are presented by means of simple  $\overline{a}$  +  $\overline{a}$  +  $\overline{a}$  +  $\overline{a}$ ,  $\overline{a}$ ,  $\overline{b}$  (b)  $\overline{a}$  $32$  quantum-mechanical arguments, the present description can be  $10^{10}$  $_{33}$  particularly valuable in the study of environmental disturbance in where we used the fact that  $\rho$  is Hermitian,  $\rho_{mn}=\rho^*_{nm}$ , and Re  $_{99}$  $_{34}$  optical quantum information processes, in which the properties of denotes the real part. of the system written directly in terms of the position and polarization Hilbert spaces. This last point is the responsible for making the method relatively simple when describing the behavior of a general ensemble of photons on propagation in free space, as well as under the action of an interacting environment. Indeed, coherence and polarization play a fundamental role.

#### **2. Theory**

39 105 To start with, we derive an expression for the degree of spatial  $_{40}$  coherence of light in a context similar to the one used to derive the  $r_0^2$  and  $41$  classical dieory [15]. In doing so, let us consider a roung s double-<br>which represents the probability density of finding a photon that  $107$  $42$  sure  $\frac{1}{2}$  and  $\frac{1}{2}$  at *P*. Similarly, the probability den-<br>emerged exclusively from  $Q_0$  at *P*. Similarly, the probability den-<br>emerged exclusively from  $Q_0$  at *P*. Similarly, the probability den-43 Battlig close to the *L*-axis which are involved by a mask with sity of finding a photon that emerged from  $Q_1$  at *P* is given by 109 44 two small openings on it. After this stage, the positions of the pho-45 tons that passed through the slits are permanently registered by a  $\rho_1(P) = \frac{\rho_{22} + \rho_{44}}{\rho_{42}}$  (8) 111 46 distant detection screen, as shown in Fig. 1. Let  $|0\rangle$  and  $|1\rangle$  denote  $\frac{P_1(1)}{P_1^2}$ .  $_{47}$  the quantum states of the photons which passed through the slits  $_{113}$  $q_0$  and  $Q_1$ , respectively, and  $|H\rangle$  and  $|V\rangle$  the states of the photons In this context, Eq. (6) can be rewritten as 49 linearly polarized along the horizontal (x-axis) and vertical (y-axis)  $Q(P) = Q_2(P) + Q_1(P) + Q_2(P) + Q_3(P) + Q_4(P)P_5Q_6(P)$  $_{50}$  directions, respectively. In this scenario, we can write the general  $P(Y) = P(0|Y) + P(1) + \sum_{i=1}^{\infty} P(i) P(i) P(i) P(i) P(i) P(i)$ <sub>51</sub> quantum state of the photons in the form **by** where the parameter  $\mu$  is given by classical theory [\[19\].](#page--1-0) In doing so, let us consider a Young's doubleslit experiment which consists in an ensemble of photons propagating close to the z-axis which are mostly blocked by a mask with quantum state of the photons in the form

$$
\begin{aligned}\n\mathbf{52} \quad |\psi\rangle &= a|H,0\rangle + b|H,1\rangle + c|V,0\rangle + d|V,1\rangle,\n\end{aligned}\n\tag{1}
$$

 $54$  with  $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$ , in order to specify simultaneously  $\sqrt{\rho_{11} + \rho_{33}\sqrt{\rho_{22} + \rho_{44}}}$ 55 121 The first two terms in Eq. (9) correspond to the sum of the indi-56 Polarization. Also, we can write the density matrix for this sys-<br>vidual probability densities of the photons which passed through 122  $57$  tentries  $p = |\psi\rangle\langle\psi|$ , which provides a  $4 \times 4$  matrix in the following each slit, and the last term is responsible for the interference pat-<br> $\epsilon$ both the slit in which the photon passes through and the state of polarization. Also, we can write the density matrix for this system as  $\hat{\rho} = |\psi\rangle \langle \psi|$ , which provides a  $4 \times 4$  matrix in the following format:

$$
\hat{\rho} = \begin{pmatrix}\n\rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\
\rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\
\rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\
\rho_{41} & \rho_{42} & \rho_{43} & \rho_{44}\n\end{pmatrix} = \begin{pmatrix}\n|a|^2 & ab^* & ac^* & ad^* \\
ba^* & |b|^2 & bc^* & bd^* \\
ca^* & cb^* & |c|^2 & cd^* \\
da^* & db^* & dc^* & |d|^2\n\end{pmatrix},
$$
\n(2) the prominence of the interference pattern in this system is  $\mu$ , the volume defined as the degree of coherence. Therefore, this parameter is  $\mu$ , and the pattern formed when both slits are open, by means of Eq. (9).

\n2) The temperature of the interference pattern in this system is  $\mu$ ,  $\$ 

where the asterisk denotes complex conjugation.

Now, if we are interested in computing the probability density

<sup>1</sup> 1 67 **in mind that horizontally polarized photons do not interfere with 67 <b>in** the structure of th e vertically polarized ones, we have that the set of the

$$
\rho(P) = \langle H, P | \rho | H, P \rangle + \langle V, P | \rho | V, P \rangle, \tag{3}
$$

<sup>6</sup> *ized at P* with horizontal and vertical polarizations, respectively. <sup>72</sup> **Assuming that the size of the slits is much smaller than the wave-**  $\frac{73}{2}$ 8 74 length of the photons, we can consider that after passing through **9 1 Mask Screen** a given slit the wavefunction of the photons are spherical waves.  $^{75}$ 10 76 Therefore, the probability amplitudes of finding a photon at *P* with <sup>11</sup> **Fig. 1.** (Color online.) Scheme of the double-slit experiment. An ensemble of photons horizontal (vertical) polarization which passed through the slit  $Q_0$ <sup>77</sup> 12 impinges a mask containing two slits,  $Q_0$  and  $Q_1$ , rendering two possible paths to  $(Q_1)$  are given respectively by horizontal (vertical) polarization which passed through the slit *Q*<sup>0</sup> (*Q*1) are given, respectively, by

$$
\psi_{H,V}^{(P)}(r_0) = \langle H, P | H, 0 \rangle = \langle V, P | V, 0 \rangle = \frac{e^{ikr_0}}{r_0}
$$
\n
$$
\psi_{H,V}^{(P)}(r_0) = \langle H, P | H, 0 \rangle = \langle V, P | V, 0 \rangle = \frac{e^{ikr_0}}{r_0}
$$
\n
$$
\tag{4}
$$

and

$$
\psi_{H,V}^{(P)}(r_1) = \langle H, P|H, 1 \rangle = \langle V, P|V, 1 \rangle = \frac{e^{ikr_1}}{r_1},\tag{5}
$$

obtain that

$$
\rho(P) = \frac{\rho_{11} + \rho_{33}}{r_0^2} + \frac{\rho_{22} + \rho_{44}}{r_1^2} + \frac{2Re[(\rho_{21} + \rho_{34})e^{ik(r_0 - r_1)}]}{r_0 r_1},
$$
\n(6)

where we used the fact that  $\rho$  is Hermitian,  $\rho_{mn} = \rho_{nm}^*$ , and *Re* denotes the real part.

 $_{35}$  coherence and polarization play a fundamental role.  $_{101}$  Let us visualize Eq. (6) under a different perspective. Observe  $_{101}$ 36 102 that if the slit *Q*<sup>1</sup> is closed, the amplitudes *b* and *d* are null in  $\text{Eq. (1), therefore, Eq. (6) reduces to}$   $\text{Eq. (6) reduces to}$ 

$$
\begin{array}{ll}\n\text{38} & \text{To start with, we derive an expression for the degree of spatial} \\
\text{40} & \text{coherence of light in a context similar to the one used to derive the} \\
\end{array} \qquad \qquad \rho_0(P) = \frac{\rho_{11} + \rho_{33}}{r_0^2},\tag{7} \qquad\n\begin{array}{ll}\n\text{50} & \text{51} \\
\text{61} & \text{62} \\
\text{71} & \text{63} \\
\text{82} & \text{64} \\
\text{93} & \text{70} \\
\text{100} & \text{100} \\
\text{101} & \text{101} \\
\text{102} & \text{102} \\
\text{103} & \text{103} \\
\text{104} & \text{106} \\
\text{108} & \text{108} \\
\text{109} & \text{101} \\
\text{100} & \text{101} \\
\text{101} & \text{102} \\
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\text{109} & \text{101} \\
\text{101} & \text{101} \\
\
$$

$$
\rho_1(P) = \frac{\rho_{22} + \rho_{44}}{r_1^2}.
$$
\n(8)

In this context, Eq.  $(6)$  can be rewritten as

$$
\rho(P) = \rho_0(P) + \rho_1(P) + 2\sqrt{\rho_0(P)}\sqrt{\rho_1(P)} Re[\mu e^{ik(r_0 - r_1)}], \quad (9)
$$

$$
\begin{array}{lll}\n\frac{62}{52} & |\psi\rangle = a|H, 0\rangle + b|H, 1\rangle + c|V, 0\rangle + d|V, 1\rangle, & (1) & & \\
\frac{63}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify simultaneously.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1, \text{ in order to specify the following.} & \\
\frac{64}{52} & \text{with } |a|^2 + |b|^2 + |c|^2 + |d|^2 =
$$

58 124 tern on the detection screen. Note that the parameter that dictates 63  $(r+1)^{n+2}$   $(r+2)^{n+3}$   $(r+3)^{n+4}$   $(r+4)^{n+5}$   $(r+4)^{n+6}$   $(r+5)^{n+1}$   $(r+6)^{n+1}$   $(r$ 

 130 By substitution of the amplitudes *a*, *b*, *c* and *d* in Eq. (10), Now, if we are interested in computing the probability density and using the Cauchy-Schwarz inequality, it is easy to show that  $131$  $\rho(P)$  to find a photon at a point P on the detection screen, keeping  $0 \leq |\mu| \leq 1$ . Since the interference term is maximum when  $|\mu| = 1$ , 132  $0 \leq |\mu| \leq 1$ . Since the interference term is maximum when  $|\mu| = 1$ ,

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