Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Transport of spin-orbit coupled Bose–Einstein condensates in lattice with defects

Kun-Qiang Zhu, Zi-Fa Yu, Ju-Kui Xue *

Key Laboratory of Atomic & Molecular Physics and Functional Materials of Gansu Province, College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou, 730070, China

ARTICLE INFO

Article history:
Received 24 February 2017
Received in revised form 5 May 2017
Accepted 13 May 2017
Available online xxxx
Communicated by V.A. Markel

Keywords:
Bose–Einstein condensate
Spin-orbit coupling
Superfluidity
Defects

ABSTRACT

We theoretically investigate the propagation properties of spin-orbit (SO) coupled Bose–Einstein condensate in an optical lattices with defects. By using the tight-binding and two-mode ansatz approximation, we find that the coupled effects of SO-coupling, Raman coupling, Zeeman field and atomic interactions can control the superfluidity of the system. Particularly, there exists a critical scattering length for crossing from a normal regime to a superfluid regime. The critical scattering length for supporting the superfluidity strongly depends on the defect type, SO-coupling, Raman coupling, Zeeman field and quasimomentum of the plane waves. The SO-coupling and quasimomentum make the system more easily entering into the superfluid regime, while the pure Raman coupling and pure Zeeman field inhibit the system entering into the superfluid regime. Interestingly, the coupled effect between Raman coupling and Zeeman field can both enhance and suppress the system entering into the superfluid regime. This engineering provides a possible means for studying the propagation properties and the corresponding dynamics of two-species SO-coupled BECs in disordered optical lattice.

1. Introduction

Spin-orbit (SO) coupling is an interaction between a quantum particle's spin and its momentum, which plays a crucial role in condensed matter physics system. In electronic systems, it leads to many interesting concepts, such as the quantum spin Hall effect and topological insulators [1–3]. In cold-atom systems, recently, the synthetic SO-coupling has been realized experimentally in neutral bosonic systems [4,5] and fermionic atomic gases [6,7]. In particular, SO-coupling with equal Rashba [8] and Dresselhaus [9] strength in a neutral atomic Bose–Einstein condensate (BEC) has been engineered by dressing two atomic spin states with a pair of counter propagation laser beams. A plenty of researches have been done on the properties of the BEC with SO-coupling, such as rich phase diagrams of ground states [10,11], tunneling dynamics [12–15], localization properties in quasiperiodic optical lattice [16–19], and nonlinear matter waves [20–23].

On the other hand, the successful control of ultracold atoms in an optical lattices has made it be an ideal playground to explore a variety of fascinating quantum phenomena [24–28]. The tunneling of atomics of the BEC in optical lattice can be controlled by the potential barrier [29–32]. Especially, the optical clock transition with Fermi gases and the spin-orbit coupled Bose–Einstein condensate loaded into a translating optical lattice have been studied [33,34]. Optical lattices provide a possible method to investigate ultracold atoms, but it always exist small random impurities or defects, which can be created with additional lasers and/or magnetic fields [35–41]. In a common physical system, the defects can be spatially localized or extended [35,36]. Particularly, the competition between defect and atomic interaction plays a crucial role in transportation property of the system and induces rich phenomena, for example, the defect can inhibits the plane waves transportation [42–44] and the propagation of plane waves experiences a crossover from a superfluid regime to a normal regime [29,35,44]. Thus, the transportation property of the disordered nonlinear discrete system has become a challenging issue. However, the transportation property of SO-coupled BEC in an optical lattices with defects has not been explored theoretically.

In this paper, we investigate the propagation properties of a SO-coupled BEC in a deep annular lattice with defects, by using the two-ansatz and tight-binding approximation. We find that the coupled effects of SO-coupling, Raman coupling, Zeeman field and atomic interactions can control the superfluidity of the system. We find that there exists a critical scattering length a_c that divides

* Corresponding author.
E-mail address: xuejk@nwnu.edu.cn (J.-K. Xue).

http://dx.doi.org/10.1016/j.physleta.2017.05.024
0375-9601/

the system into two regime: when $a > a_c$, the system is in a superfluid regime, in which a plane wave coherently passes through the defects, and when $a < a_c$, the system is in a normal regime, in which the propagation of the plane wave is reflected by the defect. Importantly, a_c and superfluidity of the system strongly depend on SO coupling, Raman coupling, Zeeman field and quasimomentum. The SO-coupling and quasimomentum can both result in the decrease of a_c, i.e. SO-coupling and quasimomentum enhance the superfluidity of the system, which means the system is more easily entering the superfluid regime. The pure Raman coupling (without Zeeman field) and pure Zeeman field (without Raman coupling) can both result in the increase of a_c, i.e. pure Raman coupling or pure Zeeman field inhibits the system entering the superfluid regime, which means the system is more difficultly entering the superfluid regime. Interestingly, the coupling between Raman coupling and Zeeman field can both enhance and suppress the system entering the superfluid regime. In addition, the superfluidity of the system also depends on the defect type.

2. Model

We consider bosons with internal spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ confined in one dimensional optical lattice with defects. Moreover, a pair of counterpropagating Raman beams couple the atomic states $|\uparrow, k_x = q\rangle$ and $|\downarrow, k_x = q + 2k_L\rangle$, which creates the effective SO-coupling. Here q and k_L are the quasimomentum and the wave vector of the Raman laser. Through Peierls substitution [16,45,46], one can get the SO-coupling strength $\gamma = k_L/k_1$ (k_1 is the wave vector of the lattice), which can be controlled by adjusting the angle between the Raman beams [16]. According to the conditions given above, the system can be described by the dimensionless Bose–Hubbard model [13,16,31,47]:

$$\begin{aligned}\hat{H} = &\sum_n[-(\hat{\varphi}_n^\dagger e^{-i\pi\gamma\hat{\sigma}_y}\hat{\varphi}_{n+1} + H.c.)] \\ &+ \frac{1}{2}\sum_{n\sigma}[a_{\sigma\sigma}\hat{\varphi}_{n\sigma}^\dagger\hat{\varphi}_{n\sigma}(\hat{\varphi}_{n\sigma}^\dagger\hat{\varphi}_{n\sigma} - 1) + a_{\sigma\overline{\sigma}}\hat{\varphi}_{n\sigma}^\dagger\hat{\varphi}_{n\sigma}\hat{\varphi}_{n\overline{\sigma}}^\dagger\hat{\varphi}_{n\overline{\sigma}}] \\ &+ \frac{\Omega}{2}\sum_n(\hat{\varphi}_{n\downarrow}^\dagger\hat{\varphi}_{n\uparrow} + \hat{\varphi}_{n\uparrow}^\dagger\hat{\varphi}_{n\downarrow}) + \frac{\delta}{2}\sum_n(\hat{\varphi}_{n\uparrow}^\dagger\hat{\varphi}_{n\uparrow} - \hat{\varphi}_{n\downarrow}^\dagger\hat{\varphi}_{n\downarrow}) \\ &+ \epsilon_n\sum_{n\sigma}\hat{\varphi}_{n\sigma}^\dagger\hat{\varphi}_{n\sigma},\end{aligned} \tag{1}$$

where $\hat{\varphi}_n = (\hat{\varphi}_{n\uparrow}, \hat{\varphi}_{n\downarrow})^T$ and $\hat{\varphi}_{n\sigma}^\dagger$ ($\hat{\varphi}_{n\sigma}$) creates (annihilates) a spin σ ($\sigma = \uparrow, \downarrow$) boson in the nth well, $n = 1, 2, \ldots, N$ (the number of sites), γ is the effective SO-coupling strength, and $\hat{\sigma}_y$ is the 2×2 Pauli matrices. Here $a_{\sigma\overline{\sigma}} = \sqrt{2}\hbar\tilde{a}_{\sigma\overline{\sigma}}/(\sqrt{\pi}ml_\perp^2 l_0)$ is effective interatomic interaction strength, with $\tilde{a}_{\sigma\overline{\sigma}}$ being the corresponding interatomic s-wave scattering length between spin σ and $\bar{\sigma}$, m being the particle mass, $l_\perp$ being the oscillator length associated with a vertical harmonic confinement, l_0 being the oscillator length of the lattice potential. Ω is the dimensionless Raman coupling strength and δ is the dimensionless Zeeman field intensity. The dimensionless defects ϵ_n may be created with additional laser and/or magnetic fields and can be spatially localized or extended [35]. In order to investigate the dynamical transportation properties, one uses the mean-field approximation, i.e., $\hat{\varphi}_{n\sigma} \simeq \langle\hat{\varphi}_{n\sigma}\rangle \equiv \varphi_{n\sigma}$ [13], where $\varphi_{n\sigma}$ ($\sigma = \uparrow, \downarrow$) are the wave functions of the SO-coupled condensates in the nth site of the lattice, while the norm $\sum_{n,\sigma}|\varphi_{n,\sigma}|^2 = N$ is conserved, and $i\partial\varphi_{n\sigma}/\partial t = \partial H/\partial\varphi_{n\sigma}^*$, then the system can be described by the dimensionless discrete Schrödinger equations [16,31] with defects as

$$\begin{aligned} i\frac{\partial\varphi_{n\uparrow}}{\partial t} = &-\cos(\pi\gamma)(\varphi_{n+1\uparrow} + \varphi_{n-1\uparrow}) \\ &+ \sin(\pi\gamma)(\varphi_{n+1\downarrow} - \varphi_{n-1\downarrow}) + \frac{\Omega}{2}\varphi_{n\downarrow} + \frac{\delta}{2}\varphi_{n\uparrow} \\ &+ a_{\uparrow\uparrow}|\varphi_{n\uparrow}|^2\varphi_{n\uparrow} + a_{\uparrow\downarrow}|\varphi_{n\downarrow}|^2\varphi_{n\uparrow} + \epsilon_n\varphi_{n\uparrow}, \\ i\frac{\partial\varphi_{n\downarrow}}{\partial t} = &-\cos(\pi\gamma)(\varphi_{n+1\downarrow} + \varphi_{n-1\downarrow}) \\ &- \sin(\pi\gamma)(\varphi_{n+1\uparrow} - \varphi_{n-1\uparrow}) + \frac{\Omega}{2}\varphi_{n\uparrow} - \frac{\delta}{2}\varphi_{n\downarrow} \\ &+ a_{\downarrow\downarrow}|\varphi_{n\downarrow}|^2\varphi_{n\downarrow} + a_{\uparrow\downarrow}|\varphi_{n\uparrow}|^2\varphi_{n\downarrow} + \epsilon_n\varphi_{n\downarrow},\end{aligned} \tag{2}$$

where the physical variables are rescaled as $x \sim x/k_1$, $t \sim t\sqrt{m/\hbar k_1^2}$, $\varphi \sim \varphi\sqrt{k_1^3/2\pi}$. The terms multiplied by $\cos(\pi\gamma)$ are the usual tunneling and the terms multiplied by $\sin(\pi\gamma)$ represent the spin-flipping tunneling which arise from the effective SO-coupling. If $\gamma = n\pi, n = 0, 1, 2, 3, \cdots$, only usual tunneling exists. If $\gamma = (n + \frac{1}{2})\pi, n = 0, 1, 2, 3, \cdots$, only spin-flipping tunneling exists while the usual tunneling is inhibited. Otherwise, the usual tunneling and spin-flipping tunneling co-exist. As is known, the interspecies interactions have much complex influence on the dynamical transportation property of SO-coupled BEC system [48–50]. Here, we assume that $a_{\uparrow\uparrow} \approx a_{\downarrow\downarrow} = a$, $a_{\uparrow\downarrow} = \chi a$ [30,51] and only consider the case of repulsive interaction. The Lagrangian of Eq. (2) is

$$\mathcal{L} = \sum_{\sigma=\uparrow,\downarrow}\sum_n i(\dot{\varphi}_{n\sigma}\varphi_{n\sigma}^* - H.c.) - H, \tag{3}$$

where the Hamiltonian of the Eq. (3) is

$$\begin{aligned} H = \sum_n\{ &-\cos(\pi\gamma)[(\varphi_{n+1\uparrow} + \varphi_{n-1\uparrow})\varphi_{n\uparrow}^* \\ &+ (\varphi_{n+1\downarrow} + \varphi_{n-1\downarrow})\varphi_{n\downarrow}^*] \\ &- \sin(\pi\gamma)[-(\varphi_{n+1\downarrow} - \varphi_{n-1\downarrow})\varphi_{n\uparrow}^* \\ &+ (\varphi_{n+1\uparrow} - \varphi_{n-1\uparrow})\varphi_{n\downarrow}^*] \\ &+ a_{\uparrow\uparrow}(\varphi_{n\uparrow}\varphi_{n\uparrow}\varphi_{n\uparrow}^*\varphi_{n\uparrow}^*) + a_{\downarrow\downarrow}(\varphi_{n\downarrow}\varphi_{n\downarrow}\varphi_{n\downarrow}^*\varphi_{n\downarrow}^*) \\ &+ 2a_{\uparrow\downarrow}(\varphi_{n\uparrow}\varphi_{n\uparrow}^*\varphi_{n\downarrow}\varphi_{n\downarrow}^*) \\ &+ \frac{\Omega}{2}(\varphi_{n\downarrow}\varphi_{n\uparrow}^* + \varphi_{n\uparrow}\varphi_{n\downarrow}^*) + \frac{\delta}{2}(\varphi_{n\uparrow}\varphi_{n\uparrow}^* - \varphi_{n\downarrow}\varphi_{n\downarrow}^*) \\ &+ \epsilon_n(\varphi_{n\uparrow}\varphi_{n\uparrow}^* + \varphi_{n\downarrow}\varphi_{n\downarrow}^*)\}.\end{aligned} \tag{4}$$

3. Superfluidity with defects

In order to investigate the dynamical transportation properties of the condensates in an optical lattice with defects ϵ_n, we assume the system is in plane wave state and study the propagation of plane wave $\varphi_{n,\sigma}(t = 0) \sim e^{ikn}$ in an annular optical lattice, i.e., with periodic boundary conditions and $k = 2\pi l/N$, l being an integer ($l = 0, \ldots, N - 1$). The angular momentum of the system is defined:

$$L(t) = i\sum_{n,\sigma}(\varphi_{n,\sigma}\varphi_{n+1,\sigma}^* - c.c.). \tag{5}$$

The angular momentum $L(t)$ oscillates between the initial values L_0 and $-L_0$, corresponding to plane waves with wave vectors k and $-k$. The different rotational states with opposite wave vectors are degenerate in the absence of impurities. However, the defects split the degeneracy by coupling the two k and $-k$ waves, which is similar to the tunneling barrier in a double-well potential between the left and right localized states. For this case, the relative population of the two waves oscillates according to an effective Josephson Hamiltonian [36]. Therefore, when the impurity ϵ is small and the energy split introduced by the impurity is smaller than the energy

Please cite this article in press as: K.-Q. Zhu et al., Transport of spin-orbit coupled Bose–Einstein condensates in lattice with defects, Phys. Lett. A (2017), http://dx.doi.org/10.1016/j.physleta.2017.05.024