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Ising percolation in a three-state majority vote model

Alexander S. Balankin [a,*], M.A. Martínez-Cruz [a], Felipe Gayosso Martínez [a], Baltasar Mena [b], Atalo Tobon [a], Julián Patiño-Ortiz [a], Miguel Patiño-Ortiz [a], Didier Samayoa [a]

[a] Grupo Mecánica Fractal, ESIME, Instituto Politécnico Nacional, México D.F., 07738, Mexico
[b] Laboratorio de Ingeniería y Procesos Costeros, Instituto de Ingeniería, Universidad Nacional Autónoma de México, Sisal, Yucatán, 97355, Mexico

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ABSTRACT

In this Letter, we introduce a three-state majority vote model in which each voter adopts a state of a majority of its active neighbors, if exist, but the voter becomes uncommitted if its active neighbors are in a tie, or all neighbors are the uncommitted. Numerical simulations were performed on square lattices of different linear size with periodic boundary conditions. Starting from a random distribution of active voters, the model leads to a stable non-consensus state in which three opinions coexist. We found that the "magnetization" of the non-consensus state and the concentration of uncommitted voters in it are governed by an initial composition of system and are independent of the lattice size. Furthermore, we found that a configuration of the stable non-consensus state undergoes a second order percolation transition at a critical concentration of voters holding the same opinion. Numerical simulations suggest that this transition belongs to the same universality class as the Ising percolation. These findings highlight the effect of an updating rule for a tie between voter neighbors on the critical behavior of models obeying the majority vote rule whenever a strict majority exists.

1. Introduction

Opinion formation through contrast and synthesis of different viewpoints has been extensively studied using models based on ideas from statistical physics (see, for review, Refs. [1–5] and references therein). Various models of opinion formation have been inspired by spin systems with short range interactions. Although these models ignore many important aspects of complex social systems, it has been recognized that simple decision-making rules allow to reveal some essential features of the opinion dynamics. Among of the most popular decision-making rules are the majority and majority-vote (MV) rules accounting for a voter's tendency to assume an opinion of a majority of its neighbors [1–21]. Specifically, in the two-state MV model on a lattice [6] each voter assumes the opinion of the majority of its neighbors with a probability $1-q$ and the opposite opinion with probability q, whereas the voter changes its initial opinions with probability $1/2$, whenever there is a tie in its neighborhood. The control parameter q plays a role of temperature in equilibrium systems. On regular lattices, the two-state MV model displays a second-order phase transition from an ordered to a disordered state at a critical value q_c. It has been recognized that the two-state MV model belongs to the Ising model universality class [6–11]. In the limiting case of $q=0$,[1] after the update $t+1$, the voter located on node i adopts the state with the spin

$$s_i(t+1) = \Theta\left[\sum_j^k s_j^i(t)\right], \tag{1a}$$

where $s_j^i(t)$ denotes the spin of the voter located on node j and the summation is over k neighbors of the voter located at node i, while

$$\Theta(x) = sign(x), \tag{1b}$$

whenever $x \neq 0$, whereas

$$\Theta(x=0) \text{ takes the values 1 and } -1 \text{ with equal probabilities.} \tag{2a}$$

Recently, Lima [9] has introduced a three-state MV model in which the third opinion is neutral ($s=0$), while the competing opinions have opposite spins ($s=\pm1$). In this three-state MV model,

* Corresponding author at: SEPI-ESIME-Zacatenco, U.P. Adolfo Lopez Mateos, Av. Politécnico Nacional s/n, México D.F. 07738, Mexico.
E-mail address: abalankin@ipn.mx (A.S. Balankin).

[1] In this case the MV model coincides with the Ising model with the zero-temperature Glauber kinetics and its ultimate fate is either a consensus or tie [4].

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each voter adopts the opinion of the majority of active voters in its neighborhood with probability $1-q$ and the minority of active voters with probability q, whenever a majority of active voters exists. Otherwise, the updating voter (active or uncommitted) assumes the state with s equal to 1 or -1 with equal probabilities. Consequently, the uncommitted voters disappear after a few cycles of N updates and the system's fate is essentially the same as in the two-state MV model. Accordingly, in numerical simulations on square lattices it was found that the three-state MV model also belongs to the Ising model universality class [9] and in the case of $q=0$ leads to the consensus or tie.

In order to model systems with a stable coexistence of minority and majority opinions, Shao et al. [22] have modified the two-state MV model such that the voter keeps its initial opinion if its neighbors are in a tie. Accordingly, the model obeys Eq. (1) with

$$\Theta(x=0) = sign\big[s_i(t)\big]. \tag{2b}$$

Consequently, voters holding the same opinion form clusters which cannot be invaded by the opposite opinion. In numerical simulations it was found that the two-state non-consensus (NCO) model defined by Eqs. (1) and (2b) exhibits a percolation transition. A spanning cluster of voters holding the same opinion appears when the initial concentration of these voters is above a certain critical value f_c [22–30]. It has been argued that the NCO model on a square lattice belongs to the same universality class as the invasion percolation [31].

In this work, we put forward a three-state MV model in which the voter becomes uncommitted if its active neighbors are in the tie, or all neighbors are uncommitted. Otherwise the voter adopts the opinion of the majority of active voters in its neighborhood. Accordingly, our three-state MV model also obeys Eq. (1) but with

$$\Theta(x=0) = 0. \tag{2c}$$

We found that this model allows for a stable coexistence of three opinions in a stable non-consensus state. The composition of the stable non-consensus state is governed by the initial composition of system and is independent of the lattice size. Furthermore, we found that the stable state undergoes a second-order percolation transition similar to the one observed in the two-state NCO model studied in Refs. [22–24,31]. However, although both model share some common features, we found that they belong to different universality classes. Specifically, the values of critical exponents found in numerical simulations suggest that the three-state MV model defined by Eqs. (1) and (2c) belongs to the same universality class as the Ising percolation. This finding highlights the effect of updating rule for the tie between voter neighbors on the critical behavior of the modified MV models.

The rest of the paper is organized as follows. In Sec. 2 we introduce the three-state MV model and describe details of numerical simulations. Sec. 3 is devoted to the results of numerical simulations. The numerical findings are discussed in Sec. 4. The main results and conclusions are outlined in Sec. 5.

2. Three-state MV model

The studied system consists of N voters that reside in nodes of a square lattice ($L \times L = N$) with four Newman neighbors ($k=4$). The voter opinions are represented by spin-like variables s_i, where index $i = 1, 2, \dots N$ denotes the node position. The active voters have spins $s_i = \pm 1$, while the uncommitted voters have spin zero ($s_i = 0$). In this work, the positive spin is initially assigned to fN randomly chosen nodes ($0 < f < 1$), while the rest of the nodes are assumed to have spin $s_i = -1$ (see Fig. 1a–c). So, the initial "magnetization" of system is equal to $m_0 = |2f-1|$, while the initial concentration of the uncommitted voters is zero.

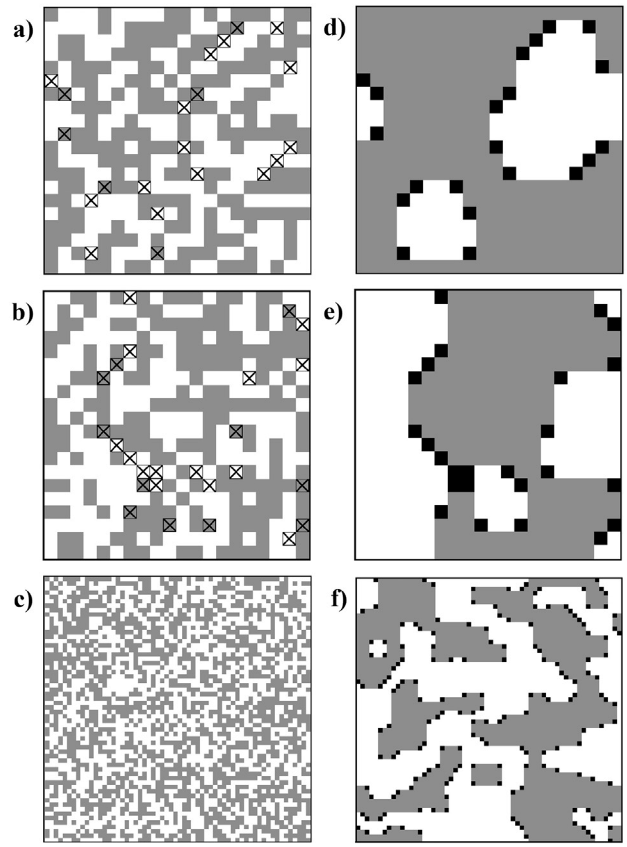

Fig. 1. Distributions of voters in: (a–c) initial ($f=0.5$) and (d–f) stable states on square lattices of size $L=20$ (a, b, d, e) and $L=60$ (c, f). White and grey nodes are occupied by active voters with $\sigma_i = 1$ and -1, respectively, whereas black nodes are associated with the uncommitted voters ($\sigma_i = 0$). In panels (a, b) the initially active voters that become the uncommitted in the final state are marked with ×.

Once the initial distribution of voters is defined, its evolution is governed by the update rule defined by Eqs. (1) and (2c). Updates are repeated until a stable non-consensus state is reached and no more changes occur. In order to study the finite size scaling, numerical simulations were performed on lattices of different size ($L = 20$, 40, 60, 80, 100, 150, 500, 1000) with periodic boundary conditions. It is a straightforward matter to understand that the probability of active voters with spin s becoming a majority in the stable state $P_s(f)$ is a monotonically increasing function of f with symmetry around $(P_s, f) = (0.5, 0.5)$, because two competing states ($s = \pm 1$) are symmetrical in the lattice of any size L. Notice that, generally, $P_1(f, L) + P_{-1}(f, L) = 1 - P_0(f, L)$, where $P_0(f, L)$ is the probability that the stable state has magnetization equal to zero. Although the majority in the initial state tends to dominate, on finite lattices the initial minority can become a majority in the stable state. However, as the lattice size increases $P_1(f, L)$ approaches to a step function. So, in the limit of $L \to 0$, the initial majority always becomes the majority in the stable state. Accordingly, we study statistical distributions of the concentrations of active ($p_{\pm 1}$) and uncommitted (p_0) voters in the stable state, while $p_1 + p_{-1} + p_0 = 1$ in each simulation. Then, we calculate the mean concentrations of the uncommitted voters $\langle p_0(f) \rangle$ and the average "magnetization" of the stable state $M(f) = \langle m(f) \rangle = \langle p_1 - p_{-1} \rangle$ on lattices of different sizes. We also calculate the standard deviations $\sigma[p_0(f), L]$ and $\sigma[m(f), L]$ obtained in 10^3 simulations with each f on lattices of each size L.

In the non-consensus stable state, active voters form clusters wherein they share the same opinion, while the uncommitted voters appear on the frontier between clusters of competing voters (see Fig. 1). We found that when the initial concentration of active