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A modified double distribution lattice Boltzmann model for axisymmetric thermal flow

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ABSTRACT

In this paper, a double distribution lattice Boltzmann model for axisymmetric thermal flow is proposed. In the model, the flow field is solved by a multi-relaxation-time lattice Boltzmann scheme while the temperature field by a newly proposed lattice-kinetic-based Boltzmann scheme. Chapman–Enskog analysis demonstrates that the axisymmetric energy equation in the cylindrical coordinate system can be recovered by the present lattice-kinetic-based Boltzmann scheme for temperature field. Numerical tests, including the thermal Hagen–Poiseuille flow and natural convection in a vertical annulus, have been carried out, and the results predicted by the present model agree well with the existing numerical data. Furthermore, the present model shows better numerical stability than the existing model.
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1. Introduction

Axisymmetric thermal flows are frequently encountered in industry, such as flow and heat transfer in pipes, turbines, solar energy equipment, etc. It is of substantial importance to understand the mechanism of the flows and heat transfers phenomena in such kind of applications. In the past few decades, lattice Boltzmann (LB) method has been evolving into a powerful numerical method to study flows and heat transfers [1–5]. Compared with the traditional computational fluid dynamics (CFD) method based on the macroscopic continuum equations, the LB method has many notable merits, such as the mesoscopic kinetic background, easy boundary treatment and inherently parallelizable computation property.

In the past few decades, many LB models for axisymmetric flows have been proposed [6–16]. Most of these LB models are the lattice Bhatnagar–Gross–Krook (LBGK) models, which employ a single relaxation time (SRT). As a matter of fact, the LBGK models have been well accepted due to their extreme simplicity [3–5]. However, one of the most well-known criticism on these LBGK models for flow field is the numerical instability at moderately low viscosity. A remedy that improves the numerical stability is employing a multiple relaxation time (MRT) instead of a single one [17–19]. By separating the relaxation rates of the hydrodynamics and kinetic moments, the numerical stability can be effectively enhanced. Compared to the studies that focused on the LBGK for axisymmetric flow, there are very few studies on the MRT model of the axisymmetric flow currently. To bridge this gap, Wang et al. [15] and Li et al. [16] proposed MRT LB models for such flow, and much better numerical stability are acquired by employing the collision matrix instead of a single collision operator.

The present MRT LB model for the flow field offers tunable parameters those help to improve the numerical stability against the LBGK model [15,16]. However, this is not the case for the currently available MRT LB model for the temperature field which employing a D2Q5 discrete velocities set. There have been several LB models for the axisymmetric thermal flows [20–23], some of which employ the SRT collision [20–22] while others take the MRT collision [23]. To the best knowledge of the authors, the present D2Q5 MRT LB models for the temperature field (both in the Cartesian and cylindrical coordinates) are numerical solvers which provide the potential for: (1) thermal flows with anisotropic diffusion cases [23,24]; (2) boundary treatment as bounce-back boundary condition is used [25,26], instead of providing a way that improves the numerical stability at low thermal diffusivity. However, in many applications with isotropic diffusion cases, the non-equilibrium extrapolation boundary treatment [27] is employed due to its simplicity, second-order accuracy, capability for different boundary conditions, and good robustness, which renders the D2Q5 MRT LB model for the temperature field futile. So it is reasonable to develop LB model that improves the numerical stability at low thermal diffusivity for these cases.

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In 2001, Inamuro [28] proposed a lattice kinetic scheme (LKS) for incompressible fluid flow with heat transfer. By adding a stress-tensor-related/temperature-related term in the equilibrium distribution function, a relaxation parameter of the stress tensor/temperature is introduced to make the dimensionless relaxation time adjustable, hence the stability of the LB model is expected to be improved. Later, this LKS scheme is extended to the LBGK models for multiphase flows [29,30], non-Newtonian fluid flows [31,32], incompressible flows and convection diffusion equations [33] and thermal flows in porous media at the representative elementary volume (REV) scale [34]. It is worth noting that in some of the literature [28,29,31], the parallel characteristic of the standard LBM is spoiled since the stress tensor (or the temperature) in the added term is calculated using a finite-difference scheme, while in some other related literature [30,32–34], a localized scheme for the stress tensor (or the temperature) is employed to retain the intrinsic merit of the standard LBM.

In this letter, based on the work of Ref. [22], a lattice Boltzmann scheme for axisymmetric temperature field is proposed and coupled with the existing MRT LB model for axisymmetric flow field. In the proposed model, the idea of the LKS is employed, and a correction term is introduced to keep the dimensionless relaxation time for the temperature field in a proper range, thus better numerical stability is expected. The rest of the paper is arranged as follows: In Section 2, the macroscopic governing equations of the axisymmetric thermal flow, as well as an MRT LB scheme for velocity field are introduced. Then a lattice-kinetic-based LB model for temperature field is proposed. Boundary treatment of the LKS-based LB model is provided in Section 3. In Section 4, some numerical tests are carried out, and finally a brief conclusion is presented in Section 5.

2. Lattice Boltzmann model for axisymmetric thermal flow

2.1. Macroscopic governing equations

The governing equations of incompressible axisymmetric thermal flow are [12,22]:

$$\frac{\partial u_x}{\partial x}+\frac{1}{r}\frac{\partial (r u_r)}{\partial r}=0 \tag{1}$$

$$\rho\frac{du_x}{dt}=-\frac{\partial p}{\partial x}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_x}{\partial r}\right)+\frac{\partial^2 u_x}{\partial x^2}\right]+\rho a_x \tag{2}$$

$$\rho\frac{du_r}{dt}=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_r)}{\partial r}\right)+\frac{\partial^2 u_r}{\partial r^2}\right]+\rho a_r \tag{3}$$

$$\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_r\frac{\partial T}{\partial r}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial r^2}\right)+\frac{k}{r}\frac{\partial T}{\partial r} \tag{4}$$

where r and x denote the radial and axial directions, respectively. u_x and u_r are the axial and radial velocities, respectively. T is the temperature, ρ the density, μ the dynamic viscosity, and k is the thermal diffusivity. a_x and a_r are the external forces in the axial and radial directions, respectively. And

$$\frac{d\varphi}{dt}=\frac{\partial \varphi}{\partial t}+\frac{\partial (u_x\varphi)}{\partial x}+\frac{1}{r}\frac{\partial (r u_r \varphi)}{\partial r} \tag{5}$$

for any variables φ.

2.2. Multi-relaxation-time lattice Boltzmann model for flow field

According to Ref. [16], the evolution equation for the distribution function can be expressed as follows:

$$f_i(\mathbf{x}+\mathbf{e}_i\delta_t,t+\delta_t)-f_i(\mathbf{x},t)=-S_{i\gamma}\left(f_\gamma-f_\gamma^{eq}\right)+\delta_t\left(\delta_{i\gamma}-\frac{S_{i\gamma}}{2}\right)F_\gamma \tag{6}$$

where $f_i(\mathbf{x},t)$ is the density distribution function at the location $\mathbf{x}$ and time t, $S_{i\gamma}$ the component of the collision matrix $\mathbf{S}$ given by $\mathbf{S}=diag(s_0,s_1,s_2,s_3,s_4,s_5,s_6,s_7)^{-1}$, $\delta_{i\gamma}$ the Kronecker delta function, $\mathbf{e}_i$ the discrete velocity specified by the standard D2Q9 lattice, δ_t the time step. $f_i^{eq}(x,t)$ is the equilibrium distribution function given by [16]

$$f_i^{eq}=r\rho w_i\left[1+\frac{\mathbf{e}_i\cdot\mathbf{u}}{c_s^2}+\frac{(\mathbf{e}_i\cdot\mathbf{u})^2}{2c_s^4}-\frac{\mathbf{u}^2}{2c_s^2}\right] \tag{7}$$

where r is the radius, ρ the density, w_i the weight coefficient given by $w_0=4/9$, $w_i=1/9$ for $i=1-4$, and $w_i=1/36$ for $i=5-8$. $c_s=c/\sqrt{3}$ is the sound speed, in which c is the lattice speed defined by $c=\delta_x/\delta_t$ with δ_x the lattice space. $\mathbf{M}$ is transformation matrix given by [16,18]:

$$\mathbf{M}=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ -4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2\\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1\\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1\\ 0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1\\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1\\ 0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1\\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \end{bmatrix}$$

Using the transformation matrix, the moments can be constructed by mapping the discrete distribution functions in the velocity space onto the moments space, which gives

$$\mathbf{m}=\mathbf{M}\mathbf{f}=r(\rho,e,\varepsilon,j_x,q_x,j_r,q_r,p_{xx},p_{xr})^T \tag{8}$$

where $\mathbf{f}=(f_0,f_1,f_2,f_3,f_4,f_5,f_6,f_7,f_8)^T$, e is the total energy, ε related to the energy square, j_x and j_r the components of the momentum $\mathbf{J}=(j_x,j_r)=\rho\mathbf{u}$, q_x and q_r related to energy flux, and p_{xx} and p_{xr} correspond to the symmetric and traceless component of the strain tensor, respectively.

$$\mathbf{m}^{(eq)}=\mathbf{M}\mathbf{f}^{(eq)}=r\left(\rho,e^{(eq)},\varepsilon^{(eq)},j_x,q_x^{(eq)},j_r,q_r^{(eq)},p_{xx}^{(eq)},p_{xr}^{(eq)}\right)^T \tag{9}$$

The conserved variables of athermal fluid are only density and momentum. The corresponding equilibrium moments for the non-conserved moments are [16–18]:

$$e^{(eq)}=\rho\left(1+3u_x^2+3u_r^2\right) \tag{10}$$

$$\varepsilon^{(eq)}=\rho\left(1-3u_x^2-3u_r^2\right) \tag{11}$$

$$q_x^{(eq)}=-\rho u_x \tag{12}$$

$$q_r^{(eq)}=-\rho u_r \tag{13}$$

$$p_{xx}^{(eq)}=\rho\left(u_x^2-u_r^2\right) \tag{14}$$

$$p_{xr}^{(eq)}=\rho u_x u_r \tag{15}$$

In the numerical implementation of the MRT LB model for the flow field, the collision step is operated in the moment space while the streaming step is executed in the velocity space as:

$$collision:\ \widetilde{\mathbf{m}}=\mathbf{m}-\mathbf{S}(\mathbf{m}-\mathbf{m}^{\mathbf{eq}})+\delta_t(\mathbf{I}-\frac{\mathbf{S}}{2})\mathbf{F} \tag{16}$$

$$streaming:\ f_i(x+e_i\delta_t,t+\delta_t)=\widetilde{f}_i(x,t) \tag{17}$$

where $\widetilde{f}=\mathbf{M}^{-1}\widetilde{\mathbf{m}}$ is the post-collision distribution function with $\widetilde{\mathbf{m}}$ representing the post-collision distribution function in the moment space. The forcing term in moment space can be explicitly expressed as:

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