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Patterns formation in diffusive excitable systems under magnetic flow effects

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ABSTRACT

We study the spatiotemporal formation of patterns in a diffusive FitzHugh–Nagumo network where the effect of electromagnetic induction has been introduced in the standard mathematical model by using magnetic flux, and the modulation of magnetic flux on membrane potential is realized by using memristor coupling. We use the multi-scale expansion to show that the system equations can be reduced to a single differential-difference nonlinear equation. The linear stability analysis is performed and discussed with emphasis on the impact of magnetic flux. It is observed that the effect of memristor coupling importantly modifies the features of modulational instability. Our analytical results are supported by the numerical experiments, which reveal that the improved model can lead to nonlinear quasi-periodic spatiotemporal patterns with some features of synchronization. It is observed also the generation of pulses and rhythmics behaviors like breathing or swimming which are important in brain researches.

1. Introduction

The complexity of the brain makes it the most important system in nature. The brain is made up of a large number of neurons grouped into functional ensembles generally called microcircuits [1]. They have a striking capacity to produce a considerable variety of coordinated patterns in response to their surrounding changes or their behavioral needs. In fact, spiking neurons have attracted the interest because many studies consider this behavior an essential component in biophysics and mathematics information processing in the nerve cell. Excitability is a common property of many physical and biological systems. As now well established, in excitable media nonlinear waves have a great importance for a better understanding of some cooperative behavior including patterns formation and synchronization since such phenomena are related to normal functioning and generation of some neural ailments [2–4]. In general, neuronal systems exemplify the properties of biologically excitable media, although relatively few investigations have been carried out on the initiation, propagation, and pathways of dynamic excitatory waves. The FitzHugh Nagumo (FHN) model is a generic model of excitability and oscillatory dynamical behavior. The model has been introduced by FitzHugh [5] with equivalent circuit by Nagumo et al. [6] is a generalization of the Van der Pol oscillator. The FHN mathematical model has been greatly contributing in nonlinear neurodynamics domain and has become prototype model for systems exhibiting excitability.

In the past years, mathematical and physical contributions devoted to neurological sciences have improved our understanding of pattern formation using the FHN mathematical model. Just to cite a few, simpler pulse solutions in the discrete FHN model have been constructed asymptotically [7–10]. Taking into account two different time constants, Panfilov and Hogeweg [11] modified the standard FHN model for excitable tissue and showed that a spiral wave can break up into an irregular spatial pattern. Dimitry et al. [12] showed that particle-like behavior can lead to formation of complex periodic and chaotic fractal-like spatiotemporal wave patterns in modified FHN network. Malevanets and Kapral [13] showed that fully developed labyrinthine pattern can be observed in a microscopic reaction model with a FHN mass action law. Very recently, Zhenga and Shena [14] showed that the FHN model has very rich dynamical behaviors, such as spotted, stripe and hexagon patterns.

However, the excitability property is much too complex and many factors should be considered as well. According to Faraday´s law of induction, the fluctuation or changes in action potentials in excitable cells (neurons) can generate magnetic field in the media;

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in that sense, the excitability of neurons will be adjusted under feedback effect [15]. Albeit the satisfactory results reported in the above mentioned studies, the appropriate mechanism and the conditions under which spatiotemporal patterns emerge and spread among coupled neurons under electromagnetic induction have not been investigated. Therefore, it is important to set more reliable FHN models, so that the effect of electromagnetic induction could be considered. This constitutes the strong motivation to this Letter. It is well known that modulational instability (MI) is a process closely related to solitons and wave patterns formation in various physical settings, where amplitude and phase modulations tend to grow exponentially due to the simultaneous effects of nonlinearity and dispersion. We show in this work that the effect of electromagnetic induction can enhance spatiotemporal information through the activation of MI. The improved FHN network can generate rhythmics behaviors like breathing or swimming which are important in brain researches [1].

The rest of the Letter is organized as follows: in Section 2, we introduce the model along with the important mathematical developments and we make use of the semi-discrete approximation to show that the equations of the system can fully be described by a single differential-difference nonlinear equation. In Section 3, the MI of plane wave solution is performed and we discuss the possibility of common regions of instability with emphasis of memristor coupling. In order to confirm our analytical predictions, we perform the numerical simulations from the generic equations of the model. Section 4 concludes the paper.

2. Model equation and asymptotic expansion

It is well known that the standard FHN model is described by two variables $v(t)$ and $w(t)$, which represent the trans-membrane potential and the slow variable for current, respectively. In this Letter, we introduce the magnetic flux variable $\phi(t)$, which is used to describe the effect of electromagnetic induction. The equations for an improved FHN model for $N = 400$ identical neurons mutually coupled to their nearest neighbors through the gap junction is now made of three ordinary differential equations for the dimensionless variable $v(t)$, $w(t)$ and $\phi(t)$ as follows:

$$\begin{aligned} \frac{dv_n}{dt} &= K(v_{n+1} - 2v_n + v_{n-1}) + v_n(v_n - a)(2 - v_n) \\ &\quad - w_n - k_1\rho(\phi_n)v_n + I_{ext}, \\ \frac{dw_n}{dt} &= \lambda(v_n - bw_n), \\ \frac{d\phi_n}{dt} &= v_n - k_2\phi_n + \phi_{ext}, \end{aligned} \tag{1}$$

with $n = 1, 2, ..., N$.

The parameter K is the coupling parameter between cells, while the parameter λ represents the ratio of the time scales for $v_n(t)$ and $w_n(t)$. The function $\rho(\phi_n) = \alpha + 3\beta\phi_n^2$ is the conductance developed from memristor [16,17] and used for memory associated with magnetic field. According to Faraday's law of electromagnetic induction and description about memristor [18], the term $k_1\rho(\phi_n)v_n$ could be regarded as additive induction current on the membrane. ϕ_{ext} is the external electromagnetic radiation which for simplicity is taken as a periodical function $\phi_{ext} = A\cos(2\pi f t)$. The ion currents of sodium, potassium contribute the membrane potential and also the magnetic flux across the membrane; thus, a negative feedback term $-k_2\phi_n$ has been introduced in the third equation of (1). I_{ext} represents the external forcing current. The parameter values used in this work are: $a = 0.3$, $b = 0.5$, $\lambda = 0.01$, $k_2 = 1$, $\alpha = 0.1$ and $\beta = 0.02$. The parameters K, k_1, I_{ext} and ϕ_{ext} will be selected so to display formation of complex patterns of the action potential.

Nonlinear equations are, owing to their complexity, typically not accessible to an analytic approach. Sometimes, the analysis involve the use of asymptotic expansions such as the multiple-scale expansion, the Fourier series expansion and the semi-discrete approximation, just to cite a few. In this Letter, we use the multiple scale analysis expansions, which implies that the first node of the network, i.e., $n = 0$, is excited at the natural frequency Ω_0. Due to nonlinear effects, that naturally affect real systems, the natural frequency will deviate, with actual frequency Ω and wave number q, to become $\Omega = \Omega_0 + \epsilon\mu$ and $q = K + \epsilon\frac{\mu}{V_g} + \epsilon^2 C_g\mu^2 + ...$, where $\frac{1}{V_g} = \frac{\partial q}{\partial\Omega}$ is the group velocity and $2C_g = \frac{\partial^2 q}{\partial\Omega^2}$ represents the group velocity dispersion. μ is a small deviation from the natural frequency Ω_0. It is therefore clear that for $\epsilon = 0$, the frequency Ω reduces to the natural frequency Ω_0 of the system. Equation (1) can be summarized in terms of the state vector $U_n(t) = \{v_n(t), w_n(t), \phi_n(t)\}$, whose unperturbed expressions can be taken in the generalized form

$$U_n(t) = \int d\Omega \hat{U}(\Omega) e^{i(qn+\Omega)}, \tag{2}$$

where $\hat{U}(\Omega) = \{\hat{v}(\Omega), \hat{w}(\Omega), \hat{\phi}(\Omega)\}$. With the expanded ω and q along with change of variables $\tau_n = \epsilon(t + n/V_g)$ and $\zeta_n = \epsilon^2 n$ and the condition $C_g = 1$, the generalized trial solutions take the form

$$U_n(t) = A(n,t)U(\zeta_n, \tau_n), \tag{3}$$

where $A(n,t) = e^{i(qn+\Omega t)}$. The method introduces a new lattice number m to support a large grid [19]. It follows that for a given lattice number n, only the set of lattice points $..., n-N, n, n+N, ...$ can be indexed in terms of the slow variable m as $\{..., (n-N) \to (m-1),\ n \to m,\ (n+N) \to (m+1)..., \}$, where $\epsilon^2 = 1/N$ is assumed due to highly pronounced discreteness effects. In so doing, the slow modulation $S(\zeta_n, \tau_n)$ of the plane wave $A(n,t)$ can be replaced by the functions $U(m,\tau)$, with $\tau = \tau_n$, and one can easily make use of the Fourier series in power of the parameter ϵ

$$U_n(t) = \sum_{p=1}^{\infty} \epsilon^p \sum_{l=-p}^{p} U_p^l(n,t). \tag{4}$$

From (4), we have $U_p^{-l}(m,\tau) = (U_p^l(m,\tau))^*$. Inserting the above solutions into Eq. (1) leads to a set of coupled equations to be solved at different orders of the small parameter ϵ, with the corresponding harmonics l. For the leading order $(1,l)$, with $l = 0$, we obtain the solution

$$\eta_1^0(m,\tau) = \psi_1^0(m,\tau) = \phi_1^0(m,\tau) = 0. \tag{5}$$

For $l = 1$ the dispersion relation

$$\Big[(i\Omega - 2K(\cos(q) - 1) + 2a + \alpha k_1)(i\Omega + \lambda b) + \lambda\Big](i\Omega + k_2) = 0 \tag{6}$$

should be satisfied for the system to admit non-trivial solutions in the form

$$\begin{aligned} \eta_1^1(m,\tau) &= \eta(m,\tau), \\ \psi_1^1(m,\tau) &= \frac{\lambda}{i\Omega + \lambda b}\eta(m,\tau), \\ \phi_1^1(m,\tau) &= \frac{\eta(m,\tau)}{(i\Omega + k_2)}. \end{aligned} \tag{7}$$