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Anderson localization in one-dimensional quasiperiodic lattice models with nearest- and next-nearest-neighbor hopping

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A B S T R A C T

We explore the reduced relative Shannon information entropies SR for a quasiperiodic lattice model with nearest- and next-nearest-neighbor hopping, where an irrational number is in the mathematical expression of incommensurate on-site potentials. Based on SR, we respectively unveil the phase diagrams for two irrationalities, i.e., the inverse bronze mean and the inverse golden mean. The corresponding phase diagrams include regions of purely localized phase, purely delocalized phase, pure critical phase, and regions with mobility edges. The boundaries of different regions depend on the values of irrational number. These studies present a more complete picture than existing works.

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1. Introduction

Anderson localization is an important and fundamental concept in condensed matter physics and disordered systems [1–3]. For electronic systems, mobility edges (MEs), which separate localized states from delocalized ones, draw a lot of attentions. For examples, MEs have been found in three-dimensional uncorrelated random disorder potential systems [1,2] and some one-dimensional (1D) disorder systems, such as random-dimer potential models [4] and long-range correlated disordered potential ones [5]. Very recently, Ganeshan, Pixley and Das Sarma have proposed a family of deterministic (i.e., no disorder) nearest-neighbor tight-binding models with exact mobility edges [6]. Some other deterministic potential models also have MEs, for instance, the Soukoulis–Economou model [7] and the slowly varying potential ones [8].

The Hamiltonian in all the above mentioned lattice models only includes hopping integral between nearest-neighbor sites. In contrast to them, Johansson and Riklund have considered both the nearest-neighbor and the next-nearest-neighbor hopping integral and proposed a quasiperiodic lattice model [9], which is a generalization of the Aubry–André model [10] and the Soukoulis–Economou model [7]. The incommensurate on-site potential in the model is $V[\cos(2\pi\alpha n) + W\cos(4\pi\alpha n)]$, where V characterizes the potential strength, W is the weight of the second cosine function and α is an irrational number. The nearest- and the next-nearest-neighbor hopping integrals are t and u, respectively. At a specific choice of parameters, i.e., the inverse bronze mean $\alpha_b = (\sqrt{13} - 3)/2$ and $W = u = 1/3$, Johansson and Riklund find that all states are delocalized, critical or localized, which depends on potential strength V. They draw a conclusion that there are no MEs in the model. On the contrary, very recently Sun et al. have studied the same model at the same specific chosen parameters except α (the inverse golden mean $\alpha_g = (\sqrt{5} - 1)/2$ is chosen) [11]. They find MEs. Subsequently, it brings a heated debate on whether there exist MEs in the model [12,13]. It is interesting that what the nature of states in the model, i.e., whether states are delocalized, critical or localized.

In contrast to existing works [9,11–13], we will illuminate the W-dependence and u-dependence results. We will calculate the Shannon information entropies to understand Anderson localization in the interesting model. The rest of the paper is organized as follows. We first introduce the model and Shannon information entropy in Section 2. Then, we provide the numerical results in Section 3. At last, we present our discussions and conclusions in Section 4.

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2. Models and Shannon information entropy

2.1. One-dimensional quasiperiodic lattice models with nearest- and next-nearest-neighbor hopping

The model is a 1D tight-binding lattice systems, which can be described by [9]

$$V[\cos(2\pi\alpha n) + W\cos(4\pi\alpha n)]c_n + t(c_{n+1} + c_{n-1}) + u(c_{n+2} + c_{n-2}) = Ec_n. \tag{1}$$

Here, V, W, t and u have been explained in Introduction. In addition, E is the eigenenergy, and c_n is the amplitude of eigenstate at the nth lattice site. If a transformation to reciprocal space is performed by

$$c_n = \sum_{-\infty}^{\infty} g_m \exp(imn2\pi\alpha), \tag{2}$$

Eq. (1) is transformed to

$$2t[\cos(2\pi\alpha m) + \frac{u}{t}\cos(4\pi\alpha m)]g_m + \frac{V}{2}(g_{m+1} + g_{m-1}) + \frac{VW}{2}(g_{m+2} + g_{m-2}) = Eg_m. \tag{3}$$

If we set $V' = 4tt'/V$, $W' = u/t$, $u' = Wt'$ and $E' = 2Et'/V$, Eq. (3) becomes

$$V'[\cos(2\pi\alpha m) + W'\cos(4\pi\alpha m)]g_m + t'(g_{m+1} + g_{m-1}) + u'(g_{m+2} + g_{m-2}) = E'g_m. \tag{4}$$

Obviously, Eq. (1) and Eq. (4) are dual to each other. The corresponding duality transformation [Eq. (2)] is a simple Fourier transformation, which maps delocalized (localized) states in position space to localized (delocalized) ones in momentum space. In other words, if the states determined by Eq. (1) are delocalized, the corresponding states by Eq. (4) are localized, and vice versa. Specifically, the model is self-dual at the condition that

$$V = 2t, W = u/t. \tag{5}$$

2.2. Shannon information entropy

For the sake of convenience, $|\beta\rangle$ denotes the eigenstate that determined by Eq. (1) and E_β denotes the corresponding eigenenergy. The position–space Shannon information entropy for $|\beta\rangle$ is defined by [14]

$$S_x = -\sum_{n=1}^{N} |c_n|^2 \ln(|c_n|^2). \tag{6}$$

Similarly, the momentum–space Shannon information entropy is

$$S_p = -\sum_{k=1}^{N} |c_k|^2 \ln(|c_k|^2). \tag{7}$$

Here $c_k = \frac{1}{\sqrt{N}}\sum_{n=1}^{N}\exp(-i2\pi nk/N)c_n$, $k = -[\frac{N}{2}], ..., [\frac{N-1}{2}]$, where $i^2 = -1$ and $[Z]$ is the integer part of Z. We define a reduced relative Shannon entropy

$$SR = (S_x - S_p)/(S_x + S_p). \tag{8}$$

We have found that $SR > 0$ and $SR < 0$ for delocalized states and localized states, respectively [14]. The larger SR, the more delocalized states are, while the smaller SR, the more localized states are. It is a simple criterion to distinguish delocalized, localized and critical states from each other.

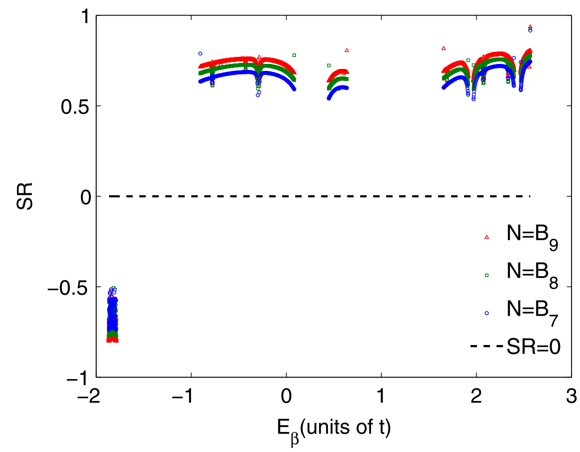

Fig. 1. (Color online) For the inverse bronze mean, the reduced relative Shannon entropy SR versus eigenenergies E_β at lattice sizes $N = B_7, B_8$ and B_9, respectively. Here, $W = u = 1/5$ and $V = 1.0$. The dash line is for the function $SR = 0$.

3. Numerical results

In calculations, we directly diagonalize Eq. (1) with the periodic boundary condition at finite system sizes and obtain all eigenenergy and corresponding eigenstates. From Eqs. (6)–(8), we obtain the reduced relative Shannon entropy SR. Without loss of generality, we choose the nearest-neighbor hopping integral t in Eq. (1) as a unit of energy. First, as examples, we study the state localization properties at two specific choices of parameters. Then, we give the phase diagram. As customary in the context of quasiperiodic system, the inverse bronze mean α_b can be approximated by the ratio of successive numbers: $B_m = 3B_{m-1} + B_{m-2}$ with $B_0 = B_1 = 1$. For examples, $B_7 = 1549$, $B_8 = 5116$ and $B_9 = 16897$. In this way, choosing $\alpha_b = B_{m-1}/B_m$ and lattice size $N = B_m$, we can obtain the periodic approximant for the quasiperiodic potential. Similarly, the inverse golden mean α_g can be approximated by the ratio of successive Fibonacci numbers: $F_m = F_{m-1} + F_{m-2}$ with $F_0 = F_1 = 1$.

3.1. State localization properties at specific parameters

At the case that the irrational number is the inverse bronze mean $[\alpha_b = (\sqrt{13} - 3)/2]$ and $W = u = 1/3$, there are no MEs in the model [9]. As a different example, we choose $W = u = 1/5$ as well as $V = 1.0$. The reduced relative Shannon entropy SR versus eigenenergies E_β is plotted in Fig. 1 at $N = B_7, B_8$ and B_9, respectively. It shows that all $SR > 0$ for eigenstates with $E_\beta > -0.9053$. The larger the lattice size N, the larger SR are. It means that these states are delocalized. At the same time, all $SR < 0$ for eigenstates with $E_\beta < -1.7912$. The larger the lattice size N, the smaller SR are. It means that these states are localized. Therefore, there exists a ME between $E_\beta = -1.7912$ and $E_\beta = -0.9053$.

As a second example, we choose $W = u = 1/5$ as well as $V = 2.0$. The corresponding reduced relative Shannon entropy SR versus eigenenergies E_β is plotted in Fig. 2. It shows that SR may be positive or negative. All the absolute value of SR, denoted by $|SR|$, are smaller than 1.0×10^{-8}, 3.0×10^{-7} and 2.5×10^{-5} at $N = B_7, B_8$ and B_9, respectively. Comparing with $|SR|$ in Fig. 1, these $|SR|$ are relatively smaller and almost near zeros. At the same time, Fig. 2 shows that there are no energy-bands for which all $SR > 0$ or all $SR < 0$, which is different from that shown in Fig. 1 for delocalized states and localized ones. Therefore, these states are critical. In fact, at $\lambda = 2.0$ and all others $W = u$, the variations of SR with E_β are similar as that in Fig. 2, thus all these states are critical. It agrees with that the model is self-dual at the parameters.

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