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Entanglement of a two-atom system driven by the quantum vacuum in arbitrary cavity size

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ABSTRACT

We study the entanglement dynamics of two distinguishable atoms confined into a cavity and interacting with a quantum vacuum field. As a simplified model for this system, we consider two harmonic oscillators linearly coupled to a massless scalar field which are inside a spherical cavity of radius R. Through the concurrence, the entanglement dynamics for the two-atom system is discussed for a range of initial states composed of a superposition of atomic states. Our results reveal how the entanglement of the two atoms behaves through the time evolution, in a precise way, for arbitrary cavity size and for arbitrary coupling constant. All our computations are analytical and only the final step is numerical.

1. Introduction

The entanglement phenomenon is one of the most exciting topics of quantum theory, due to its non-locality property, which is considered as the central idea in quantum information processing. During the last two decades there has been a growing interest in generating of entanglement, driven by the fast developments of experimental processes in quantum control. There are many potential applications in almost all quantum communication and computation protocols [1], such as quantum teleportation [2,3], quantum secure direct communication [4], quantum computation [5,6] among others.

Though entanglement is recognized as a fundamental ingredient for quantum information technology [1] it suffers from one drawback. Since quantum systems are always in contact with the surrounding environment, in general the quantum entanglement decays in time, and in some situations can display the phenomenon of sudden death, where entanglement ceases to exist at a finite time [7–9]. Consequently, the search for mechanisms to control the loss of entanglement has been of extreme importance and attracted the interest of diverse investigations, [10–14]. In general, the manipulation of quantum systems to create or maintain entanglement for sufficiently long times involves the manipulation of quantum systems in cavities [14–16]. Therefore, a precise understanding of the cavity effects on the entanglement dynamics could be of usefulness to future studies on quantum systems in cavities.

In this work, we will be mainly interested in studying how the cavity size affects the entanglement dynamics of a quantum system. Specifically, we will consider the dynamics of the entanglement of a two-atom system driven by the quantum vacuum field, where the entire system is enclosed in a spherical cavity of radius R. As is well-known, an atom placed in a cavity changes its spontaneous decay rate in a dramatic manner going from the exponential decay in free space to an almost stability for a sufficiently small cavity [17]. Therefore it is expected that the entanglement dynamics of two atoms confined in a cavity will be very different in relation to the free case.

Frequently in the literature, atoms are approximated by two level systems and in the context of the entanglement of two atoms system many works were devoted in the past (see for example [18–27] and references therein). However, to our best knowledge, the question regarding the cavity size dependence on the entanglement dynamics of such systems has not been addressed before from a theoretical point of view. It is the purpose of this work to fill partially this gap. However, an analytical treatment of this situation even in a perturbative scheme could be extremely hard, as one can conclude from Ref. [28], where the authors considered in perturbation theory the cavity effects on the radiative processes of two entangled two-level system. Therefore, instead of using two-level systems coupled to the electromagnetic field, we use a simplified model of atoms considering each one as a single harmonic oscillator and the electromagnetic field is taken to be a massless scalar field. Of course, real atoms do not have equally spaced energy levels, and they are not one-dimensional systems either. Also, in real situations, the entanglement dynamics is strongly influenced by the surrounding environment, and therefore we should

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include, for example, temperature effects. However our main purpose in this work is not to understand how the nature of the atoms or the environment affects the entanglement dynamics, but to determine what is its precise dependence on the cavity size.

To model the atoms trough harmonic oscillators, with regard to the stability of the ground state, we will work in the dressed coordinates and states approach, which were introduced some time ago in the context of an harmonic oscillator coupled to a massless scalar field [29] and subsequently developed and applied in many situations [30–39]. From the physical point of view, these concepts allowed to make a clear distinction between the physical oscillator (atom) and the field degrees of freedom. Also the model accounted for the experimental observations of the decaying process of atoms in cavities [33]. On the other hand, the dressed coordinates and states approach has proven to be an advantageous technique to perform analytical calculations. As will be shown in this work, when the method is applied to the study of the entanglement of two harmonic oscillators (atoms), we will get analytical results for cavities of arbitrary radius or coupling constant, and only the final step will require to perform a numerical analysis. We will focus on the study of the first excited states of the harmonic oscillators-field system, because as will be shown in section 2 the behavior of the spontaneous decaying processes is similar to one of two level atoms systems. Thus, the study of the entanglement dynamics of the first excited states will be sufficient for our principal purpose.

It is worth pointing out that there are several works in the literature wherein the entanglement dynamics for two harmonic oscillators has been extensively studied (see [40–52] and references therein), however, to our best knowledge, the explicit cavity size dependence has not been considered before, as it will be done in the present work.

This paper is organized as follows. In section 2, we describe the model of the two harmonic oscillators (atoms) coupled to a massless scalar field, define the dressed coordinates, and states and compute some probability amplitudes that will be useful for further studies on the entanglement of the two-atom system inside the spherical cavity. In section 3, we discuss the entanglement of the two-atom system through the quantity called concurrence, illustrating the dynamics of the entanglement as a function of arbitrary coupling constant and the cavity radius size. Finally, in section 4 we present our conclusions. We will consider natural units $\hbar = c = 1$.

2. The model

Let us consider two identical atoms inside a spherical cavity linearly coupled to a massless scalar field. Roughly approximating the atoms by two harmonic oscillators of frequency ω_0, the system can be described by the following Hamiltonian,

$$H = \tfrac{1}{2}(p_A^2 + \omega_0^2 q_A^2) + \tfrac{1}{2}(p_B^2 + \omega_0^2 q_B^2) + \tfrac{1}{2}\sum_{k=1}^{N}(p_k^2 + \omega_k^2 q_k^2) + + \frac{1}{\sqrt{2}}\sum_{k=1}^{N} c_k q_k (q_A + q_B) + \sum_{k=1}^{N}\frac{c_k^2}{4\omega_k}(q_A + q_B)^2, \quad (1)$$

where the coordinates and momenta q_A, p_A and q_B, p_B correspond to the harmonic oscillators A and B respectively, the coordinates and momenta q_k and p_k are related to the field modes of frequencies ω_k and c_k is the coupling strength between the harmonic oscillators and the modes of the scalar field. Since the system is enclosed in a spherical cavity of radius R, it is not difficult to find that the field frequencies and coupling strength are given respectively by [29]

$$\omega_k = \frac{\pi k}{R}, \quad k = \{1, 2, ...\} \quad (2)$$

and

$$c_k = \frac{\omega_k}{\pi\sqrt{2}}\sqrt{g\Delta\omega_k}, \quad \Delta\omega_k = \frac{\pi}{R}, \quad (3)$$

where g is frequency dimensional coupling constant. The last term in Eq. (1) is introduced to guarantee the positiveness of the Hamiltonian and can be understood as a renormalization of the particle oscillator frequencies [53]. At the end of calculations in Eq. (1), we will take $N \to \infty$. Also, note that we can recover the free space case taking the limit $R \to \infty$.

The model Hamiltonian given by Eq. (1) has been introduced in Ref. [54] and used to study the entanglement between two single harmonics oscillators in Ref. [51]. In order to diagonalize the Hamiltonian (1), we introduce the relative and the center of mass coordinates q_- and q_0 respectively, and given by

$$q_- = \tfrac{1}{\sqrt{2}}(q_A - q_B), \quad q_0 = \tfrac{1}{\sqrt{2}}(q_A + q_B). \quad (4)$$

Substituting the above relations in Eq. (1), we have

$$H = \tfrac{1}{2}(p_-^2 + \omega_0^2 q_-^2) + \tfrac{1}{2}(p_0^2 + \omega_0^2 q_0^2) + \tfrac{1}{2}\sum_{k=1}^{N}(p_k^2 + \omega_k^2 q_k^2) + + \sum_{k=1}^{N} c_k q_k q_0 + \sum_{k=1}^{N}\frac{c_k^2}{2\omega_k} q_0^2. \quad (5)$$

Note that in the Hamiltonian (5), the relative coordinate q_- is decoupled, thus, one can diagonalize the Hamiltonian ignoring the first term. For this purpose, we introduce, the collective coordinates and momenta, Q_r and P_r, given by

$$q_\mu = \sum_r t_\mu^r Q_r, \quad p_\mu = \sum_r t_\mu^r P_r, \quad (6)$$

where $\mu = 0, k$, and t_μ^r is given by

$$t_k^r = \frac{c_k}{\omega_k^2 - \Omega_r^2} t_0^r, \quad t_0^r = \left(1 + \sum_{k=1}^{N}\frac{c_k^2}{(\omega_k^2 - \Omega_r^2)^2}\right)^{-\frac{1}{2}}. \quad (7)$$

It is worth to mention that the matrix $\{t_\mu^r\}$ is orthogonal and satisfies the following relations

$$\sum_\mu t_\mu^r t_\mu^s = \delta_{rs}, \text{ and } \sum_r t_\mu^r t_\nu^r = \delta_{\mu\nu}. \quad (8)$$

The Hamiltonian (5) can be rewritten in collective coordinates, which simply reduce to

$$H = \tfrac{1}{2}(p_-^2 + \omega_0^2 q_-^2) + \tfrac{1}{2}\sum_r (P_r^2 + \Omega_r^2 Q_r^2), \quad (9)$$

where the normal frequencies Ω_r are the solutions of the equation

$$\omega_0^2 - \Omega_r^2 = \sum_{k=1}^{N}\frac{c_k^2\Omega_r^2}{\omega_k^2(\omega_k^2 - \Omega_r^2)}. \quad (10)$$

Now, we are ready to write the eigenfunctions and energy eigenvalues of the system. The eigenfunctions are given by

$$\phi_{n_-, n_0, n_1, ..}(q_-, Q) = \phi_{n_-}(q_-)\prod_{r=0}\phi_{n_r}(Q_r), \quad (11)$$

where $\phi_{n_-}(q_-)$, $\phi_{n_r}(Q_r)$ are one dimensional harmonic oscillator eigenfunctions of frequencies ω_0 and Ω_r respectively. Whereas the corresponding energy eigenvalues are,

$$E_{n_-, n_0, n_1, ...} = \left(n_- + \tfrac{1}{2}\right)\omega_0 + \sum_{r=0}\left(n_r + \tfrac{1}{2}\right)\Omega_r, \quad (12)$$

where $n_-, n_r = \{0, 1, 2, ...\}$.