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Thermodynamic properties of the blackbody radiation: A Kaniadakis approach

Imene Lourek [a], Mouloud Tribeche [a,b,*]

[a] *Faculty of Physics, Theoretical Physics Laboratory (TPL), Plasma Physics Group (PPG), University of Bab-Ezzouar, and USTHB, B.P. 32, El Alia, Algiers 16111, Algeria*
[b] *Algerian Academy of Sciences and Technologies, Algiers, Algeria*

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A B S T R A C T

The thermodynamic properties of the blackbody radiation are revisited, for the first time, within the theoretical framework of the κ-statistics introduced by Kaniadakis. Using the κ-counterpart of the Bose–Einstein distribution, generalized expressions for the free energy, the entropy, the specific heat, and the pressure are obtained. All quantities are shown to recover their standard expressions in the limit $\kappa \to 0$. The reexamination of the thermodynamic properties of the blackbody radiation shows that it emits more energy with an increase of the value of $|\kappa|$ in comparison with the standard Planck radiation law. Moreover, the effects of the deformed Kaniadakis statistics are shown to be more appreciable for high temperatures. Our results could be used as a theoretical support for experimental studies implying blackbody radiation such as the study of microwave background radiation.

1. Introduction

The Maxwell–Boltzmann distribution is the distribution of an amount of energy between identical and distinguishable particles. In spite of its great success, the statistical mechanics based on the Boltzmann–Gibbs (BG) entropy measure does not seem to be adequate in dealing with many physical systems. In fact, a lot of theoretical problems present difficulties when treated within the standard BG statistical mechanics. One can cite, as an example, the failure to explain the spectrum of the Cosmic rays – *one of the most important relativistic particle systems*. In the last few decades, many attempts to generalize the BG statistical mechanics have been undertaken. Nonextensive q-statistics suggested for the first time by Tsallis [2], has been the most investigated in the literature, and many physical processes under different physical conditions and within different approaches have been addressed. Due to its historical significance and regarding its importance to model different physical processes, the blackbody radiation has been naturally investigated in Tsallis nonextensive statistics framework [3–12]. The Tsallis entropy (which is a one-parameter generalization of the BG entropy with a power-law behavior) has shown a good agreement with observations and experimental measurements [13–15], etc. However, other generalizations of the entropy also lead to power-law distributions which are different from those generated by the Tsallis entropy. Hence, it is of interest to study the implication of such alternative distributions and their ability to describe key phenomena.

Another possible generalization of statistical mechanics, known as Kaniadakis κ-statistics [16], has been introduced more recently. The latter presents many interesting properties (maximum entropy principle, thermodynamic stability, Lesche stability, continuity, etc.) allowing it to be a possible generalization of the BG statistical mechanics, especially in the context of special relativity, and the resulting distributions have been observed in a variety of physical, natural and artificial systems. For these reasons, Kaniadakis κ-statistics has attracted an increasing interest in the last decade. The different applications of Kaniadakis statistics include the study of cosmic rays [17], relativistic [18] and classical [19] plasmas in the presence of external electromagnetic fields, the relaxation in relativistic plasmas under wave–particle interactions [20,21], quark–gluon plasma formation [22], kinetics of interacting atoms and photons [23], stellar distributions in astrophysics [24–26], fractal systems [27], field theories [28], random matrix theory [29–31], error theory [32], game theory [33], the theory of complex networks [34], classical information theory [35] and quantum information theory [36,37]. It has also been considered in nonlinear kinetics [38–40] and in a lot of economic systems [41–43].

* Corresponding author at: Faculty of Physics, Theoretical Physics Laboratory (TPL), Plasma Physics Group (PPG), University of Bab-Ezzouar, and USTHB, B.P. 32, El Alia, Algiers 16111, Algeria. Fax: +213 21 24 73 44.
E-mail address: mouloudtribeche@yahoo.fr (M. Tribeche).

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Regarding the success of the formalism in the relativistic context, we have recently reconsidered the blackbody radiation using the counterpart of the Bose–Einstein distribution in the κ-statistics arising from the Kaniadakis entropy [44]. The generalized Planck radiation law has been presented and compared to the usual law, to which it reduces in the limiting case $\kappa \to 0$. Effective Einstein's coefficients of emission and absorption have been defined in terms of the Kaniadakis parameter κ. In Ref. [44], the suggestion has been made that it may be of interest to reconsider and generalize the thermodynamical quantities of the blackbody radiation within the theoretical framework of the κ-statistics arising from the Kaniadakis entropy. In this paper, we explore this suggestion, within the theoretical framework of Ref. [44] to which the reader is referred. This issue appears as one of the interesting prospects that can be envisaged to complement and provide new insight into our published results [44]. In addition, it is legitimate to wonder about the effects of the Kaniadakis κ-statistics on the generalized thermodynamical quantities of the blackbody radiation.

2. The Planck law

Before proceeding further, let us first briefly review the Planck radiation law. The latter is based upon the Bose–Einstein (BE) distribution

$$n_i = \frac{g_i}{\exp(\frac{E_i-\mu}{k_B T}) - 1} \tag{1}$$

where g_i is the degeneracy of the energy level E_i, n_i is the corresponding number of states, μ is the chemical potential, k_B is the Boltzmann constant, and T is the temperature. One of the most important applications of the Bose–Einstein distribution is the emission of electromagnetic radiation by a body heated to a temperature T (that is the blackbody radiation). A blackbody is defined as an empty and closed cavity of volume V, whose walls are held at a temperature T. This radiation is composed of photons, i.e., bosons with a spin of unity. The energy carried by a single photon with a certain electromagnetic wavelength λ and frequency ν is given by

$$E = h\nu = \hbar\omega = \frac{hc}{\lambda} \tag{2}$$

where $\nu = \frac{\omega}{2\pi} = \frac{c}{\lambda}$ and h ($\hbar = \frac{h}{2\pi}$) is the Planck constant (reduced Planck constant). Due to the wall absorption, the number of photons N is not stored. It turns out that

$$\left(\frac{\partial F}{\partial N}\right)_{T,V} = \mu = 0 \tag{3}$$

where F is the Helmholtz free energy. The average number of photons $\langle n_i \rangle$ which are in a microstate of energy $\varepsilon_i = \hbar\omega_i$, given by the BE distribution in the particular case where $\mu = 0$

$$\langle n_i \rangle = \frac{1}{\exp(\frac{\hbar\omega_i}{k_B T}) - 1} \tag{4}$$

is called the Planck law.

3. The blackbody radiation in the framework of the Kaniadakis κ-statistics

To reexamine the Planck law in the framework of the Kaniadakis distribution, we introduce the deformed exponential and logarithm which are based on a one continuous parameter κ

$$\exp_\kappa(x) = \left[\sqrt{1+\kappa^2 x^2} + \kappa x\right]^{\frac{1}{\kappa}} \tag{5}$$

$$\ln_\kappa = \frac{x^\kappa - x^{-\kappa}}{2\kappa} \tag{6}$$

where $-1 \le \kappa \le 1$. The most interesting property of this function is its asymptotic power law behavior given by

$$\exp_\kappa(x)_{x\to\pm\infty} \sim (|2\kappa x|)^{\pm\frac{1}{|\kappa|}} \tag{7}$$

The deformed BE distribution in the framework of the κ-statistics [45] is given by

$$\langle n_i \rangle = \frac{1}{\exp_\kappa(\frac{\varepsilon_i-\mu}{kT}) - 1} \tag{8}$$

The deformed Planck law [44]

$$dN_\kappa = \langle n_i \rangle \frac{V\omega^2 d\omega}{\pi^2 c^3} = \frac{V}{\pi^2 c^3} \frac{\omega^2 d\omega}{\exp_\kappa(\frac{\hbar\omega}{k_B T}) - 1} \tag{9}$$

that expresses the number of photons dN_κ with a pulsation between ω and $\omega + d\omega$, is obtained by multiplying $\langle n_i \rangle$ by the number of microstates.

The energy corresponding to this radiation is then given by

$$dE_\kappa = \frac{V}{\pi^2 c^3} \frac{\hbar\omega^3 d\omega}{\exp_\kappa(\frac{\hbar\omega}{k_B T}) - 1} \tag{10}$$

The spatial density energy per frequency interval is then defined as

$$\rho_\kappa = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\exp_\kappa(\frac{\hbar\omega}{k_B T}) - 1} \tag{11}$$

The generalized Planck law describes the distribution of the electromagnetic energy (or the distribution of the photon density) radiated by a blackbody at a given temperature T. Planck law can be presented in different variants involving parameters such as intensity, flux density or spectral distribution. Two limiting cases, viz., $\hbar\omega \ll k_B T$ and $\hbar\omega \gg k_B T$, deserve special attention. When $\hbar\omega \ll kT$ (high-temperature or low-frequency limit), the classical limit [44]

$$dE_\kappa = \frac{V\omega^2}{\pi^2 c^3} k_B T d\omega \tag{12}$$

is recovered. The latter can be perceived as a κ-generalization of the Rayleigh–Jeans formula, where the light is regarded as a superposition of harmonic oscillators (the number of oscillators is given by $\frac{V\omega^2}{\pi^2 c^3} d\omega$) of frequency ω and energy $k_B T$. For $\hbar\omega \gg kT$, Eq. (10) becomes [44]

$$dE_\kappa = \frac{V\hbar}{\pi^2 c^3} \omega^3 \exp_\kappa^{-\frac{\hbar\omega}{k_B T}} d\omega \tag{13}$$

which can be viewed as a generalization of the Wien's law.

Integrating (10) over all the frequencies, we obtain the following generalized total energy radiated by the cavity

$$E_\kappa = \frac{V\hbar}{\pi^2 c^3} \int_0^\infty \frac{\omega^3 d\omega}{\exp_\kappa(\frac{\hbar\omega}{k_B T}) - 1} \tag{14}$$

Introducing the following change of variable $x = \frac{\hbar\omega}{k_B T}$, (14) becomes

$$E_\kappa = \frac{V\hbar}{\pi^2 c^3} \left(\frac{k_B T}{\hbar}\right)^4 \int_0^\infty \frac{x^3 dx}{\exp_\kappa(x) - 1} \tag{15}$$

Having in mind that

$$J_n^\kappa(t) = \int_0^\infty \frac{x^n dx}{\exp_\kappa(x+t) - 1} \tag{16}$$