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Oscillating collision of the granular chain on static wall

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ABSTRACT

Collision of the granular chain on static wall is investigated by discrete element method. Collision time and traveling time are proposed on the basis of the characteristics of the collision of a single grain with a wall and the propagation of interaction force wave in a granular chain to explain the collision process. Simulation results show that an oscillating collision force is generated when the force waves successively arrive at the wall. For the collision of a mono-dispersed chain, the simulation data are in good agreement with the predicted relationship between the maximum chain length of $n_{\max}$ and the first maximum collision force F_{A}. Rigid wall and soft wall are defined as $n_{\max} = 1$ and $n_{\max} \geq 2$, respectively. Two similar processes of oscillating collisions occur when a light or a heavy impure grain is introduced. In these processes, two maximum collision forces, namely, F_{A} and F_{B}, are observed, respectively. The simulation results about the influence of the mass and position of light impure grain on the collision force F_{B} further confirm our theoretical predictions.

1. Introduction

Granular materials are ubiquitous in our daily lives and in the industrial powder processing [1,2]. Nonlinear interactions among the individual grains lead to various phenomena, such as granular gas [3], dilute-to-dense flow transition [4], and long-lived solitary wave [5]. A container packed with granular materials is always used as shock protection and energy reservoir [6,7]. The application and its mechanism involved in the control of collision behavior of a granular chain on a wall have been extensively investigated in the scientific and engineering community [8–10].

One simple collision case occurs between a completely elastic spherical grain and a static wall. The maximum collision force is obtained at the moment when incident kinetic energy is completely transformed into elastic potential energy. Then, the grain bounces back at the same incident speed. Hertz's law describes collision dynamics in detail [11]. Conversely, a complex collision phenomenon appears when the number of grains is more than two [12–14]. For example, a super ball effect is observed when the bottom of a chain contains a larger ball and the top of the chain comprises a smaller one [15]. The smaller ball can obtain a reflected speed greater than incident speed. In the experiments and simulations, a train of oscillating collisions generate when the chain is composed of mono-dispersed grains [16]. Furthermore, more grains participate in the beginning of collision when the elastic coefficient of wall is decreased. For a longer chain, the collision of the beginning becomes independent of additional grains.

Theoretical and numerical investigations have been conducted to describe the collision dynamics of the granular chain with a wall. A classic start is used to explain the observations on the basis of energy and momentum conservation through binary collision approximation [12,16]. However, these conservation laws are insufficient to describe the dynamic behaviors of a chain containing more than two grains because of multi-grain interactions [17–21]. The effect of interaction force wave on the collision dynamics should be considered because of a finite duration between adjacent grains. Our simulation results are consistent with the theoretical predictions of Nesterenko on the long wavelength approximation [22,23]. The dynamic features of wave propagation of interaction force are highly sensitive to the structure of granular chain [24–27]. The introduction of impure grain can result in the generation of secondary waves. Likewise, the presented simulations demonstrate that a second collision occurs when the decorated chain containing an impure grain collides with the static wall.

Herein this study mainly focuses on the collision forces of the mono-dispersed granular chain and the decorated granular chain

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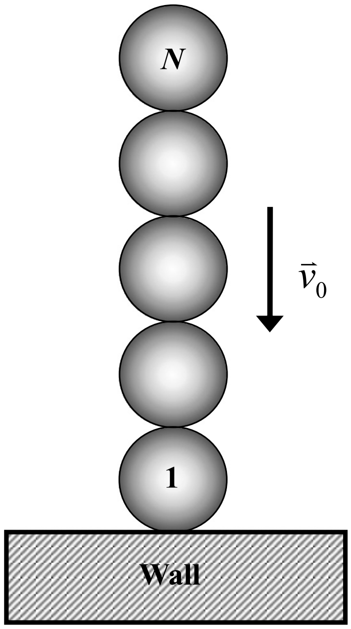

Fig. 1. A snapshot of system at the beginning of simulation.

with a light or a heavy impure grain with the static wall. The simulation model and method are established in Section 2, and theoretical analysis is given in Section 3. Also, it is proposed in this study to explain the collision mechanical properties of granular chain with the wall combining by the collision time of a single grain with the wall and the traveling time of force wave passing across two adjacent grains. In Section 4, the theoretical predictions are verified by simulations. In the end, the conclusions are presented in section 5.

2. Simulation model

The simulation system was schematically shown in Fig. 1. Wherein, a granular chain collides with the static wall. All of the grains are spherical and are placed in a line, and every grain is assigned with an initial incident velocity v_0 to initiate the collision. At the beginning of simulation, the grains are arranged in the way of barely coming in contact with each other. Discrete element method (DEM) is applied, and the dynamics of each grain is governed by Newton's equations similarly implemented in our previous studies [28,29]. In a simulation time step, the position and velocity of grains are updated in turn. The contact among grains is completely in a central line and only the translational motion is considered in the simulation. The normal interaction of two contact grains is determined by Hertz's law in the absence of dissipation [31,32].

$$F_{i,i+1} = \begin{cases} kz_{i,i+1}^{3/2}, & z_{i,i+1} \geq 0 \\ 0, & z_{i,i+1} < 0 \end{cases} \quad 0 \leq i < N \tag{1}$$

In this study, the overlap of two grains in contact is denoted as $z_{i,i+1} = d - (x_{i+1} - x_I)$, wherein the grain diameter is $d = 5.0$ mm, and x_i and x_{i+1} are the positions of grain i and grain $i+1$ at the time of t, respectively. The wall is denoted as $i = 0$, and the overlap between the wall and grain 1 is calculated by $z = d/2 - x_1$. The elastic coefficient k is calculated as follows: $k = \frac{4}{3}\frac{Y_i Y_{i+1}}{Y_i + Y_{i+1}}\sqrt{\frac{R_i R_{i+1}}{R_i + R_{i+1}}}$, $Y = \frac{E}{1-\nu^2}$. k_0 and k_g are the elastic coefficients of grain 1 with the wall and the grains in the chain, respectively. The grain is made of the steel with a mass density of $\rho = 7.9 \times 10^3$ kg/m^3. The elastic parameters involve in the Young's modulus $E = 200$ GPa and Poisson's ratio $\nu = 0.28$ [30]. In the simulation, the mass and elastic coefficient of grains in the chain are as follows: $m = 0.517$ g and $k_g = 5.1 \times 10^9$ N/m$^{3/2}$. The mass of impure grain is denoted as m_I. The wall is deemed as an infinite mass in the simulation. The Verlet-velocity algorithm is applied to update the position and velocity of grains. The simulation time step is $dt = 1.0 \times 10^{-8}$ s.

3. Theoretical analysis

Under the ideal conditions, a granular chain is composed of completely elastic spherical grains with the same diameter and mass. For the collision of the granular chain with static wall, two characteristic times play a critical role in the collision dynamics. One of the characteristic times is the collision time τ_0 of a unique chain containing one grain. In this case, the collision can be divided into two equal parts which are the incident compression process and reflection dilation process. The maximum collision force F_0 occurs at the time of τ_0. The collision time is strictly derived on the basis of Hertz's theory [11].

$$\tau_0 = 1.47(\frac{4k_0}{5m})^{-2/5} v_0^{-1/5} \tag{2}$$

The second characteristic time is the traveling time τ_w determined by the mechanical force wave passing across two adjacent grains. The propagation dynamics of force wave in a Hertz chain has been extensively investigated [23,33]. In this study, Nesterenko's theory is reviewed and in this theory the long wavelength approximation is successfully applied. By Eq. (1), the grain motion in the chain can be expressed as follows:

$$m\frac{d^2 x_I}{dt^2} = k_g[d - (x_i - x_{i-1})]^{3/2} - k_g[d - (x_{i+1} - x_i)]^{3/2} \tag{3}$$

As shown in Nesterenko's analysis [23], the coordinate x_I is replaced by the displacement relative to its original position u_I. Thus the equation of grain motion is transformed as follows:

$$m\frac{d^2 u_I}{dt^2} = k_g[(u_i - u_{i-1})^{3/2} - (u_{i+1} - u_i)^{3/2}] \tag{4}$$

In the long wavelength approximation ($L \gg d$, where L is the characteristic spatial size of the perturbation), by the fourth order expansion algorithm, the nonlinear equation is derived as below.

$$u_{tt} = c^2\{\frac{3}{2}(-u_x)^{1/2}u_{xx} + \frac{d^2}{8}(-u_x)^{1/2}u_{xxx} - \frac{d^2}{8}\frac{u_x u_{xxx}}{(-u_x)^{1/2}} - \frac{d^2}{64}\frac{(u_{xx})^3}{(-u_x)^{3/2}}\} \tag{5}$$

In this study, $-u_x > 0$ and $c^2 = \frac{k_g}{m}d^{5/2}$. Eq. (5) is further simplified and addressed in a previous study [34]. Therefore, the traveling time τ_w is obtained by using the following equation:

$$\tau_w = (\frac{4k_g}{5m})^{-2/5} v_{max}^{-1/5} \tag{6}$$

Wherein, v_{max} is the maximum grain velocity of mechanical force wave.

In Eq. (2) and Eq. (6), the maximum chain length of n_{max} can be obtained. At the time of $\tau_0 = (n_{max} - 1)\tau_w$, the generated wave of interaction force between the grain $n_{max} - 1$ and grain n_{max} just reach the wall. For the longer chain of $N > n_{max}$, the wave of interaction force of grains far away which are deemed as i and $i+1$ ($i > n_{max}$) cannot reach the wall. Thus, the following equation is given:

$$n_{max} = 1 + 1.47(\frac{v_0}{v_{max}})^{-1/5}(\frac{k_0}{k_g})^{-2/5} \tag{7}$$