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Conclusive identification of quantum channels via monogamy of quantum correlations

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ABSTRACT

We investigate the action of global noise and local channels, namely, amplitude-damping, phase-damping, and depolarizing channels, on monogamy of quantum correlations, such as negativity and quantum discord, in three-qubit systems. We discuss the monotonic and non-monotonic variation, and robustness of the monogamy scores. By using monogamy scores, we propose a two-step protocol to conclusively identify the noise applied to the quantum system, by using generalized Greenberger–Horne–Zeilinger and generalized W states as resource states. We discuss a possible generalization of the results to higher number of parties.

1. Introduction

An important characterization of a composite quantum system is by the correlations, both classical and quantum, between its constituting parts. Quantum information theory provides a collection of measures of quantum correlations [1–3], which can broadly be categorized into two classes. One is the "entanglement-separability" class, encompassing various measures of quantum entanglement in both bipartite and multipartite domain [1]. The other is the information-theoretic regime [2,3], consisting of quantum correlations such as quantum discord [4,5]. Both entanglement as well as information theoretic quantum correlation measures have been proposed to be resources for several quantum protocols [1–3]. However, quantum correlations, especially entanglement, have been found to be fragile under decoherence [6,7]. Naturally, due to their immense importance in quantum information processing tasks, investigating the behavior of quantum correlations under various kinds of environmental noise has been a topic of utmost importance in quantum information theory.

Most of the available literature that deals with decoherence of quantum correlations consider bipartite quantum correlation measures due to their relative computational simplicity. It has been shown that the bipartite entanglement measures tend to decay rapidly with increasing noise, and vanish when a threshold noise level is crossed. This phenomena is known as "entanglement sudden death", and has been studied extensively in the case of bipartite systems under different types of environments [8]. In stark contrast to this behavior, information theoretic measures like quantum discord have been found to undergo an asymptotic decay with increasing noise strength [9,10], indicating a higher robustness against noise than that of entanglement. It has also been shown that special two- as well as multiqubit mixed quantum states can be engineered for which "discord-like" quantum correlations may remain frozen over a finite range of noise strength [11–14], while the entanglement measures for those states exhibit no such property (cf. [15]). Although behavior of bipartite quantum correlations under decoherence is a well-investigated topic, similar studies in the multipartite scenario [16–21] are limited due to the lack of computable measures of quantum correlations for mixed multipartite states.

Recent developments on the monogamy relation of quantum correlations [22–24] have provided an effective tool to investigate the multipartite nature of quantumness present in a composite quantum system. Qualitatively, monogamy of a quantum correlation measure corresponding to a multipartite state is the property

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that allows a chosen party to share only limited amount of quantum correlation with all the other parties except one, to which it is highly quantum correlated. Interestingly, such monogamy constraints can be quantified via the "monogamy score" [25], leading to multipartite quantum correlation measures that use bipartite measures of quantum correlations, thereby reducing the difficulty in the computation of the measures for multipartite states. The monogamy property of quantum correlations has been shown to be important in several aspects in quantum mechanics and quantum information, like foundations of quantum mechanics [26], quantum cryptography [27–29], teleportation [30,31], quantum dense coding [32], quantum steering [33,34], many-body physics [35–37], and black-hole information theory [38,39]. Experimental investigation of this property has also been initiated [40]. Therefore, it has become important to investigate the behavior of the monogamy property of quantum correlations when the system is subjected to noisy environments.

This paper has two different objectives that are complementary to each other. In one, we study the dynamics of monogamy of quantum correlations, denoted by $\mathcal{Q}$. As measures of quantum correlations, we use the monogamy scores of two bipartite quantum correlation measures, namely, the negativity, $\mathcal{N}$, [41–44] a measure of bipartite entanglement, and quantum discord, $\mathcal{D}$, [4,5], a quantum correlation measure from the information-theoretic domain. We choose a global noise, and three local noisy channels, namely, the amplitude-damping (AD), the phase-damping (PD), and the depolarizing (DP) channels as different models of environmental noise [45]. We demonstrate how the dynamics of monogamy, in the case of three-qubit systems, exhibit qualitatively different behavior depending on whether the input quantum state is chosen from the family of generalized Greenberger–Horne–Zeilinger (gGHZ) state [46], or the generalized W (gW) states [47,49,48], which are not equivalent under stochastic local operations and classical communication (SLOCC). More specifically, we show that monogamy scores of negativity as well as quantum discord exhibit a monotonic decay with respect to the corresponding noise parameter, when gGHZ state is subjected to these noise models, while there exist non-monotonic dynamics when the input state is the gW state. We also investigate the trends of monogamy scores against noise, when arbitrary three-qubit pure states belonging to the two inequivalent SLOCC classes of three-qubit pure states, namely, the GHZ and the W classes [48], are chosen as inputs. Moreover, we introduce a concept called the "dynamics terminal", which quantify the durability of quantum correlation measures under decoherence, and show that it can distinguish between different quantum correlation measures as well as different types of noise. The study also reveals that for the PD channel, the negativity monogamy score can exhibit a more robust behavior against noise strength than that observed for the monogamy score of quantum discord, which we call the "discord monogamy score".

Besides characterizing the dynamical features of quantum correlations under decoherence, it is also interesting to address the reverse question as to whether the modes of environmental noise can be identified by using the properties of quantum correlations. Although a few studies have been motivated by similar goal [50,51], the literature regarding this issue is extremely limited. While most of the studies have tried to distinguish different types of noise by the different dynamical behavior of different quantum correlations, concrete protocol to conclusively identify the type of noise to which the quantum state is exposed is yet to be introduced. As the second objective of this paper, we use the highly entangled gGHZ and any gW states as resources, and design a two-step protocol involving the monogamy relations of negativity and quantum discord to conclusively distinguish the type of noise applied to the quantum state, where the noise models include a global noise, and several local channels, namely, AD, PD, and DP channels.

The paper is organized as follows. In Sec. 2, we discuss the dynamical behavior of the negativity and discord monogamy scores, when the gGHZ and the gW states are subjected to different types of noise. The behavior of monogamy against noise, when arbitrary three-qubit pure states are considered as input, is also studied in this section. In Sec. 3, the two-step channel discrimination protocol with monogamy scores is presented. Sec. 4 presents the concluding remarks.

2. Monogamy of quantum correlations under decoherence

In this section, we investigate the behavior of monogamy scores corresponding to negativity and quantum discord for three-qubit quantum states under the influence of global as well as local noise. Brief descriptions of the quantum correlation measures, namely, negativity and quantum discord, and monogamy of quantum correlations can be found in [52]. Discussions on different types of noise considered in this paper are provided in [52]. Before considering arbitrary three-qubit pure states, we examine the generalized GHZ (gGHZ), and the generalized W (gW) states as the input states to various types of noise.

2.1. Generalized GHZ states

The generalized GHZ state, shared between three qubits, 1, 2, and 3, reads as

$$|\Psi_3\rangle = a_0|000\rangle + a_1|111\rangle, \tag{1}$$

where a_0 and a_1 are the complex parameters satisfying $|a_0|^2 + |a_1|^2 = 1$. In this paper, we consider qubit 1 as the nodal observer while computing monogamy scores for negativity ($\delta_\mathcal{N}$), and quantum discord ($\delta_\mathcal{D}$). Note that the monogamy scores for the gGHZ state, in the noiseless scenario, is always positive for all quantum correlation measures including negativity and quantum discord. This is due to the fact that the two-qubit reduced density matrix $\rho_{12} = \rho_{13} = |a_0|^2|00\rangle\langle 00| + |a_1|^2|11\rangle\langle 11|$, obtained from the gGHZ state, is a classically correlated two-qubit state having vanishing quantum correlations, while the state $|\Psi_3\rangle$ in the $1:23$ bipartition always has a non-zero value of quantum correlation for $a_0, a_1 \neq 0$. Even for the noise parameter $p \neq 0$, $\delta_\mathcal{N}$ and δ_D are given by $\delta_\mathcal{N} = \mathcal{N}(\rho^{\text{gGHZ}}_{1:23})$ and $\delta_\mathcal{D} = \mathcal{D}(\rho^{\text{gGHZ}}_{1:23})$ respectively, when the gGHZ state is subjected to the four types of noise considered in this paper (see [52]). Hence, both negativity and quantum discord are always monogamous in the present scenario, which can be applied to discriminate channels, as we shall see in Sec. 3. Note that the entire discussion also holds for gGHZ states of an arbitrary number of parties subjected to the different types of local and global noise considered here.

The above discussion helps one to determine analytical expressions for negativity monogamy score as a function of the noise parameter in the case of different types of noise [52]. On the other hand, analytically determining the discord monogamy score, $\delta_\mathcal{D}$, for all the types of noise, is in general hard due to the optimization required to compute quantum discord for ρ^{gGHZ} in the $1:23$ split [53]. So far, analytical determination of quantum discord has been possible only for very restricted class of mixed states [54,55]. Hence, we employ numerical optimization over the complete set of rank-1 projective measurements involved in the definition of quantum discord. The behavior of the monogamy scores corresponding to negativity and quantum discord for different types of noise are depicted in Fig. 1, where the top panels are for $\delta_\mathcal{N}$, and the bottom panels correspond to $\delta_\mathcal{D}$. For all the noise models considered in this paper, $\delta_\mathcal{N}$ and $\delta_\mathcal{D}$ monotonically decreases with increasing values of p for a fixed value of $|a_0|$, and vanishes