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¹⁹ ARTICLE INFO ABSTRACT ⁸⁵

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²¹ Article history: **Example 21** Mestudy the dynamics of the planar circular restricted three-body problem in the context of a pseudo-22 Received 15 September 2016 Newtonian approximation. By using the Fodor–Hoenselaers–Perjés procedure, we perform an expansion 88 23 Acceptive 12 December 2016 **1990 The mass of a static massive spherical source up to the first non-Newtonian term, giving place** 89 24 Communicated by D. Palach and the second to a gravitational potential that includes first-order general relativistic effects. With this result, we model 90 25 91 a system composed by two pseudo-Newtonian primaries describing circular orbits around their common 26 92 center of mass, and a test particle orbiting the system in the equatorial plane. The dynamics of the new 27 Nonimear dynamics and chaos
₂₉ Classical general relativity **200** where the introduction of a new parameter *ε*, allows us to observe the transition from the Newtonian 28 Edissical general relativity

to the pseudo-Newtonian regime. We show that when the Jacobian constant is fixed, a chaotic orbit in 29 95 the Newtonian regime can be either chaotic or regular in the pseudo-Newtonian approach. As a general 30 96 result, we find that most of the pseudo-Newtonian configurations are less stable than their Newtonian α and β 97 β system of equations is studied in terms of the Poincaré section method and the Lyapunov exponents, equivalent.

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1. Introduction

One of the simplest and most frequently studied version of the general three-body problem, is the planar circular restricted threebody problem (henceforth CRTBP), which can be stated as follows:

- Two primaries, \mathcal{M}_1 and \mathcal{M}_2 at positions X_1 and X_2 , respectively, follow a circular orbit around their common center of mass keeping a fixed distance *r*, while moving at constant angular velocity ω₀.
- A third body M , that is much smaller than either M_1 or M_2 , remains in the orbital plane of the primaries.
- The equations of motion are derived only for the test particle M , whose motion does not affect the primaries.

The basic formulation of the CRTBP dates back to Euler, who proposed the use of synodical coordinates *(x, y)* instead of the inertial coordinate system *(X, Y)*, in order to simplify this problem [\[1\].](#page--1-0) The transformation between these two systems can be performed by means of the rotation matrix,¹

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65 131 0375-9601/© 2016 Elsevier B.V. All rights reserved.66 and the contract of the con

 37 and 103 and 103 and 103 and 103 and 103 and 103 and 103 38 **1.** Introduction (a) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{b$ $\langle y \rangle$ $\langle \sin \omega_0 t \cos \omega_0 t \rangle / \langle t \rangle$ $\langle 105 \cos \omega_0 t \rangle$ - *x y* \setminus = $\int \cos \omega_0 t \quad -\sin \omega_0 t$ $\sin \omega_0 t$ cos $\omega_0 t$ - *X Y* \setminus *.* (1)

 40 One of the simplest and most frequently studied version of the Using this transformation, Lagrange proved the existence of five 106 ⁴¹ general three body problem is the planar circular restricted three equilibrium points for the system, named Lagrangian points. The ¹⁰⁷ 42 body problem (banceforth CPTBP) which can be stated as follows:
Body problem (banceforth CPTBP) which can be stated as follows: Subject of equilibrium points in the CRTBP has been studied ex-43 109 tensively in the literature (see *e.g.* [\[2\]](#page--1-0) and references therein). The ⁴⁴ 110 **110** 110 110 110 110 110 110 110 110 discovery of the Trojan asteroids around the Lagrangian points L_4 110 45 111 and *L*⁵ in the Sun–Jupiter system [\[3\],](#page--1-0) and the recent observations 46 112

of asteroids around *L*₄ for the Sun–Earth system [\[4\],](#page--1-0) increased the-

mass keeping a fixed distance rubile moving at constant and the Sun–Earth system [4], increased the-47 113 oretical research on the subject (see *e.g.* [\[5\]\)](#page--1-0). It should be noted 48 $\frac{1}{2}$ $\frac{1}{2}$ that, in spite of the fact that the CRTBP is much simpler than the 114 49 **•** A thild body *i*. That is much smaller than entier \mathcal{M}_1 or \mathcal{M}_2 , general three-body problem, it is non-integrable, which opened the 115 50 116 possibility to analyze systematically the orbits [\[2,6\].](#page--1-0)

 $_{51}$ \bullet The equations of motion are derived only for the test parti-
Under the assumption of weak fields and low velocities, and as $_{117}$ $_{52}$ cle \mathcal{M} , whose motion does not affect the primaries. $_{\text{a} \text{ first approximation to the relativistic CRTBP, in 1967 [7] Krefetz 118}}$ $_{\text{a} \text{ first approximation to the relativistic CRTBP, in 1967 [7] Krefetz 118}}$ $_{\text{a} \text{ first approximation to the relativistic CRTBP, in 1967 [7] Krefetz 118}}$ 53 119 considered for the first time the post-Newtonian equations of mo-54 The basic formulation of the CRTBP dates back to Euler, who pro- tion for the CRTBP, using the Einstein–Infeld–Hoffmann (EIH) for- 120 $_{55}$ posed the use of synodical coordinates (x, y) instead of the inertial $_{\text{malign}}[8]$. The Lagrangian for this system was explicitly presented 121 $_{56}$ coordinate system (X, Y), in order to simplity this problem [1]. The by Contopoulos in 1976 [\[9\]](#page--1-0) and some typos for the Jacobian con-
 57 transformation between these two systems can be performed by stant were corrected by Maindl et al. [\[10\],](#page--1-0) who also studied the de- 123 58 124 viations due to the post-Newtonian corrections on the Lagrangian 59 125 points [\[11\].](#page--1-0) Concerning analytical solutions to the general relativis-60 126 tic three-body problem, Yamada et al. [\[12\]](#page--1-0) obtained a collinear 61 $\overline{F_{t}}$ $\overline{F_{t}}$ $\overline{F_{t}}$ and addresses: fidules bem@unal edu co (**F1** Dubeibe) fadulora@uis edu co solution by using the EIH approximation up to the first order. In 127 62 (F.D. Lora-Clavijo), guillermo.gonzalez@saber.uis.edu.co (G.A. González). **a later paper, they studied the post-Newtonian triangular solu-** 128 63 ¹ Along the paper $G = M = \omega_0 = r = 1$, is understood. $\qquad \qquad$ tion for three finite masses, showing that such configuration is \qquad 129

E-mail addresses: fldubeibem@unal.edu.co (F.L. Dubeibe), fadulora@uis.edu.co (F.D. Lora-Clavijo), guillermo.gonzalez@saber.uis.edu.co (G.A. González).

¹ Along the paper $G = M = \omega_0 = r = 1$, is understood.

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unstable.

⁷ To avoid the cumbersome equations of motion that take place that the state of the state ⁸ in the post-Newtonian formalism, Steklain and Letelier used the $\bar{\rho} = \frac{p}{\bar{p}} = \frac{$ ⁹ Paczyński–Wiita pseudo-Newtonian potential to study the dynam- $\rho^2 + z^2$ $\rho^2 + z^2$ $\rho^2 + z^2$ ¹⁰ ics of the CRTBP in the Hill's approximation [\[15\],](#page--1-0) finding that \sim 10 and 11 some pseudo-Newtonian systems are more stable than their New-
11 some pseudo-Newtonian systems are more stable than their New-12 tonian counterparts. Following this idea, and considering that the $\frac{1}{2}$ and $\frac{1}{2}$ as ¹³ Jacobian constant is not preserved in the post-Newtonian approxi- ∞ ¹⁴ mation (which limits the dynamical studies), in the present paper $\tilde{\xi} = \sum a_{ij} \bar{\rho}^i \bar{z}^j$. (7) ⁸⁰ ¹⁵ we shall use an alternative approach to studying the dynamics $\frac{1}{100}$ and $\frac{1}{100}$ ¹⁶ of the pseudo-Newtonian CRTBP. To do so, we derive an approx-¹⁷ imate potential for the gravitational field of two uncharged spin- and the coefficients a_{ij} are calculated by the recursive relations a_{ij} ¹⁸ less particles modeled as sources with multipole moment *m*, by $\lfloor 17 \rfloor$ 19 85 using the Fodor–Hoenselaers–Perjés (FHP) procedure [\[16\]](#page--1-0) (taking ²⁰ into account the corrections made by Sotiriou and Apostolatos $(r+2)^2 a_{r+2,s} = -(s+2)(s+1)a_{r,s+2}$ 21 [\[17\]\)](#page--1-0). Abusing astrophysical terminology, we call the new potential $\sum_{n=1}^{\infty} (a_n a_n^* - b_n b_n^*) a_n$ 22 pseudo-Newtonian, due to the fact that in this kind of approaches $\frac{1}{1-\epsilon}$ $\frac{1}{2}$ $\frac{1}{2}$ 23 the common Newtonian formulas are used even when the result- \mathbb{R}^3 and \mathbb{R}^4 and \mathbb{R}^5 and \mathbb{R}^5 and \mathbb{R}^8 and \mathbb{R}^8 and \mathbb{R}^8 and \mathbb{R}^8 and \mathbb{R}^8 and \mathbb{R}^8 and $\mathbb{R}^$ ²⁴ ing potentials do not satisfy the Laplace equation. Unlike other $\times (p^2 + g^2 - 4p - 5g - 2pk - 2gl - 2)$ 25 91 pseudo-Newtonian approaches, the final expressions are not ad-26 hoc proposals but are derived directly from the multipole structure $T^{up+2,g-2(p+2)}$ ²⁷ of the source. 93 imate potential for the gravitational field of two uncharged spinof the source.

²⁸ The paper is organized as follows: In section 2, by means of the $\frac{94}{100}$ 29 FHP procedure, we calculate the gravitational pseudo-potential for $H_{p-2,g+2(g+2)(g+1-2l)}$, (8) 95 ³⁰ each primary; then we write down the Lagrangian of the CRTBP where $m-r-k-n$ $0 < k < r$ $0 < n < r-k$ with k and n even 31 with their respective equations of motion for a test particle un-
31 with their respective equations of motion for a test particle un-
31 and $\pi s = 1 - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = -\frac{1}{2} - \frac{1}{2} = -\frac{1}{2} = -\frac{1}{2} = -$ 32 der the influence of this potential. In section [3,](#page--1-0) we analyze the *gravitational multipole moments* P_i of the source are computed 33 gradual transition of the dynamics for the FHP pseudo-Newtonian $\frac{5}{100}$ from their values on the symmetry axis $m_i = a_0$; by means of the 34 approximation to the classical regime. The analysis is made using ϵ_0 following relationships 35 101 Poincaré surfaces of section and the variational method for the cal-³⁶ culation of the largest Lyapunov exponent [\[18\].](#page--1-0) Finally, in section $\frac{4}{P_0 - m_0}$ $\frac{4}{P_0 - m_0}$ $\frac{4}{P_0 - m_0}$ $\frac{P_1 - m_1}{P_2 - m_1}$ $\frac{P_2 - m_2}{P_3 - m_2}$ 37 we summarize our main conclusions. 103

39 105 **2. Pseudo-Newtonian equations of motion**

41 The Fodor–Hoenselaers–Perjés procedure is an algorithm to de- $\frac{3^{11}}{2}$ $\frac{3^{11}}{2}$ $\frac{3^{11}}{2}$ $\frac{1^{11}}{2}$ ⁴² termine the multipole moments of stationary axisymmetric elec-
⁴² termine the multipole moments of stationary axisymmetric elec-
 w _{here} $M_{ii} = m_{ii} + m_{ii} + m_{ii}$ ⁴³ trovacuum space–times [\[16\].](#page--1-0) The method can be stated as follows:
From the previous it can be inferred that once we know the ⁴⁴ In the Ernst formalism [\[19,20\],](#page--1-0) Einstein field equations are reduced σ *exalizational multipole moments* and following the inverse proce- 45 to a pair of complex equations through the introduction of the $\frac{1}{111}$ dure it is possible to determinate approximate expressions for the ⁴⁶ complex potentials \mathcal{E} and Ψ , which can be defined in terms of gravitational potential ξ , in terms of the physical parameters of the

$$
\mathcal{E} = \frac{1-\xi}{1+\xi}, \quad \Psi = \frac{5}{1+\xi}, \tag{2}
$$
\n
$$
P_0 = m, P_i = 0 \quad \text{for} \quad i > 1, \tag{10}
$$
\n
$$
\mathcal{E} = \frac{1-\xi}{1+\xi}, \tag{11}
$$

51 117 satisfying the alternative representation of the Einstein–Maxwell 52 118 such that *m* denotes the mass of the source. From Eq. (8) and field equations,

$$
(\xi \xi^* - \zeta \zeta^* - 1)\nabla^2 \xi = 2(\xi^* \nabla \xi - \zeta^* \nabla \zeta) \cdot \nabla \xi,
$$
\n(3)

$$
56 \quad (\xi \xi^* - \zeta \zeta^* - 1) \nabla^2 \zeta = 2(\xi^* \nabla \xi - \zeta^* \nabla \zeta) \cdot \nabla \zeta.
$$
\n(4)

netic potentials in a very direct way,

$$
\begin{array}{lll}\n60 & \xi = \phi_M + i\phi_J, & \zeta = \phi_E + i\phi_H, \\
\delta_1 & \delta_2 = \phi_M + i\phi_J, & \zeta = \phi_E + i\phi_H,\n\end{array} \tag{11}
$$

62 where $φα$ with $α = M$, J , E , H , are analogous to the Newtonian 128 63 mass, angular momentum, electrostatic and magnetic potentials, It is important to note that the infinite sum of terms of the poten- 129 64 respectively (see *e.g.* [\[21\]](#page--1-0) and [\[17\]\)](#page--1-0). Hereafter, for the sake of con- tial ξ corresponds to the Schwarzschild solution, while the lower 130 65 venience, we consider $\phi_E = \phi_H = 0$, which implies $\zeta = \Psi = 0$, *i.e.* order of approximation, *i.e.* by taking the first term, corresponds to ¹³¹ ⁶⁶ the absence of electromagnetic fields. The Newtonian potential. In order to stress that the approximations ¹³² the absence of electromagnetic fields.

¹ not always equilateral [\[13\].](#page--1-0) Recently, as a first study of chaos in Mow, according to Geroch and Hansen [21,22], the multipole 67 ² the post-Newtonian CRTBP, Huang and Wu [\[14\]](#page--1-0) studied the influ- moments of a given space–time are defined by measuring the de- 68 ³ ence of the separation between the primaries, concluding that if viation from flatness at infinity. Following this idea, the initial 69 4 it is close enough, the post-Newtonian dynamics is qualitatively 3 -metric h_{ij} is mapped to a conformal one, that is $h_{ij} \to h_{ij} = ~^{70}$ ⁵ different. In particular, some Newtonian bounded orbits become $\Omega^2 h_{ii}$. The conformal factor Ω transforms the potential ξ into $^{-71}$ ⁶ unstable.
⁷²
⁷ To avoid the sumbersome equations of motion that take place $\tilde{\xi} = \Omega^{-1/2}\xi$, with $\Omega = \bar{r}^2 = \bar{\rho}^2 + \bar{z}^2$, and Now, according to Geroch and Hansen [\[21,22\],](#page--1-0) the multipole moments of a given space–time are defined by measuring the deviation from flatness at infinity. Following this idea, the initial 3-metric h_{ij} is mapped to a conformal one, that is $h_{ij} \rightarrow h_{ij} =$ ²*hij* . The conformal factor transforms the potential *ξ* into

$$
\bar{\rho} = \frac{\rho}{\rho^2 + z^2}, \quad \bar{z} = \frac{z}{\rho^2 + z^2}, \quad \bar{\varphi} = \varphi.
$$
 (6)

On the other hand, the potential $\tilde{\xi}$ can be written in a power series of $\bar{\rho}$ and \bar{z} as

$$
\tilde{\xi} = \sum_{i,j=0}^{\infty} a_{ij} \bar{\rho}^i \bar{z}^j,
$$
\n(7)

[\[17\]](#page--1-0)

$$
(r+2)^2 a_{r+2,s} = -(s+2)(s+1)a_{r,s+2}
$$

+
$$
\sum_{k,l,m,n,p,g} (a_{kl}a_{mn}^* - b_{kl}b_{mn}^*) [a_{pg}
$$

$$
\times (p^2 + g^2 - 4p - 5g - 2pk - 2gl - 2)
$$

+
$$
a_{p+2,g-2}(p+2)
$$

$$
\times (p+2-2k)
$$

$$
p+2-2k
$$

$$
+ a_{p-2,g+2}(g+2)(g+1-2l)], \qquad (8)
$$

where $m = r - k - p$, $0 \le k \le r$, $0 \le p \le r - k$, with *k* and *p* even, and $n = s - l - g$, $0 \le l \le s + 1$, and $-1 \le g \le s - l$. Finally, the gravitational multipole moments *Pi* of the source are computed from their values on the symmetry axis $m_i \equiv a_{0i}$, by means of the following relationships

38 104 *P*⁰ = *m*0*, P*¹ = *m*1*, P*² = *m*2*, ^P*³ ⁼ *^m*3*, ^P*⁴ ⁼ *^m*⁴ [−] ¹ *m*∗ ⁰*M*20*,*

40 106 7 *^P*⁵ ⁼ *^m*⁵ [−] ¹ 3 *m*∗ ⁰*M*³⁰ [−] ¹ 21 *m*∗ ¹*M*²⁰ (9)

where $M_{ij} = m_i m_j - m_{i-1} m_{i+1}$.

47 113 the new potentials *ξ* and *ς*, through the definitions $\frac{48}{1-k}$ a concrete example, a source whose multipole structure is given by From the previous, it can be inferred that once we know the gravitational multipole moments, and following the inverse procedure, it is possible to determinate approximate expressions for the gravitational potential *ξ* , in terms of the physical parameters of the source (see *e.g.* [\[23\]\)](#page--1-0). Hence, let us apply the outlined procedure to

$$
P_0 = m, P_i = 0 \text{ for } i \ge 1,
$$
 (10)

 53 119 119 119 the seed $a_{00} = m$, it can be noted that the only non-vanishing 119 54 $(\xi \xi^* - \zeta \zeta^* - 1) \nabla^2 \xi = 2(\xi^* \nabla \xi - \zeta^* \nabla \zeta) \cdot \nabla \xi,$ coefficients are $a_{2n,2m}$ with $n, m \in \mathbb{N}$. Thus the potential ξ is re-55 55 1/1, $\zeta = 2(\zeta + \zeta)$ 5 1/2², and is explicitly given by $\zeta = 2$
(1)

The fields
$$
\xi
$$
 and ζ are related to the gravitational and electromagnetic field ξ and ζ are related to the gravitational and electromagnetic field ξ and ζ are related to the gravitational and electromagnetic field ξ and ζ are the electric potential in a very direct way,

$$
\frac{1}{59} \text{ m}^5 \rho^2 (3\rho^2 - 4z^2)
$$

$$
\xi = \phi_M + i\phi_J, \quad \zeta = \phi_E + i\phi_H, \tag{11}
$$
\n
$$
\xi = \phi_M + i\phi_J, \quad \zeta = \phi_E + i\phi_H, \tag{12}
$$
\n
$$
\xi = \phi_M + i\phi_J, \quad \zeta = \phi_E + i\phi_H, \tag{13}
$$
\n
$$
\xi = \phi_M + i\phi_J, \quad \zeta = \phi_E + i\phi_H, \tag{14}
$$

It is important to note that the infinite sum of terms of the potential *ξ* corresponds to the Schwarzschild solution, while the lower order of approximation, *i.e.* by taking the first term, corresponds to

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