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Classical understanding of electron vortex beams in a uniform magnetic field

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ABSTRACT

Recently, interesting observations on electron vortex beams have been made. We propose a classical model that shows vortex-like motion due to suitably-synchronized motion of each electron's cyclotron motion in a uniform magnetic field. It is shown that some basic features of electron vortex beams in a uniform magnetic field, such as azimuthal currents, the relation between energy and kinetic angular momentum, and the parallel-axis theorem are understandable by using this classical model. We also show that the time-dependence of kinetic angular momentum of electron vortex beams could be understood as an effect of a specific nonuniform distribution of classical electrons.

1. Introduction

Free electron vortex states have recently been predicted by considering semiclassical (paraxial) wave packets [1] and observed in electron microscopy [2–4]. The free electron vortex is practically equivalent to an optical vortex beam; however, in the presence of a magnetic field, the properties of electron vortex beams become different from those of its optical counterpart. The electron vortex beam with angular momentum has a magnetic moment and interacts with an external magnetic field, which gives interesting physics and applications [1,5–11].

In the experimental realization of electron vortex beams in a transmission electron microscope (TEM) and their studies [2,3], a magnetic lens field is mandatory. The Larmor image rotation in a magnetic lens field was indicated by studying the electron trajectories [12], which implies the possibility of electron vortex beams. However, ray optics given by electron trajectory is insufficient to explain the wave optical properties of vortex beams such as diffraction and interference effects. The rotational dynamics of electron vortex beams was shown theoretically and experimentally to be quite different by using wave-optical method [10,11,13,14].

For electron vortex beams, the interactions between electrons are assumed to be so small that can be neglected. Therefore, the problem of electron vortex beams in a magnetic field seems to be reduced to the problem of one electron in the same magnetic field. The physics of an electron in a uniform magnetic field is well understood both classically and quantum mechanically [15]. The motion of a classical electron in a uniform magnetic field is circular with constant speed, known as the cyclotron motion; hence it is expected that the kinetic angular momentum of such motion about its center is constant. However, an interesting issue, which seems to be contrary to classical cyclotron motion, was pointed out by Greenshields et al. [16]. They showed, using quantum vortex solutions, that the kinetic angular momentum of an electron vortex beam in a uniform magnetic field is time-varying in general, although the system has a rotational symmetry. This fact seems to be contradictory to the rotational symmetry of the system and they showed that the conservation of kinetic angular momentum is recovered by including the angular momentum of the electromagnetic field.

It is, however, still intriguing to determine how rotationally symmetric vortex solutions can have a time-varying radius in quantum vortex solutions; as this is in contrast to the constant radius of the rotationally symmetric classical cyclotron motion. It has been found that the average radial position of the electron vortex state expands and contracts [9], and it was also recently shown that the orbital angular momentum of an electron vortex beam can be decomposed into separate angular momenta according to parallel-axis theorem; although this seems to be only meaningful in an extended probability distribution [17].

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As can be seen from the above, the characteristics of electron vortex beams are different from those of the classical cyclotron motion of one electron and are mainly considered to be quantum aspects of electron vortex beams. In this paper, we propose a classical model that shows a vortex-like motion due to suitably-synchronized motion of electrons in their classical cyclotron motions. Using this model, some basic features of electron vortex beams such as azimuthal currents and the relation between energy and kinetic angular momentum could be understood. In addition, we show that the time-varying behavior of kinetic angular momentum and parallel-axis theorem could also be understood using this model. These results suggest that our classical model is helpful to intuitively understand some aspects of the physics of electron vortex beams.

2. Motion of one electron in a uniform magnetic field

The motion of one electron in a uniform magnetic field is a well-known problem both classically and quantum mechanically [15]. However, little attention has been focused on the time-dependence of angular momenta, particularly from a classical perspective. In our model, the classical motion of one electron is used as the building block of synchronized apparent motion to enable understanding of electron vortex beams. Therefore, we briefly review the problem of the motion of one electron in a uniform magnetic field as we focus on the time dependence of angular momenta.

The Lagrangian for an electron in a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$, in a cylindrical coordinate (ρ, ϕ, z) and the symmetric gauge with $\mathbf{A}_s = (B\rho/2)\hat{\boldsymbol{\phi}}$ is as follows

$$\mathcal{L} = \frac{1}{2}m\mathbf{v}^2 + e\mathbf{v}\cdot\mathbf{A}_s = \frac{1}{2}m\dot{\rho}^2 + \frac{1}{2}m\rho^2\dot{\phi}^2 + \frac{1}{2}m\dot{z}^2 + \frac{e}{2}B\rho^2\dot{\phi}, \tag{1}$$

where e, m, and $\mathbf{v}$ are charge, mass, and velocity of the electron, respectively. The canonical conjugate momenta can then be defined as

$$p_\rho = \frac{\partial\mathcal{L}}{\partial\dot{\rho}} = m\dot{\rho}, \quad p_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = m\rho^2\dot{\phi} + \frac{e}{2}B\rho^2. \tag{2}$$

This Lagrangian has a rotational symmetry about the z-axis, i.e., there is no ϕ-dependence, so that the corresponding conjugate momentum p_ϕ is a constant of motion and the z-component of the canonical angular momentum, i.e., $p_\phi \equiv L_z = (\mathbf{r}\times\mathbf{p})_z$.

Kinetic angular momentum is defined as

$$L_z^{\text{kin}} = \mathbf{r}\times(\mathbf{p} - e\mathbf{A}_s) = L_z + \frac{m\omega_c}{2}\rho^2, \tag{3}$$

where $\mathbf{r} = \rho\hat{\rho} + z\hat{z}$ and $\omega_c = -eB/m$ is the classical cyclotron frequency of the electron and the second term in Eq. (3) is known as the diamagnetic angular momentum. Conservation of canonical angular momentum is guaranteed by the rotational symmetry of the Lagrangian, and thus the conservation of kinetic angular momentum depends on the time dependence of ρ^2, which is related to choice of the coordinate origin.

The Cartesian coordinate is useful for studying the effect of the choice of the coordinate's origin, and the Hamiltonian $\mathcal{H} = \mathbf{v}\cdot\mathbf{p} - \mathcal{L}$ is written in the Cartesian coordinate as

$$\mathcal{H} = \frac{1}{2m}\mathbf{p}^2 + \frac{\omega_c}{2}L_z + \frac{m}{8}\omega_c^2(x^2+y^2), \tag{4}$$

where $\mathbf{p}^2 = p_x^2 + p_y^2 + p_z^2$. The classical equation of motion for the electron from the Hamiltonian (4) becomes the usual Lorentz force equation,

$$m\frac{d^2}{dt^2}\mathbf{r} = e\mathbf{v}\times\mathbf{B}. \tag{5}$$

As the z-directional motion of the electron described by Eq. (5) is trivial because there is a translation symmetry along z-direction in the Hamiltonian (4), we thus focus on the motion of the electron in the xy-plane.

Equation (5) describes cyclotron motion in the xy-plane and the general solutions are

$$x(t) = x_0 + R\cos(\omega_c t + \theta), \quad y(t) = y_0 + R\sin(\omega_c t + \theta), \tag{6}$$

where (x_0, y_0) is the center of the cyclotron orbit with radius R and θ is the phase shift. The squared 2-dimensional radial distance $\rho^2 = x^2 + y^2$ of the electron from the origin of the coordinate then becomes

$$\rho^2 = x_0^2 + y_0^2 + R^2 + 2x_0R\cos(\omega_c t + \theta) + 2y_0R\sin(\omega_c t + \theta), \tag{7}$$

which shows time dependence. The following dynamical equation for ρ^2 is obtained by direct calculation as

$$\frac{d^2}{dt^2}\rho^2 = -\omega_c^2\rho^2 - 2\frac{\omega_c}{m}L_z + \frac{4}{m}E, \tag{8}$$

where E is the 2-dimensional energy (classical Hamiltonian) $\frac{1}{2}m(v_x^2 + v_y^2) = \frac{1}{2}mR^2\omega_c^2$. It is then easily possible to confirm that Eq. (8) is equivalent to the quantum mechanical equation (Eq. (11) of Ref. [16]) for the squared radius in Heisenberg formalism.

These results suggest that the oscillations of ρ^2 and the resultant time-varying diamagnetic response could be understood to have originated from a mismatch between the coordinate origin and the center of the cyclotron orbit. It is possible to confirm that the classical torque $\mathbf{r}\times(e\mathbf{v}\times\mathbf{B})$ is not zero for cyclotron motion of the electron whose centers are different from the origin of the coordinate. This torque is responsible for the change of the kinetic angular momentum of the electron. If the coordinate origin and the center of the cyclotron orbit match, i.e., $x_0 = y_0 = 0$, then ρ^2 $(= R^2)$ becomes a constant of motion. As a result, the kinetic angular momentum is also conserved, in addition to the canonical angular momentum. Time dependence of the kinetic angular momentum of the electron vortex beam is discussed in Section 3.

3. Classical model of electron vortex beam in a uniform magnetic field

In this section we construct a classical model that shows a vortex-like motion due to the suitably-synchronized motion of two-dimensional classical electrons in a uniform magnetic field. We call this apparent synchronized motion "classical electron vortex".

We consider here that electrons move at the same speed within a uniform magnetic field and that there is no Coulomb repulsion between electrons; these conditions are assumed in usual electron vortex beams. Therefore all electrons rotate in their cyclotron orbits with the same radius R as given by the solutions in Eq. (6) and can be classified using the canonical angular momentum that is a constant of motion.

Canonical angular momentum L_z, is the z-component of $\mathbf{r}\times(m\mathbf{v} + e\mathbf{A}_s)$ and is calculated as

$$L_z = \frac{m}{2}\omega_c(R^2 - R_{\text{cen}}^2), \tag{9}$$

where R_{cen} is the distance from the coordinate origin to the center of the cyclotron orbit, i.e., $R_{\text{cen}}^2 = x_0^2 + y_0^2$. Hence, there are three categories of cyclotron motion of the electrons according to $L_z > 0$ $(R^2 > R_{\text{cen}}^2)$, $L_z = 0$ $(R^2 = R_{\text{cen}}^2)$, and $L_z < 0$ $(R^2 < R_{\text{cen}}^2)$ as in Fig. 1.