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Transmission and reflection of vector Bessel beams through an interface between dielectrics

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ABSTRACT

Simple full vectorial analytical exact results are obtained for the propagation of Bessel beams through an interface separating different media characterized by material parameters (ϵ_1, μ_1) and (ϵ_2, μ_2). A real space description is used and all the results are written in terms of the transverse wavevector k_t of the incident beam, taken as an input parameter. It is shown that perfect transmissions are obtained when the incident wave is either in a medium of larger or lower index of refraction, compared to the medium of the transmitted wave, and we provide the particular values, k_t^F, that the transverse wavevector must obey in order to observe such effects. The phenomenon of total internal reflection is also verified for normal incidence.

1. Introduction

One of the main concerns of physical optics is to understand the classical interaction and propagation of arbitrary waves through general materials. By considering the propagation of fully linearly polarized waves, Fourier optics adequately provides an explanation for various effects by treating the electromagnetic field as a scalar quantity [1]. Nevertheless the electromagnetic field is characterized by a vector field and must obey Maxwell's equations, so that any consideration about the polarization of the waves in question cannot be taken into account by using a pure scalar approximation. In 1987, Durnin discovered such a scalar solution of the Helmholtz equation, namely, the Bessel beam [2]. The search began for providing a vectorial formal solution of Maxwell's equations and at the same time trying to preserve essential features of Durnin's solution, such as the nondiffractive character and orbital angular momentum properties [3–5]. Unfortunately, there is no general approach to this problem and the interaction of Bessel beams (also known as X waves) with ordinary matter is studied with several distinct formalisms. Phenomena such as superluminal propagation properties through slabs [6,7], total internal reflection [8], changes on the size of the Bessel beam rings under reflection [9], interaction with absorbing media [10,11] are well established in the literature. We cite in particular the more general formalism developed in [12,13] which closely resembles ours. The importance of understanding these solution stems from the remarkable range of important applications that Bessel beams can provide, in particular, optical micromanipulation of microsized particles [14], three dimensional imaging of living cells [15] and optical levitation [16] to cite a few.

Once vectorial solutions are obtained, the next logical step is to make the interaction of the beam with some material. In a previous paper [5] we have obtained a vectorial Bessel beam solution of Maxwell's equations and it was shown that it possesses properties resembling Durnin's solution. In this paper we take a step forward in asking what happens to this vector beam as it interacts with an interface separating dielectric materials characterized by material parameters (ϵ_1, μ_1) and (ϵ_2, μ_2) taken to be real positive quantities. In particular, we have found special configurations in this system such that total internal reflection and total transmission are obtained even when the Bessel beam undergoes normal incidence.

2. Theory

The analysis begins by presenting full vectorial analytic solutions of Maxwell's equations for two types of polarization modes TE and TM. The derivation was established in a previous paper [5] and therefore only the essential results will be presented in the following. By considering time harmonic propagation $\exp(-i\omega t)$ and an electric field and magnetic field induction of the form $\mathbf{E}(x, y, z, t) = \mathbf{E}(x, y)\exp(ik_z z - i\omega t) = [\mathbf{E}_\perp(x, y) + E_z(x, y)\hat{z}]\exp(ik_z z - i\omega t)$ and $\mathbf{B}(x, y, z, t) = \mathbf{B}(x, y)\exp(ik_z z - i\omega t) = [\mathbf{B}_\perp(x, y) + B_z(x, y)\hat{z}]\exp(ik_z z - i\omega t)$, where z is the propagation direction, ω the angular frequency, k_z the wavevector in the z

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direction and $\mathbf{E}_\perp(x,y) = \hat{z} \times [\mathbf{E}(x,y) \times \hat{z}]$ (same for $\mathbf{B}_\perp$), which is the transverse part of the electric field ($\hat{z} \cdot \mathbf{E}_t = 0$), it can be demonstrated that

$$\mathbf{E}(x,y,z,t) = \exp(ik_z z - i\omega t)\left[-\frac{i\omega}{k_\perp^2}\hat{z} \times \nabla_\perp B_z\right], \tag{1}$$

$$\mathbf{B}(x,y,z,t) = \exp(ik_z z - i\omega t)\left[\frac{ik_z}{k_\perp^2}\nabla_\perp B_z + \hat{z}B_z\right], \tag{2}$$

for TE polarization, where $k_\perp^2 = \epsilon\mu\omega^2 - k_z^2$, where ϵ and μ are the (position independent) dielectric constant and permeability of the medium where the optical beam propagates, respectively. The field component $B_z(x,y)$ must be a solution of $(\nabla_\perp^2 + k_\perp^2)B_z(x,y) = 0$ where $\nabla_\perp^2 = \nabla^2 - \partial_z^2$. Transforming to cylindrical coordinates one may choose $B_z(\rho,\phi) = B_0 J_m(k_\perp\rho)\exp(im\phi)$ where B_0 is the amplitude (taken to be real), $J_m(k_\perp\rho)$ is the m-order Bessel function of the first kind and $m = 0, 1, 2\ldots$ denotes the order of the solution. By substituting B_z in Eqs. (1) and (2), the following equations for TE polarized Bessel beam are obtained

$$\mathbf{E}(\rho,\phi,z,t) = E_0\exp(ik_z z + im\phi - i\omega t)\left[\hat{\phi}J^- - i\hat{\rho}J^+\right], \tag{3}$$

$$\mathbf{B}(\rho,\phi,z,t) = B_0\exp(ik_z z + im\phi - i\omega t) \times \left[\frac{ik_z}{2k_\perp}[\hat{\rho}J^- + i\hat{\phi}J^+] + J_m(k_\perp\rho)\hat{z}\right], \tag{4}$$

where $J^\pm = J_{m-1}(k_\perp\rho) \pm J_{m+1}(k_\perp\rho)$ and $E_0 = -i\omega B_0/2k_\perp$. The TM solutions can be obtained quite easily by following the preceding analysis and we will study them later. By doing a simple calculus problem it can be demonstrated that $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$ as required by Maxwell's equations without sources. With these full vectorial solutions one is ready to impose electromagnetic boundary conditions.

3. Applications and discussion

To this end, consider that the half-space $z \geq 0$ is occupied by a material characterized by (ϵ_2, μ_2), $z \leq 0$ by a material characterized by (ϵ_1, μ_1) and a Bessel beam of order m is normally incident from negative to positive z. After using the usual Maxwell's equations boundary conditions at $z = 0$ one may deduce the following relations:

$$\frac{B_0'}{B_0} = \frac{E_0'}{E_0} = \left[\frac{\mu_2 k_z - \mu_1 k_z''}{\mu_2 k_z + \mu_1 k_z''}\right], \tag{5}$$

$$\frac{B_0''}{B_0} = \frac{E_0''}{E_0} = \left[\frac{2\mu_2 k_z}{\mu_2 k_z + \mu_1 k_z''}\right], \tag{6}$$

where B_0 (E_0), B_0' (E_0') and B_0'' (E_0'') are the amplitudes of the incident, reflected and transmitted magnetic (electric) fields, respectively, $k_z = (\epsilon_1\mu_1\omega^2 - k_\perp^2)^{1/2}$ and $k_z'' = (\epsilon_2\mu_2\omega^2 - k_\perp^2)^{1/2}$ are the z components of the wavevector for $z < 0$ and $z > 0$. In deriving Eqs. (5) and (6) the continuity of the transversal wavevector was used. We also assumed that initially all three, i.e., the incident, reflected and transmitted waves, had different values of m, but after substitution into the original equations for the fields, it can be demonstrated that $m = m' = m''$, which shows that the reflected and transmitted beams must have the same order as the incident one. All the relevant equations can be written in terms of the single transverse wavevector $k_\perp$ parameter of the incident beam (for a given set of material parameters and angular frequency ω). As we are concerned with propagating incident waves, $k_\perp$ must always be smaller than $(\epsilon_1\mu_1)^{1/2}\omega$. Now, either $\epsilon_1\mu_1 < \epsilon_2\mu_2$ or $\epsilon_1\mu_1 > \epsilon_2\mu_2$. For the first condition (a Bessel beam traveling from air to glass, for instance) let $r_1 = (\epsilon_1\mu_1)^{1/2}\omega$ and $r_2 = (\epsilon_2\mu_2)^{1/2}\omega$

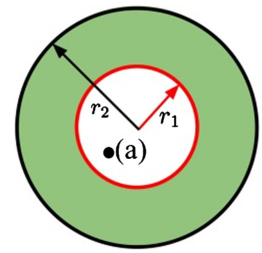

Fig. 1. Diagram showing the radius $r_1 = (\epsilon_1\mu_1)^{1/2}\omega$ and $r_2 = (\epsilon_2\mu_2)^{1/2}\omega$ for the case $r_2 > r_1$. For the region (a) it is shown propagating waves on both sides of the structure. The region outside the white circle is forbidden for propagating incident waves. The point (a) indicates a value $k_\perp$ for the incident wave.

which corresponds to the radius shown in Fig. 1. As $k_\perp$ must always be smaller than r_1 for an incident propagating wave (which corresponds to the white region in Fig. 1), $k_\perp < r_2$ and, consequently, k_z'' is real and the transmitted wave propagates without decaying. Geometrically, it is clear that for every point inside the white circle of radius r_1 it will also be inside of the larger circle with radius r_2. We conclude that it is impossible to have evanescent behavior for $z > 0$ if $\epsilon_1\mu_1 < \epsilon_2\mu_2$. Now, for a single TE polarized plane wave incident onto the interface, there is an angle, called Brewster's angle, for which there is no reflected wave [9]. Making the analogy with our system, we ask if there is such behavior for the vector Bessel beam. By looking at Eq. (5) it is seen that if $\mu_2 k_z = \mu_1 k_z''$ the amplitudes of the reflected electric and magnetic waves are zero. This happens for a $k_\perp$ given by

$$k_\perp^F = \omega\left[\left(\frac{\mu_1\mu_2}{\mu_2^2 - \mu_1^2}\right)(\epsilon_1\mu_2 - \epsilon_2\mu_1)\right]^{1/2}. \tag{7}$$

Given $(\epsilon_{1,2}, \mu_{1,2})$ parameters, the transverse wavevector for full transmission can be calculated using Eq. (7). Be aware that the condition $k_\perp^F < r_1$ must also be obeyed. We will give examples later showing that the conditions can be fulfilled, though. Note that if the frequency is in the optical spectrum where $\mu_1 \approx \mu_2 \approx \mu_0$, this effect of full transmission probably will not be observable at all, for $k_\perp^F$ becomes much larger than r_1. We conclude that an analogous Brewster's angle exists for this situation but it is a little more subtle than the plane wave solution. Referring to Fig. 1 it may be said that there are some points inside the circle with radius r_1 for which there is a full transmission of the incident beam. Later we will demonstrate a specific example where these points can be visualized.

Consider now $\epsilon_1\mu_1 > \epsilon_2\mu_2$ ($r_1 > r_2$) which can be thought of as a Bessel beam propagating from glass to air, for instance. The diagram representing r_1 and r_2 is shown in Fig. 2. For this situation, $k_\perp$ still can have values lying inside both circles, as represented by the green shaded area. This represents propagating waves in both materials. But now there are points satisfying $r_2 < k_\perp < r_1$, such as point (a) in Fig. 2. For these points k_z'' becomes pure imaginary and an evanescent wave appears in the region $z > 0$. This is analogous to the total internal reflection phenomenon of a polarized plane wave incident upon a material from a higher to a lower index of refraction. What plays the role of the critical angle here is the critical wavevector $k_\perp^C = \omega(\epsilon_2\mu_2)^{1/2}$, i.e., when the incident transverse wavevector matches the value of r_2. We conclude that as $k_\perp$ acquires large values and becomes closer to r_2, the state of the transmitted electromagnetic field goes through a transformation from a propagating to an evanescent state. It must be pointed out that Eq. (7) can still be satisfied for the green shaded area of Fig. 2 (this time we must have $k_\perp^F < r_2$). Full transmission of the incident wave through the boundary from the medium with larger refraction index to the lower (and vice versa) is possible for the vectorial Bessel beam.