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Study of the skin effect in superconducting materials

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The skin effect is analyzed to provide the numerous measurements of the penetration depth of the electromagnetic field in superconducting materials with a theoretical basis. Both the normal and anomalous skin effects are accounted for within a single framework, focusing on frequencies less than the superconducting gap. The emphasis is laid on the conditions required for the penetration depth to be equal to London's length, which enables us to validate an assumption widely used in the interpretation of all current experimental results.

1. Introduction

Superconductivity is characterized by two prominent properties [1–5], namely persistent currents in vanishing electric field and the Meissner effect [6], which latter highlights the rapid decay of an applied magnetic field in bulk matter in a superconductor. Early insight into the Meissner effect was achieved thanks to London's equation [7]

$$B + \mu_0 \lambda_L^2 \mathrm{curl}\, j = 0 \quad ,$$

where μ_0, j, λ_L stand for the magnetic permeability of vacuum, the persistent current, induced by the magnetic induction B and London's length, respectively. London's equation, combined with those of Newton and Maxwell, entails [1–5,7] that the penetration depth of the magnetic field is equal to λ_L

$$\lambda_L = \sqrt{\frac{m}{\mu_0 \rho e^2}} \quad , \tag{1}$$

where e, m, ρ stand for the charge, effective mass and concentration of superconducting electrons. Thus the measurement of λ_L is all the more important, since there is no other experimental access to ρ.

Pippard [8–10] carried out the first measurements of electromagnetic energy absorption at a frequency $\omega \approx 10$ GHz in superconducting Sn, containing impurities, and interpreted his results within the framework of the anomalous skin effect. In normal conductors, the real part of the dielectric constant $\epsilon_R(\omega)$ being negative for $\omega < \omega_p$, where $\omega_p \approx 10^{16}$ Hz stands for the plasma frequency, causes the electromagnetic field to remain confined within a thin layer of frequency dependent thickness $\delta(\omega)$, called the skin depth, and located at the outer edge of the conductor. δ is well known [11,12] to behave like $\omega^{-1/2}$ at low frequency and to reach, in very pure metals at low temperature, a ω-independent, lower bound δ_a, characteristic of the anomalous skin effect [13,14]. Actually, in the wake of Pippard's work, all current determinations of λ_L, made in superconducting materials [15–20], including high T_c compounds (T_c stands for the critical temperature), consist of measuring the penetration depth of the electromagnetic field at frequencies $\omega \in [10\text{ MHz}, 100\text{ GHz}]$, while assuming $\lambda_L = \delta_a$.

As the latter assumption has hardly been questioned, the main purpose of this work is to ascertain its validity by working out a comprehensive analysis of the skin effect, including both the usual and anomalous cases. The treatment of the electrical conductivity at finite frequency in a superconducting material runs into further difficulty, because, according to the mainstream model [1–5], the conduction electrons make up, for $T < T_c$, a two-component fluid, comprising normal and superconducting electrons. This work, which is intended at deriving the respective contributions of the two kinds of electrons, is based solely on Newton and Maxwell's equations.

The outline is as follows: Section 2 deals with the skin effect; the results are used to work out the conduction properties of the two-fluid model in Section 3, while contact is made with

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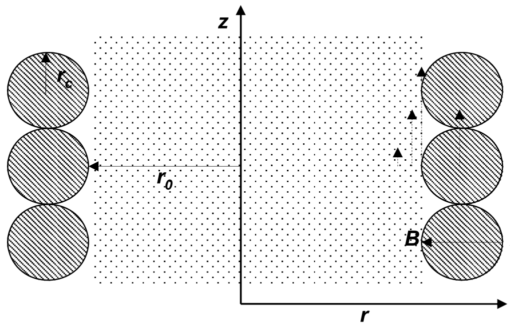

Fig. 1. Cross-section of the superconducting sample (dotted) and the coil (hatched); E_θ and j_θ are both normal to the unit vectors along the r and z coordinates; vertical arrows illustrate the r dependence of $B_z(r)$; r_c has been magnified for the reader's convenience; Eq. (13) has been integrated from A ($B_z(r_0 + 2r_c) = 0$) to B.

the experimental results in Section 4. The conclusions are given in Section 5.

2. Skin effect

Consider as in Fig. 1 a superconducting material of cylindrical shape, characterized by its symmetry axis z and radius r_0 in a cylindrical frame with coordinates (r, θ, z). The material is taken to contain conduction electrons of charge e, effective mass m, and total concentration ρ. It is subjected to an oscillating electric field $E(t,r) = E_\theta(r)e^{i\omega t}$, with t referring to time. As $E_\theta(r)$ is normal to the unit vectors along the r and z coordinates, there is $\mathrm{div}E = 0$. E induces a current $j(t,r) = j_\theta(r)e^{i\omega t}$ along the field direction, as given by Newton's law

$$\frac{dj}{dt} = \frac{\rho e^2}{m}E - \frac{j}{\tau} \quad , \tag{2}$$

where $\frac{\rho e^2}{m}E$ and $-\frac{j}{\tau}$ are respectively proportional to the driving force accelerating the conduction electrons and a generalized friction term, which is non-zero in any superconducting material provided $\frac{dj}{dt} \neq 0$.

The existence of a *friction force* in superconductors, carrying an *ac current*, is known experimentally (see [4] p. 4, 2nd paragraph, line 9). For example, the measured ac conductivity for the superconducting phase of $BaFe_2(As_{1-x}P_x)_2$ has been found (see [18] p. 1555, 3rd column, 2nd paragraph, line 11) to be $\approx .03\sigma_n$, where σ_n stands for the normal conductivity [21] measured just above the critical temperature T_c. However because the current is carried by electrons, making up either a BCS state [22] or a Fermi gas [2] in a superconducting and normal metal, respectively, the physical sense of τ in Eq. (2) for superconductors may be different from that given by the Drude model [2] for a normal metal. To understand this difference and to model the new τ, we shall next work out the equivalent of Ohm's law for a superconducting material when submitted to an electric field.

The superconducting state, carrying no current in the absence of external fields, is assumed to comprise two subsets of equal concentration $\rho/2$, moving in opposite directions with respective mass center velocity $v, -v$, which ensures $j = 0, \quad p = 0$, where p refers to the average electron momentum. Under a driving field E, an ensemble $\delta\rho/2$ of electrons is transferred from one subset to the other, so as to give rise to a finite current $j = \delta\rho e v = e\delta p/m$, where δp stands for the electron momentum variation. The generalized friction force is responsible for the reverse mechanism, whereby electrons are transferred from the majority subset of concentration $\frac{\rho+\delta\rho}{2}$ back to the minority one ($\frac{\rho-\delta\rho}{2}$). It follows from flux quantization and the Josephson's effect [1–5,23] that the elementary transfer process involves a pair [24] rather than a single electron. Hence if τ^{-1} is defined as the transfer probability per unit time of one electron pair, the net electron transfer rate is equal to $\frac{\rho+\delta\rho-(\rho-\delta\rho)}{2\tau} = \frac{\delta\rho}{\tau}$. By virtue of Newton's law, the resulting generalized friction term is $mv\delta\rho/\tau = \delta p/\tau \propto j/\tau$, which validates Eq. (2). Furthermore for $\omega\tau << 1$, the inertial term $\propto \frac{dj}{dt}$ in Eq. (2) is negligible, so that we can write the equivalent of Ohm's law for the superconducting material as

$$j = \sigma E \quad , \quad \sigma = \frac{\rho e^2 \tau}{m} \quad . \tag{3}$$

Thus both Ohm's law and σ are seen to display [2] the same form in normal and superconducting metals, as well.

E induces a magnetic induction $B(r,t) = B_z(r)e^{i\omega t}$, parallel to the z axis. B is given by the Faraday–Maxwell equation as

$$-\frac{\partial B}{\partial t} = \mathrm{curl}E = \frac{E}{r} + \frac{\partial E}{\partial r} \quad . \tag{4}$$

The displacement vector D, is parallel to E and is defined as

$$D = \epsilon_0 E + \rho e u \quad , \tag{5}$$

where ϵ_0, u refer to the electric permittivity of vacuum and displacement coordinate of the conduction electron center of mass, parallel to E. The term $\rho e u$ represents the polarization of conduction electrons [25]. Because $\mathrm{div}E = 0$ entails that $\mathrm{div}D = 0$, Poisson's law warrants the lack of charge fluctuation around ρe. Thence since there is by definition $j = \rho e\frac{du}{dt}$, the displacement current reads

$$\frac{\partial D}{\partial t} = j + \epsilon_0 \frac{\partial E}{\partial t} \quad . \tag{6}$$

Finally the magnetic field $H(t,r) = H_z(r)e^{i\omega t}$, parallel to the z axis, is given by the Ampère–Maxwell equation as

$$\mathrm{curl}H = -\frac{\partial H}{\partial r} = j + \frac{\partial D}{\partial t} = 2j + \epsilon_0 \frac{\partial E}{\partial t} \quad . \tag{7}$$

Replacing $E(t,r)$, $j(t,r)$, $B(t,r)$, $H(t,r)$ in Eqs. (2), (4), (7) by their time-Fourier transforms $E_\theta(\omega,r)$, $j_\theta(\omega,r)$, $B_z(\omega,r)$, $H_z(\omega,r)$, while taking into account

$$B_z(\omega,r) = \mu(\omega) H_z(\omega,r) \quad ,$$

where $\mu(\omega) = \mu_0 (1 + \chi_s(\omega))$ ($\chi_s(\omega)$ is the magnetic susceptibility of superconducting electrons at frequency ω) yields

$$\begin{aligned} &E_\theta(\omega,r) = \frac{1+i\omega\tau}{\sigma} j_\theta(\omega,r) \\ &i\omega B_z(\omega,r) = -\left(\frac{E_\theta(\omega,r)}{r} + \frac{\partial E_\theta(\omega,r)}{\partial r}\right) \\ &\frac{\partial B_z(\omega,r)}{\partial r} = -\mu(\omega)\left(2j_\theta(\omega,r) + i\omega\epsilon_0 E_\theta(\omega,r)\right) \end{aligned} \tag{8}$$

Eliminating $E_\theta(\omega,r)$, $j_\theta(\omega,r)$ from Eqs. (8) gives

$$\frac{\partial^2 B_z(\omega,r)}{\partial r^2} = \frac{B_z(\omega,r)}{\delta^2(\omega)} - \frac{\partial B_z(\omega,r)}{r\partial r} \quad . \tag{9}$$

The skin depth δ and plasma frequency ω_p are defined [2,11,12] as

$$\delta(\omega) = \frac{\lambda_L}{\sqrt{(1+\chi_s(\omega))\left(\frac{2i\omega\tau}{1+i\omega\tau} - \frac{\omega^2}{\omega_p^2}\right)}} \quad ,$$

$$\omega_p = \sqrt{\frac{\rho e^2}{\epsilon_0 m}}$$

Note that the above formula of $\delta(\omega)$ retrieves indeed both, the usual [11,12] expression $|\delta| = \frac{1}{\sqrt{2\mu_0\sigma\omega}}$, valid for $\omega\tau << 1$, and the $\omega\tau >> 1$ limit $\delta_a = \frac{\lambda_L}{\sqrt{2}}$, typical of the anomalous skin effect [13,14] and widely used in the interpretation of the experimental work [8–10,15–20].