Physics Letters A 381 (2017) 639–645

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Nonlinear wave breaking in self-gravitating viscoelastic quantum fluid

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ARTICLE INFO

Article history:
Received 26 July 2016
Received in revised form 23 November 2016
Accepted 11 December 2016
Available online 21 December 2016
Communicated by F. Porcelli

Keywords:
Quantum hydrodynamics
Symmetry breaking instability
Soliton
Quasiperiodic wave

ABSTRACT

The stability of a viscoelastic self-gravitating quantum fluid has been studied. Symmetry breaking instability of solitary wave has been observed through 'viscosity modified Ostrovsky equation' in weak gravity limit. In presence of strong gravitational field, the solitary wave breaks into shock waves. Response to a Gaussian perturbation, the system produces quasi-periodic short waves, which in terns predicts the existence of gravito-acoustic quasi-periodic short waves in lower solar corona region. Stability analysis of this dynamical system predicts gravity has the most prominent effect on the phase portraits, therefore, on the stability of the system. The non-existence of chaotic solution has also been observed at long wavelength perturbation through index value theorem.

1. Introduction

Self gravitating fluid has significant importance in astrophysics, condensed matter physics, plasma physics and many other branches of physics. Self gravitating fluid can support many different nonlinear waves. The knowledge of breaking and stability of those waves is a fundamental key to explore the structure and dynamics of the fluids. Astrophysical bodies like, molecular clouds, galaxies, etc., can be correctly represented through self-gravitating fluids. Therefore, the condensation and star formation mechanisms can be studied through self gravitating fluid model aided with appropriate initial and boundary conditions [1,2].

In linear regime, the influence of viscoelasticity, quantum statistical correction [3] and magnetic field [4,5] on gravitational instability (Jeans instability) for quantum fluid has been studied. However, the intrinsic nonlinearities of hydrodynamic equations produce many striking phenomena like dispersion of soliton, quasi-periodic short wave generation, symmetry breaking instability of solitary waves, very large amplitude wave (rouge wave), chaos, turbulence, etc. into the system. Therefore, attention has gradually shifted from linear mode study to nonlinear analysis.

The existence and stability of nonlinear waves specifically, soliton, have been studied for self gravitating molecular clouds [6]. Adams et al. [7] studied the existence and nonexistence of solitary wave in a gravitating fluid via 'charge density' theory, and concluded that, solitary waves exist only for those systems which are gravitationally stable against arbitrary large perturbation. Whereas, if the self gravitating fluid is made up with at least two species with different thermal velocities, then it can support solitary waves [6]. The existence of soliton has been studied for weakly magnetized molecular clouds by several authors [7,8]. The collision of two compact stars or collapse of a single star can produce 'solitary gravitational sound' waves, which carries information about the (colliding or collapsing) stars and the propagating medium. These types of solitary wave have been studied for self gravitating fluid consisting of weakly interacting massive particles [9]. Effect of non-uniformity of the medium on the propagation and modulation of nonlinear waves has been studied extensively with reductive perturbation method [10]. The quantum hydrodynamic (QHD) formulation for different physical system has been studied extensively. Minguzzi et al. [11] studied trapped BEC via QHD and obtained a microscopic Landau equation for the inhomogeneous system. The scattering problem of particles has been attacked through QHD by Nassar [12] and derived a formula for the transmission coefficient of the scattered particles. Propagation and collision of soliton ring in quantum plasma have also been studied via QHD [13]. Ghosh et al. [14] studied the evolution of solitary waves in Bose–Einstein gravitational condensate gas in QHD regime and concluded that, rarefactive solitary wave exists in this system. Similarity between Bose–Einstein condensation in the canonical ensemble, and the gravitational collapse of classical self-gravitating gas has been established by Sopik et al. [15]. Conditions

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http://dx.doi.org/10.1016/j.physleta.2016.12.026

of existence for dust-acoustic solitary wave in a self-gravitating opposite polarity dust-plasma system have been derived by Mamumn et al. [16]. Nonlinear plane periodic acoustic wave and its connection to the galaxy formation have been discussed by Liang [17]. The evolution of dust-ion-acoustic waves has been studied through self-gravitating fluid model by Masood et al. [18]. A very interesting short asymmetric wave exists in solar corona region, which seems to take part in heating mechanism of the region. Robert et al. [19] predicts that these types of asymmetric waves can be initiated in the corona by flare. These short waves are quasi-periodic waves. Nakariakov et al. [20] studied these waves through wavelet image method. They state that, the quasiperiodicity results from the geometric dispersion of the modes. The instantaneous period of oscillations decreases with time.

Phase space analysis is one of the most useful tool to study the stability criterion and the existence of chaotic state in a given system. Sola and Pettini [21] have studied the existence and the dynamics of chaos in self-gravitating N-body systems through Geometric analysis in configuration space. They have discussed the relationship between the Riemannian geometric description of chaos and Lyapunov exponents.

In this paper, we studied the evolution of nonlinear disturbances in a self gravitating viscoelastic quantum fluid. Generally solid material exhibits elastic properties much more profoundly than fluids. However, there is a certain limit [22], above which fluids also can show elastic properties. On the other hand, internal frictions are present in the real fluids, which, as a consequence produce viscosity. The presence of organic components in molecular clouds makes the clouds viscoelastic [22]. In exotic environments of high density and temperature, such as core of neutron stars, pulsars, magnetars, and white dwarfs, matter can only exist in the form of fluid (plasma) for which, quantum effects play a significant role because of the very high densities involved. Therefore, a quantum hydrodynamic (QHD) approach is very relevant and fruitful to analyze such astrophysical objects. The presence of quantum pressure (or Bohm potential) in momentum equation of QHD indicates the possibility of quantum tunneling of particles through potential barrier. This tunneling phenomenon gives rise to quantum dispersion in the medium. In general, we can name our approach as 'viscoelastic QHD under the action of gravity'.

We address mainly two points in this paper, namely, (I) evolution of a nonlinear disturbance (solitonic and Gaussian pulse) in a self gravitating quantum viscoelastic fluid and (II) the dynamics of a self gravitating system in phase space.

The paper has been organized as follows.

Physical assumptions and basic equations are stated in section 2. In section 3, the equation to study the dynamical evolution of the system has been derived. Evolutions of nonlinear disturbances in the system have been studied in section 4. The phase space dynamics (stability analysis) of the system is given in section 5. The results of the study have been discussed in section 6.

2. Physical assumptions and basic equations

The continuity and momentum equations for a compressible viscoelastic fluid with quantum potential are as follows:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho\vec{u}) = 0, \tag{1}$$

and

$$\left(1+\tau\frac{d}{dt}\right)\left[\rho\left(\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\vec{\nabla})\vec{u}\right)+\vec{\nabla}p-\rho\vec{g}-\frac{h^2}{8\pi^2m^2}\rho\vec{\nabla}\left(\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}\right)\right]=\eta\nabla^2\vec{u}+\left(\eta+\frac{\zeta}{3}\right)\vec{\nabla}(\vec{\nabla}.\vec{u}) \tag{2}$$

The Poisson equation for gravity is:

$$\nabla^2\phi = 4\pi G(\rho-\rho_0) \tag{3}$$

and the equation of state is

$$p = c^2\rho$$

where η and ζ are the shear viscosity and bulk viscosity, respectively. τ is the relaxation time of the system. m is the mass of the constituent particles of the system. c is the speed of sound through the system. ϕ is gravitational potential defined as $\vec{g} = -\vec{\nabla}\phi$.

3. Dynamical evolution of the system

To investigate the dynamical evolution of the system, we consider one spatial dimension (generalization to more spatial dimension is trivial), namely, $\bar{x} = x/L$ (L is the length scale of the system), $\bar{t} = ct/L$, $\bar{\rho} = \frac{\rho}{\rho_0}$, $\bar{u} = u/c$. Hereafter we shall use these new variables and remove all the bars for simplicity of notation. From (1) and (2), we obtain

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho\vec{u}) = 0, \tag{4}$$

$$\left(1+\tau_m\frac{d}{dt}\right)\left[\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)+\frac{\partial \ln\rho}{\partial x}-H^2\rho\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{\rho}}\frac{\partial^2\sqrt{\rho}}{\partial x^2}\right)\right]-\frac{1}{R}\frac{\partial^2 u}{\partial x^2}=-\left(\frac{L}{\lambda_j}\right)^2\left(1+\tau_m\frac{d}{dt}\right)\frac{\partial\phi}{\partial x} \tag{5}$$

where $\tau_m = \frac{\tau}{cL}$; $\frac{1}{R} = \frac{12\eta+\zeta}{3cL\rho_0}$; $H^2 = \frac{h^2}{8\pi^2m^2c^2L^2}$; $\lambda_j = \frac{2\pi c}{\sqrt{4\pi G\rho_0}}$.

To study the nonlinear propagation characteristic of perturbation, we employ the reductive perturbation technique and introduce the following stretched coordinates:

$$\xi = \epsilon^{\frac{1}{2}}(x-Mt),\ \ \tau = \epsilon^{\frac{3}{2}}t, \tag{6}$$

where M determines the (normalized) group velocity of the linear wave and ϵ characterizes the strength of the nonlinearity. The physical variables ρ, u are expanded in the power series of ϵ as

$$\begin{pmatrix}\rho\\u\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}+\epsilon\begin{pmatrix}\rho^{(1)}\\u^{(1)}\end{pmatrix}+\epsilon^2\begin{pmatrix}\rho^{(2)}\\u^{(2)}\end{pmatrix}+\cdots \tag{7}$$

Now taking divergence of equation (5) and using equation (3) we have

$$\frac{\partial}{\partial x}\left\{\left(1+\tau_m\frac{d}{dt}\right)\left[\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)+\frac{\partial\ln\rho}{\partial x}-H^2\rho\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{\rho}}\frac{\partial^2\sqrt{\rho}}{\partial x^2}\right)\right]-\frac{1}{R}\frac{\partial^2u}{\partial x^2}\right\}=\left(\frac{L}{\lambda_j}\right)^2\left(1+\tau_m\frac{d}{dt}\right)\left(\rho-1\right) \tag{8}$$

To include the gravitational effect and viscoelasticity and also for the consistent perturbation, we consider the following scaling

$$\frac{L}{\lambda_j}\sim\bigcirc(\epsilon);\ \frac{1}{R}\sim\bigcirc(\sqrt{\epsilon}) \tag{9}$$

Now substituting the stretching coordinates (6), perturbation expansions (7) and the scalings (9) into (4) and (8), we obtain the following relations in the lowest powers of ϵ:

$$M = 1;\ \rho^{(1)} = u^{(1)} \tag{10}$$