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Superconducting qubit in a nonstationary transmission line cavity: Parametric excitation, periodic pumping, and energy dissipation

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ABSTRACT

We consider a superconducting qubit coupled to the nonstationary transmission line cavity with modulated frequency taking into account energy dissipation. Previously, it was demonstrated that in the case of a single nonadiabatical modulation of a cavity frequency there are two channels of a two-level system excitation which are due to the absorption of Casimir photons and due to the counterrotating wave processes responsible for the dynamical Lamb effect. We show that the parametric periodical modulation of the resonator frequency can increase dramatically the excitation probability. Remarkably, counterrotating wave processes under such a modulation start to play an important role even in the resonant regime. Our predictions can be used to control qubit-resonator quantum states as well as to study experimentally different channels of a parametric qubit excitation.

1. Introduction

Quantum electrodynamics (QED) of superconducting circuits is one of fast and intensively developing fields of a modern physics. The interest to superconducting circuits, which consist of Josephson qubits and transmission line cavities [1], is heated by the possibility of implementation of quantum computation [2], observation of new phenomena of quantum optics in GHz frequency domain [3], as well as an engineering of sub-wavelength quantum metamaterials [4]. An outstanding feature of superconducting circuits is that their parameters are tunable *in situ*: excitation frequencies of qubits can be varied externally, while both the frequency of fundamental mode of a resonator and qubit-resonator coupling energy can be modulated in GHz range by means of auxiliary SQUIDs embedded in the circuit's architecture or using more sophisticated methods. Particularly, superconducting quantum circuits can be used as a unique platform to investigate nonstationary cavity QED phenomena, such as the dynamical Casimir effect [5].

In the series of papers [6,7] dealing with optical systems there was considered a behavior of a two-level atom in a nonstationary high-Q cavity, which experiences a single nonadiabatic change of its frequency. One of the channels of a parametric atom excitation in this situation is through a nonadiabatic change of its Lamb shift, which was termed the "dynamical Lamb effect" [7]. It is produced by counterrotating wave processes leading to a modulation of the atom's dressing by virtual photons and can be considered as the new effect in the nonstationary cavity QED. There is another mechanism of atom excitation in this system which is due to the absorption of photons generated by the cavity dynamical Casimir effect [7]. The absorption is governed by resonant (Jaynes–Cummings) processes. This mechanism is, in general, dominant for the case of nonstationary cavity and therefore it "screens" the dynamical Lamb effect.

In our recent papers [8] (see also Ref. [9]), we suggested an idea how to make the dynamical Lamb effect dominant. It is attractive to use a superconducting system which consists of a *stationary* resonator having a tunable coupling with the qubit. No Casimir photons are generated in this case, while the only one channel of qubit excitation is through the dynamical Lamb effect. Although a proposed idea allows for the observation of this effect, its unambiguous experimental realization may be not so easy.

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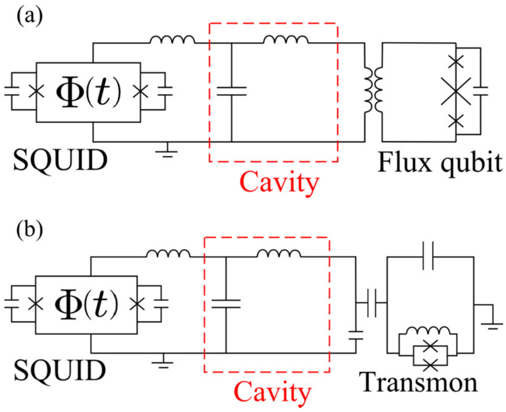

Fig. 1. (Color online.) Equivalent electric superconducting circuits of the systems under consideration. Both setups (a, b) consist of: 1) auxiliary SQUID, 2) single mode cavity, represented as LC-contour (red dashed), and 3) flux qubit (a) or transmon (b). Crosses stand for Josephson junctions with the sizes being related to the values of Josephson energies. SQUID's loop is subjected to the rapidly tunable magnetic flux $\Phi(t)$. The interaction between the electromagnetic field in the cavity and qubits can be organized via (a) inductive coupling in case of the flux qubit or (b) via capacitive coupling in case of the transmon.

Therefore, it is of interest to come back to a simpler scheme with variable resonator frequency, which is more straightforward to implement. In this article, we concentrate on the effect of a *periodic* modulation of the cavity mode frequency. We show that it provides a tool to distinguish between different channels of qubit excitation even near the resonance as well as to enhance the effect as a whole. We also take into account both energy dissipation and pure dephasing, which always exist in real systems and are able to suppress quantum effects. In contrast to most of other studies, we mainly focus on the analysis of different mechanisms of a parametric qubit excitation, i.e., due to rotating wave processes and counterrotating wave processes and under the variation of only the resonator frequency.

2. System

The effect under consideration can be implemented in tunable superconducting circuits, see, e.g., Ref. [10]. As it is shown in Fig. 1 (a, b), the basic components of possible setups involve single mode cavity (superconducting coplanar waveguide), which has auxiliary SQUID embedded into one of its ends, and an artificial macroscopic atom, such as flux qubit (a) or transmon (b), coupled inductively or capacitively to the cavity. Equivalent electric circuit of the resonator is associated with LC-contour inside the red dashed sector in Fig. 1. Alternating external flux $\Phi(t)$, threading the SQUID loop, provides an effective modulation of the resonator inductance at the desired frequency. As a consequence, such a modulation via SQUID plays a role of a non-stationary boundary conditions for the electromagnetic field in the cavity. Eventually, this results in modulation of the photon mode frequency.

3. Model

The full non-stationary Hamiltonian of the system under consideration can be represented as

$$H(t) = H_0(t) + H_{\mathrm{Cas}}(t) + V. \tag{1}$$

The Hamiltonian of non-interacting qubit and cavity is given by

$$H_0(t) = \hbar\omega(t)a^\dagger a + \frac{1}{2}\epsilon(1+\sigma_3), \tag{2}$$

where $a^\dagger$ and a are secondary quantized operators of photon creation and annihilation in the transmission line cavity of non-stationary frequency $\omega(t)$. Pauli operators $\sigma_3 = 2\sigma_+\sigma_- - 1$, σ_+, σ_- act in the space of qubit excited and ground states. The non-stationary term $H_{\mathrm{Cas}}(t)$ in (1) is responsible for the dynamical Casimir effect, i.e., the photon generation from vacuum [11–14]

$$H_{\mathrm{Cas}}(t) = i\hbar\frac{\partial_t\omega(t)}{4\omega(t)}(a^2 - a^{+2}). \tag{3}$$

The last term V in (1) describes a qubit-cavity interaction

$$V = g(a + a^\dagger)(\sigma_- + \sigma_+), \tag{4}$$

where $(a + a^\dagger)$ and $(\sigma_- + \sigma_+)$ can be associated with the electric field and dipole moment, respectively, while g is the coupling energy. This interaction term can be divided into two parts $V = V_1 + V_2$, where $V_1 = g(a\sigma_+ + a^\dagger\sigma_-)$ yields the well known rotating wave approximation (RWA) or Jaynes–Cummings model, provided V_2 is dropped, while V_2 is given by $V_2 = g(a^\dagger\sigma_+ + a\sigma_-)$. RWA terms conserve the total excitations number, whereas counterrotating wave contributions produce and annihilate pairs of excitations.

As it was shown in [7], in the case of a single instantaneous switching of cavity frequency ω from ω_1 to ω_2, the qubit excitation probability at $t \to \infty$ due to the Jaynes–Cummings processes (absorption of Casimir photons generated by $H_{\mathrm{Cas}}(t)$) strongly depends on ω_2 as

$$w_e^{(C)} \simeq \frac{g^2}{(\varepsilon - \omega_2)^2}\frac{(\omega_2 - \omega_1)^2}{4\omega_1\omega_2}, \tag{5}$$

when $|\varepsilon - \omega_2| \gg g$. It turns out that in the opposite case $|\varepsilon - \omega_2| \ll g$ the maximum value $w_e^{(C)} \sim (\omega_2 - \omega_1)^2/\omega_2^2$ is achieved in the resonance between ε and ω_2 [7]. Note that this last value is independent on g and, in the case of a weak modulation is small.

The qubit excitation probability due to the counterrotating wave processes, i.e., the dynamical Lamb effect is not so strongly dependent on ω_2 [7]:

$$w_e^{(L)} \simeq g^2\frac{(\omega_2 - \omega_1)^2}{(\omega_2 + \varepsilon)^2(\omega_1 + \varepsilon)^2}, \tag{6}$$

which in principle allows for the separation of the two effects: $w_e^{(L)}$ becomes of the order of $w_e^{(C)}$ at large detuning $|\varepsilon - \omega_2| \sim \omega_2$. But $w_e^{(L)}$ is small as $\sim (\omega_2 - \omega_1)^2 g^2/\omega_2^4$. At $g/\omega_2 \ll 1$, this quantity is much smaller than the maximum value of $w_e^{(C)}$ attained near the resonance, where the excitation probability is controlled by Jaynes–Cummings processes. These circumstances make it problematic to probe the mechanism of qubit excitation linked to counterrotating terms.

Now we consider a *periodic* modulation of resonator frequency

$$\omega(t) = \omega_0 + d\cos(\Omega t). \tag{7}$$

There appear several controlling parameters: the time-averaged detuning $\Delta = \varepsilon - \omega_0$, modulation frequency Ω, and its amplitude d. We hereafter concentrate on the limits of a small-amplitude variations, $d \ll \omega_0$, and a weak qubit-cavity coupling, $g \ll \omega_0$. We then address system's dynamics by solving numerically the Lindblad equation

$$\partial_t\rho(t) - \Gamma[\rho(t)] = -i[H(t), \rho(t)], \tag{8}$$

where $\rho(t)$ is a density matrix of qubit and photon mode. Dissipation in the cavity of the rate κ and qubit decoherence γ are described through the matrix $\Gamma[\rho] = \kappa(a\rho a^\dagger - \{a^\dagger a, \rho\}/2) + \gamma(\sigma_-\rho\sigma_+ - \{\sigma_+\sigma_-, \rho\}/2) + \gamma_\varphi(\sigma_z\rho\sigma_z - \rho)$. In superconducting qubits the pure decoherence rate γ_φ is typically of the same order as relaxation γ. Both quantities are significantly larger than the relaxation rate of a cavity, $\gamma \gg \kappa$.